Seminar Faisceaux Pervers
Mixed Complexes

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1 Introduction and motivating examples

We start with some motivating examples to illustrate the general theory that follows.

Example 1. Let E_0 be an elliptic curve over a finite field $k = \mathbb{F}_q$ with an algebraic closure K. Let $E = E_0 \times_k K$, then the l-adic Tate Module of E_0 is defined as

$$T_l(E_0) := \varprojlim_n E_0[l^n](K) \otimes_{\mathbb{Z}_l} \mathbb{Q}_l$$

together with the action of $\mathrm{Gal}(K/k)$. The number of (rational) points of the elliptic curve is given by

$$E_0(\mathbb{F}_{q^n}) = q^n + 1 - \alpha^n - \beta^n$$

where α and β are complex numbers with $|\alpha| = |\beta| = q^{1/2}$. In fact, α and β are the eigenvalues of the Frobenius endomorphism of the Tate module and are algebraic numbers (rather than l-adic integers).

Remark 1. The previous example can be interpreted as follows: The étale cohomology of the elliptic curve E_0 is

$$H^i(E, \mathbb{Q}_l) := \begin{cases} \mathbb{Q}_l & \text{if } i = 0 \\ T_l(E_0)^\vee & \text{if } i = 1 \\ \mathbb{Q}_l(-1) & \text{if } i = 2 \end{cases}$$

and by the Lefschetz fixed point formula we find

$$E_0(\mathbb{F}_{q^n}) = \sum_{i=0}^{2} (-1)^i \operatorname{tr}\left(\mathrm{Frob}_{q^n} | H^i(E, \mathbb{Q}_l)\right) = q - \operatorname{tr}(\mathrm{Fr}_q | H^1(E, \mathbb{Q}_l)) + 1.$$

It is good to note that the eigenvalues of Frobenius on $H^i(E, \mathbb{Q}_l)$ are algebraic numbers of absolute value $q^{i/2}$, this means that the cohomology is <u>pure</u>. This is true for any smooth projective variety over $\mathbb{F}_q$ by the Weil conjectures (proven by Deligne in [Del74]).

Example 2. Let $E_0/\mathbb{F}_q$ be an elliptic curve and let $A_0 = E_0 \setminus \{P, Q\}$ where P, Q are distinct $\mathbb{F}_q$-rational points. Then the long exact sequence in compactly supported cohomology shows that there is a short exact sequence

$$0 \to \mathbb{Q}_l \to H^1_c(A_0, \mathbb{Q}_l) \to H^1_c(E_0, \mathbb{Q}_l) \to 0. \tag{1}$$

The first cohomology group $H^1_c(A_0, \mathbb{Q}_l)$ is not pure of weight 1, but it is <u>mixed</u> of weight ≤ 1: It admits a filtration

$$0 \to \mathbb{Q}_l \to H^1_c(A_0, \mathbb{Q}_l).$$

whose graded pieces are pure of weight ≤ 1.

Remark 2. In fact, if we take E_0 over $\mathbb{Q}$ it is possible to choose P and Q such that the sequence (1) does not split.

2 Weights and mixed sheaves

Definition 1. *An algebraic number α (algebraic over $\mathbb{Q}$) is called a q-Weil number of weight w if for all morphisms $\sigma : \mathbb{Q}(\alpha) \to \mathbb{C}$ we have $|\sigma(\alpha)| = q^{w/2}$.*

Definition 2. *Let X_0 be a scheme over $\mathbb{F}_q$ and let $\mathcal{F}_0$ be an l-adic sheaf on X_0. We call F_0 pointwise pure of weight w if for all $n \geq 1$ and all $x \in X(\mathbb{F}_{q^n})$ the geometric Frobenius automorphism*

$$\phi_x^* : \mathcal{F}_x \to \mathcal{F}_x$$

has q^n-Weil numbers of weight w as eigenvalues.

Definition 3. *Let X_0 be a scheme over $\mathbb{F}_q$ and let $\mathcal{F}_0$ be an l-adic sheaf on X_0. We call F_0 mixed of weight $\leq w$ if F_0 admits a filtration*

$$0 = G_0 \subset G_1 \subset \cdots \subset G_n = F_0$$

such that for all i the quotient G_{i+1}/G_i is pointwise pure of weight $w_i \leq w$. We call $\mathcal{F}_0$ mixed if there exists a $w \in \mathbb{Z}$ such that $\mathcal{F}_0$ is mixed of weight $\leq w$.

