II-1. Let $F = \mathbf{F}_{29}$.

(i) Find a primitive 4-th root of unity $\zeta \in F$ and compute its inverse $\zeta^{-1} \in F$.

(ii) Find the matrices V_ζ and $V_{\zeta^{-1}}$ and verify that their product if $4I$.

II-2. Let $F = \mathbf{F}_{17}$ and $f = 5x^3 + 3x^2 - 4x + 3$, $g = 2x^3 - 5x^2 + 7x - 2$ in $F[x]$.

(i) Show that $\zeta = 2$ is a primitive 8-th root of unity in F, and compute ζ^{-1}.

(ii) Compute $h = f \cdot g \in F[x]$.

(iii) For $0 \leq i < 8$ compute $\alpha_i = f(\zeta^i)$, $\beta_i = g(\zeta^i)$, and $\gamma_i = \alpha_i \cdot \beta_i$. Compare γ_i to $h(\zeta^i)$.

(iv) Find V_ζ and $V_{\zeta^{-1}}$ and compute their product.

(v) Use the fast Fourier transform as in the example to compute $f \cdot g$.

II-3. [**Huffman code**] Read the attached explanation on Huffman coding.

(i) Implement the algorithm to build an optimal Huffman code.

(ii) Apply this to a brief sentence of your choice (with some different frequencies), for example "The computer algebra courses in Amsterdam".

(iii) Prove that the algorithm produces optimal Huffman codes!