Example 3. Let $X_0/\mathbb{F}_q$ be any scheme, then the constant l-adic sheaf $\mathbb{Q}_l$ is pure of weight 0. The Tate twist $\mathbb{Q}_l(-n)$ is pure of weight $2n$.

Example 4. Let S_0 be a scheme over $\mathbb{F}_q$ and let $\pi : X_0 \to S_0$ be a family of (elliptic) curves over S_0. Then the sheaf

$$\mathcal{F}_0 = R^1\pi_*\mathbb{Q}_l$$

is pure of weight 1. Indeed, if $s \in S_0$ then the fibre of X_0 over s is a(n elliptic) curve E_0 and the corresponding stalk of $\mathcal{F}_0$ is the first étale cohomology $H^1(E, \mathbb{Q}_l)$ of E_0.

Definition 4. *Let X_0 as before and let $K \in D^b_c(X_0, \mathbb{Q}_l)$, then we call K mixed of weight $\leq w$ if $H^i(K)$ is mixed of weight $\leq w+i$ for all $i \in \mathbb{Z}$. The full subcategory of mixed complexes is denoted by $D^b_m(X_0, \mathbb{Q}_l) \subset D^b_c(X_0, \mathbb{Q}_l)$.*

Proposition 1. *Let X_0 be a scheme of finite type over $\mathbb{F}_q$. Then the category $D^b_m(X_0, \mathbb{Q}_l)$ is a triangulated subcategory stable under the six functors.*

Proof. We start by proving a lemma:

Lemma 1. *The category of mixed sheaves on X_0 is closed under subquotients and extensions.*

Proof. Omitted. □

To show that $D^b_m(X_0, \mathbb{Q}_l)$ is a triangulated subcategory we have to check the following three things:

- *The subcategory $D^b_m(X_0, \mathbb{Q}_l)$ is stable under shifts.* If K_0 is pure of weight $\leq w$, then $K_0[1]$ is pure of weight $\leq w+1$ since $H^i(K_0[1]) = H^{i+1}(K_0)$.
- *The subcategory $D^b_m(X_0, \mathbb{Q}_l)$ is stable under isomorphisms.* Isomorphisms in $D^b_c(X_0, \mathbb{Q}_l)$ induce isomorphisms on cohomology from which the claim follows.
- *If there is a distinguished triangle $X \to Y \to Z \to X[1]$ in $D^b_c(X_0, \mathbb{Q}_l)$ with $X, Z \in D^b_m(X_0, \mathbb{Q}_l)$ then also $Y \in D^b_m(X_0, \mathbb{Q}_l)$.*

  Let $i \in \mathbb{Z}$ and consider the long exact sequence in cohomology

  $$\cdots \longrightarrow H^i(X) \xrightarrow{f} H^i(Y) \xrightarrow{g} H^i(Z) \longrightarrow \cdots$$

  and note that this induces a short exact sequence

  $$0 \to H^i(X)/\ker f \to H^i(Y) \to \operatorname{Im} g \to 0.$$

  The outer terms of this short exact sequence are subquotients of mixed sheaves, hence mixed by Lemma 1. This then shows that $H^i(Y)$ is an extension of mixed sheaves, and we conclude that $H^i(Y)$ is mixed by Lemma 1.

Next, we check that the six functors preserve the category $D^b_m(X_0, \mathbb{Q}_l)$. Let Y_0 be a finite type $\mathbb{F}_q$-scheme and let $f : X_0 \to Y_0$ be a morphism of schemes. We will need the following two results by Deligne:

Theorem 1 (Deligne, Thm 3.3.1 [Del80])**.** *Assume that f is separated and let $\mathcal{F}_0$ be a mixed sheaf of weight $\leq w$ on X_0. Then $R^i f_! \mathcal{F}_0$ is a mixed sheaf of weight $\leq w+i$ on Y_0.*

Theorem 2 (Deligne, Thm 6.12 [Del80])**.** *Let $\mathcal{F}_0$ be a mixed sheaf on X_0, then $R^i f_* \mathcal{F}_0$ is a mixed sheaf on Y_0.*

This allows us to prove that f_* and $f_!$ preserve mixedness (we really need $f_! = Rf_!$ here, so we work with the derived convention). Let $K_0 \in D^b_m(X_0, \mathbb{Q}_l)$ be a mixed complex, then we want to show that $H^i Rf_! K_0 = R^i f_! K_0$ is mixed. There is a hypercohomology spectral sequence

$$E_2^{p,q} = (R^p f_!) H^q K_0 \Rightarrow H^i(f_! K_0)$$

and Theorem 1 tells us that the terms on the E_2 page are mixed sheaves. Applying Lemma 1 shows that the terms on the E_∞ page are mixed, giving us a filtration of $H^i f_! K_0$ by mixed sheaves. Since a filtration is the same thing as an iterated extension, the lemma implies mixedness of $H^i f_! K_0$ which means that $f_! K_0 \in D^b_m(Y_0, \mathbb{Q}_l)$. The result for f_* follows from Theorem 2 in the same way.

Now let $L \in D^b_m(Y_0, \mathbb{Q}_l)$, then we want to show that $f^* L$ is again mixed. This follows from the fact that the stalk of $f^* H^i L$ at a point x is 'the same' as the stalk of $H^i L$ at the point $f(x)$. The fact that $f^!$ preserves mixedness follows from the fact that

$$D \circ f^! = f^* \circ D$$

and the fact that D preserves mixedness (which is shown below). $\square$

It is clear that the tensor product of pure *sheaves* is again pure. This comes down to the fact that the 'eigenvalues of the tensor product are just the pairwise products of the eigenvalues'. Since we are working with $\mathbb{Q}_l$ sheaves, the tensor product is exact, and it follows with some work that mixed sheaves are again mixed. Now we can apply the fact

$$H^n(K_0 \otimes L_0) = \bigoplus_{i+j=n} H^i(K_0) \otimes H^j(L_0)$$

Lemma 2. *The category of mixed complexes is closed under duality $D(-)$.*

Proof. We end by showing that the category of mixed complexes is closed under duality. This is clearly true when $\dim X_0$ is 0 so we can induct on the dimension of X_0. Let $K_0 \in D^b_m(X_0, \mathbb{Q}_l)$ be a mixed complex and let $U_0 \overset{j}{\hookrightarrow} X$ be a smooth dense open subscheme such that $j^* K_0$ is a smooth complex. Let $Z \overset{i}{\hookrightarrow} X$ be the closed complement and note that $\dim Z < \dim X_0$. We note that by smoothness of U, the complex $D(j^* K_0)$ is again mixed, hence also the complex

$$j_* D(j^* K_0) = D(j_! j^* K_0).$$

By induction, we know that $D(i^* K_0)$ is again mixed and so also

$$D(i_* i^* K_0) = i_* D(i^* K_0).$$

Now we have the usual triangle

$$j_! j^* K_0 \to K_0 \to i_* i^* K_0$$

and since the duals of the outer two terms are mixed, so is the dual of the middle term. Indeed, the category of mixed complexes forms a triangulated subcategory.

$\square$

3 Mixed perverse sheaves

3.1 The perverse t-structure on mixed complexes

Now that we have introduced the (triangulated) subcategory $D^b_m(X_0, \mathbb{Q}_l) \subset D^b_c(X_0, \mathbb{Q}_l)$ for any $X_0/\mathbb{F}_q$ we would like to show that the t-structure on $D^b_c(X_0, \mathbb{Q}_l)$ 'restricts' to one on $D^b_m(X_0, \mathbb{Q}_l)$.

Proposition 2. *The perverse truncation functors ${}^p\tau_{\geq 0}$ and ${}^p\tau_{\leq 0}$ preserve the subcategory $D^b_m(X_0, \mathbb{Q}_l) \subset D^b_c(X_0, \mathbb{Q}_l)$.*

Proof. Since the perverse t-structure satifies $D(D^{\leq 0}_c(X_0, \mathbb{Q}_l)) = D^{\geq 0}_c(X_0, \mathbb{Q}_l)$ by definition, it suffices to check that ${}^p\tau_{\leq 0}$ preserves mixedness. Indeed, by Lemma 2 we know that mixedness is preserved under $D(-)$.

The proof goes by Noetherian induction on X. On a point $\{x\} \subset X$ the perverse t-structure is the same as the ordinary one so there the result holds. Now let $K_0 \in D^b_m(X_0, \mathbb{Q}_l)$ and let $j : U \to X$ be an essentially smooth open subset such that j^*K_0 is a smooth complex.

We know that the perverse truncation of j^*K_0 is again mixed, since the perverse truncation of j^*K_0 is just a shift of the ordinary truncation of j^*K_0. By the Noetherian induction hypothesis, we also know that the perverse truncation of i^*K_0 (where $i : X \setminus U = Z \to X$ is the inclusion of the closed complement) is mixed. Now, we claim that i^* and j^* 'commute with perverse truncation', i.e., that

$$
\begin{aligned}
{}^p\tau_{\leq 0} i^* K_0 &= i^* \, {}^p\tau_{\leq 0} K_0 \\
{}^p\tau_{\leq 0} j^* K_0 &= j^* \, {}^p\tau_{\leq 0} K_0.
\end{aligned}
$$

The claim follows from the fact that the following two diagrams commute and therefore the 'adjoint diagram' (the same diagram replacing all the functors with the adjoint functor in the other direction) also commutes, by uniqueness properties of adjoint functors.

$$
\begin{tikzcd}
{}^pD^{\leq 0}_b(U, \mathbb{Q}_l) \arrow[r, hook] \arrow[d, "j_!"] & D^c_b(U, \mathbb{Q}_l) \arrow[d, "j_!"] \\
{}^pD^{\leq 0}_b(X_0, \mathbb{Q}_l) \arrow[r, hook] & D^c_b(X_0, \mathbb{Q}_l)
\end{tikzcd}
\qquad
\begin{tikzcd}
{}^pD^{\leq 0}_b(Z, \mathbb{Q}_l) \arrow[r, hook] \arrow[d, "i_*"] & D^c_b(Z, \mathbb{Q}_l) \arrow[d, "i_*"] \\
{}^pD^{\leq 0}_b(X_0, \mathbb{Q}_l) \arrow[r, hook] & D^c_b(X_0, \mathbb{Q}_l)
\end{tikzcd}
$$

The fact that the diagrams commute follows from t exactness of i_* and t right-exactness of $j_!$. Finally, we remark that mixedness is a stalkwise condition, so we should be able to check it after pulling back to a cover of X_0 (like $X_0 = U \cup Z$). To make this precise, we remark that pullback is t-exact with respect to the ordinary t-structure, so it commutes with taking cohomology, and this gives the result since we have shown mixedness of j^*K_0 and i^*K_0. $\square$

3.2 Weights

We have already defined a notion of being of weight $\leq w$ for mixed sheaves $\mathcal{F}_0$ and now we want to extend this notion to mixed complexes. Theorem 1 motivates the following definition:

Definition 5. *We call a complex $K_0 \in D^b_m(X_0, \mathbb{Q}_l)$ mixed of weight $\leq w$ if H^iK_0 is mixed of weight $\leq w + i$ for all i. We denote the full subcategory of mixed complexes of weight $\leq w$ by $D^c_{\leq w}(X_0, \mathbb{Q}_l)$. We call K mixed of weight $\geq w$ if $D(K)$ is of weight $\leq -w$ and similarly write $D^c_{\geq w}(X_0, \mathbb{Q}_l)$.*

Remark 3. It is good to note that the subcategories $D^c_{\leq, \geq w}(X_0, \mathbb{Q}_l)$ are not triangulated subcategories. The notion of being of weight $\geq w$ agrees with the naive definition when X_0 is smooth (since then we know the dualizing complex) but not in general. In particular, it is possible that $w(K_0) \geq w$ but $K_0 \notin D^b_{\geq w}(X_0, \mathbb{Q}_l)$. Here $w(K_0)$ is defined as the maximum of the weights $w(H^i(K_0)) - i$.

Theorem 3 (Permanence properties)**.** *Let $f : X_0 \to Y_0$ be a morphism of schemes of finite type over $\mathbb{F}_q$, then*

1. *The functors $f_!$ and Rf^* preserve $D^b_{\leq w}$.*
2. *The functors f_* and $f^!$ preserve $D^b_{\geq w}$.*
3. *The functor $\bigotimes$ sends $D^b_{\leq w} \times D^b_{\leq w'}$ into $D^b_{\leq w+w}$.*
4. *The functor $\underline{Hom}$ sends $D^b_{\leq w} \times D^b_{\geq w'}$ into $D^b_{\leq -w+w'}$.*
5. *Verdier duality D exchanges $D^b_{\leq w}$ and $D^b_{\geq w}$.*

Proof.

1. The result for f^* follows from the fact that pullback commutes with taking (ordinary) cohomology. If $K_0 \in D^b_m(X_0, \mathbb{Q}_l)$, then there is a hypercohomology spectral sequence
$$E_2^{p,q} = (R^p f_!) H^q K_0 \Rightarrow H^{p+q}(f_! K_0).$$
By the assumption that $H^p K_0$ is mixed of weight $\leq w + p$ and Theorem 1 we find that the $E_2^{p,q}$ term is mixed of weight $\leq w + p + q$ and this is easily seen to then also hold for $E_\infty^{p,q}$. This implies that $H^{p+q} f_! K_0$ is mixed of weight $\leq w + p + q$ hence that $f_! K_0$ is mixed of weight $\leq w$.
2. This follows by duality from part 1.
3.
4. This follows from part 3, the formula
$$\underline{Hom}(K_0, L_0) = D(K_0 \otimes D(L_0))$$
and part 5.
5. This holds by definition.

□

4 On Proposition 5.1.2 in [BBD82]

Let X_0 be a scheme of finite type over $\mathbb{F}_q$ and $\mathcal{F}_0$ a $\mathbb{Q}_l$-sheaf on X_0. Then there is a canonical isomorphism $F_q^* : \mathrm{Fr}_q^* \mathcal{F} \to \mathcal{F}$. Similarly, if $K_0 \in D^b_c(X_0, \mathbb{Q}_l)$ then there is a canonical isomorphism $F_q^* : \mathrm{Fr}_q^* K_0 \to K_0$ and the statement also holds if $K_0 \in \mathrm{Per}(X_0)$.

Proposition 3 (Proposition 5.1.2 [BBD82])**.** *Let $\mathrm{Per}_{\mathrm{Fr}}(X)$ denote the category of perverse sheaves on X equipped with an isomorphism to their Frobenius pullback . The functor $\mathrm{Per}(X_0) \to \mathrm{Per}_{\mathrm{Fr}}(X)$ sending $\mathcal{F}_0 \mapsto (\mathcal{F}, \mathrm{Fr}_q^*)$ is fully faithful and the essential image is stable under extensions and subquotients.*

Proof. We are in the following situation

$$\begin{array}{ccc} X & \longrightarrow & X_0 \\ \downarrow{\scriptstyle a'} & & \downarrow{\scriptstyle a} \\ \mathrm{Spec}(\mathbb{F}) & \longrightarrow & \mathrm{Spec}(\mathbb{F}_q). \end{array}$$

Let $M_0 = \underline{Hom}(K_0, L_0) \in D^b_c(X_0, \mathbb{Q}_l)$ and note that $M = \underline{Hom}(K, L)$ (this follows from the fact that pullback commutes with tensor products). The abelian group $\hom(K_0, L_0)$ is equal to $H^0(\Gamma_{X_0} M_0)$, where Γ_{X_0} is the global sections functor. There is a commutative diagram of triangulated categories (the fact that the square commutes is explained well in [(ht]).

$$\begin{tikzcd}
D^b_c(X, \mathbb{Q}_l) \arrow[d, "a'_*"] & D^b_c(X_0, \mathbb{Q}_l) \arrow[l, "p_X"'] \arrow[d, "a_*"] \arrow[dr, "\Gamma_{X_0}"] & \\
D^b_c(\operatorname{Spec} \mathbb{F}, \mathbb{Q}_l) & D^b_c(\operatorname{Spec} \mathbb{F}_q, \mathbb{Q}_l) \arrow[l, "p"'] \arrow[r, "\Gamma"] & D^b(\mathrm{Ab})
\end{tikzcd}$$

In particular this implies that

$$\begin{aligned} p^* a_* \underline{Hom}(K_0, L_0) &= a'_* p_X^* \underline{Hom}(K_0, L_0) \\ &= a'_* \underline{Hom}(K, L). \end{aligned}$$

Now write $N_0 = a_* \underline{Hom}(K_0, L_0)$ and $N = p^* N_0$ as usual. The hypercohomology spectral sequence for Γ reads

$$E_2^{l,k} H^l(\operatorname{Spec} \mathbb{F}_q, H^k N_0) \Rightarrow H^{l+k}(\Gamma N_0).$$

We note that

$$H^l(\operatorname{Spec} \mathbb{F}_q, H^k N_0) = H^l_{\mathrm{Gal}}(\operatorname{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q), H^k N)$$

and that $G \cong \hat{\mathbb{Z}}$ which has cohomological dimension one. This means that the spectral sequence degenerates at the second page. Moreover, we know that the zeroth cohomology is taking invariants, and the first cohomology is taking co-invariants giving us short exact sequences

$$0 \to (H^{i-1} N)_G \to H^i(\Gamma N_0) \to (H^i N)^G \to 0$$

for all n. Now let $N_0 = a_* \underline{Hom}(K_0, L_0)$, then $N = a'_* \underline{Hom}(K, L)$ and the short exact sequences read

$$0 \to \operatorname{Ext}^{i-1}(K, L)_F \to \operatorname{Ext}^i(K_0, L_0) \to \operatorname{Ext}^i(K, L)^F \to 0.$$

Choosing $i = 0$ gives a short exact sequence

$$0 \to \hom^{-1}(K, L) \to \hom(K_0, L_0) \to \hom(K, L)^G \to 0.$$

The fact that K, L lie in the heart of the perverse t-structure, means that the group $\hom^{-1}(K, L)$ vanishes, giving us the fully faithfulness (this also shows that the statement doesn't hold for general $K_0 \in D^b_c(X_0, \mathbb{Q}_l)$. When $i = 1$, we get a short exact sequence

$$0 \to \hom(K, L)_F \to \operatorname{Ext}^1(K_0, L_0) \to \operatorname{Ext}^1(K, L)^F \to 0.$$

It is important to note that $\operatorname{Ext}^1(K, L)^F \neq \operatorname{Ext}^1((K, \phi), (L, \phi))$, i.e., it is not the ext-group in the category $\operatorname{Per}_{\mathrm{Fr}}(X)$. There is however a forgetful map

$$\operatorname{Ext}^1((K, \phi), (L, \phi)) \to \operatorname{Ext}^1(K, L)$$

which factors over the inclusion $(\mathrm{Ext}^1(K,L))^F \to \mathrm{Ext}^1(K,L)$ and the induced map $f : \mathrm{Ext}^1((K,\phi),(L,\phi)) \to (\mathrm{Ext}^1(K,L))^F$ is surjective. The kernel consists of extensions

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathrm{Fr}^* K & \longrightarrow & \mathrm{Fr}^* K \oplus \mathrm{Fr}^* L & \longrightarrow & \mathrm{Fr}^* L_{\phi_L} & \longrightarrow & 0 \\ & & \Big\downarrow{\scriptstyle \phi_K} & & \Big\downarrow{\scriptstyle \psi} & & \Big\downarrow & & \\ 0 & \longrightarrow & K & \longrightarrow & K \oplus L & \longrightarrow & L & \longrightarrow & 0 \end{array}$$

that are nontrivial in $\mathrm{Per}_{\mathrm{Fr}}(X)$. In other words, the map ψ is not equal to the map $\left(\begin{smallmatrix} \phi_K & 0 \\ 0 & \phi_L \end{smallmatrix}\right)$ but of the form

$$\begin{pmatrix} \phi_K & U \\ 0 & \phi_L \end{pmatrix}$$

for some nontrivial $U \in \hom(K,L)$. It can be shown that this determines an isomorphism between $\ker f$ and $\hom(K,L)_F$, which I will not explain here. For the fact that the essential image is closed under subquotients we refer to Lemma 5.1.3 in [BBD82]. $\square$

References

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