Local Langlands seminar

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IMPORTANT On Thursday, November 20 we cannot use the Hilbert space. We have to use HG02.802.

1 Smooth Representations

Speaker: Milan Lopuhaä

Treat: §1–3 of [1]. Quick review of locally profinite groups. Careful treatment of smooth representations. (See also [3, §1.2–1.4].) Haar Measures: End with statement of the Duality Theorem in §3. (See also [3, §2.1] and parts of [2].)

Abstract: In this talk, I will give a short introduction to the local Langlands conjecture for GL_2. The largest part of the seminar will be dedicated to the study of irreducible smooth representations of $\mathrm{GL}_2(F)$ of a local field F. In this talk, we will start by defining locally profinite fields and their smooth representations. Furthermore, we will define basic notions, such as induced representations, duality, and Haar measures, that will be of use in the rest of the seminar.

2 The Hecke Algebra, Linear Groups over Finite Fields

Speaker: Peter Badea

Treat: §4–6 of [1]. Smooth representations are precisely modules over the Hecke algebra (analogous with the group ring). Classify representations of finite linear groups.

Abstract: In this talk, we define and discuss the Hecke Algebra $\mathcal{H}(G)$, as well as the equivalence beween smooth modules over $\mathcal{H}(G)$ and smooth representations of a locally profinite group. In the second part of the talk, we will work out the irreducible representations of the group $\mathrm{GL}_2(k)$ over a finite field k. This classification will be useful later on in the seminar when we will wish to examine the representations of $\mathrm{GL}_2(F)$ over a non-Archimedean local field F.

3 Linear Groups over Local Fields and the Mirabolic Group

Speaker: Ben Moonen

Treat: [1, §7,8]. Show that GL_2 is unimodular ([1, §7.5 Prop.]). State the Duality Theorem ([1, §7.7 Prop.]). Define the Mirabolic group and classify its irreducible smooth representations.

4 Jacquet Modules and Cuspidal Representations

Speaker: Julius Witte

Treat [1, §9–10]. It is possible to leave out Appendix §10a.

State (but not necessarily prove) the Irreducibility Criterion ([1, §9.6]). State (and if possible prove) the Classification Theorem ([1, §9.10,9.11 Thm]). This classifies non-cuspidal representations.

Continue with a start of the classification of cuspidal representations. Define cuspidal representations. Prove [1, §10.2 Thm].

5 Classification of Cuspidal Representations I

Speaker: Maarten Solleveld

Treat [1, §11–13]. State and prove both theorems of §11.

Introduce strata, and classify irreducible smooth representations in terms of strata ([1, §13.3 Cor].

6 Classification of Cuspidal Representations II

Speaker: Maarten Solleveld

Treat [1, §14–15]. State and prove the Exhaustion Theorem (a characterization of irreducible cuspidal representations in terms of strata) [1, §14.5 Thm]. It is possible to skip the proof of [1, §14.3 Lem].

State the Classification Theorem ([1, §15.5 Cor].

Black box stuff from §16,17

§16 is used in §15.1 and §15.6. It functions as appendix of §15 (finishing some proofs).

§17 is used in §14.4 (proof of §14.3). The main result is:

Theorem. The Steinberg representation of G is square-integrable. Non-cuspidal representations are square-integrable if and only if they differ from the Steinberg representation by a character.

7 Tame Cuspidals

Speaker: Gert Heckman

Treat [1, §18–22]. State and prove the Tame Parametrization Theorem ([1, §20.2 Thm].

8 Functional Equation for GL_2 and Cuspidal Local Constants

Speaker: Johan Commelin

Treat [1, §24,25], borrowing parts of §23. The main results are on functional equations for cuspidal representations. State and prove [1, §24.2 Thms] and [1, §25.2 Thm]. (Maybe look at [3, §3.3] for a summary.)

Most probably treat [1, §25.7] as black box.

9 Functional Equation for Non-Cuspidal Representations

Speaker: ?

Treat [1, §26,27]. (Maybe look at [3, §3.3] for a summary.) State and prove [1, §26.1 Thm], the functional equation for non-cuspidal representations.

State and prove [1, §27.1, Thm]: irreducible smooth representations are determined by their L-function and ε-factor.

10 Weil Groups

Speaker: ?

Treat [1, §28–30]. (Maybe take a look at [3, §3.1,3.2].) We now come to the second part of objects in the Langlands correspondence. Define Weil groups and show some properties.

Recall the statement of Local Class Field Theory ([1, §29.1]). State [1, §29.4 Thm] and prove as much as you like of it (see §30 for the proof).

11 Deligne Representations

Speaker: ?

Treat [1, §31–32]. (Maybe take a look at [3, §3.4].) Define Deligne representations and motivate them using §32. In particular state [1, §32.7 Thm].

12 Local Langlands for GL_2

Speaker: ?

Treat [1, §33–35]. State and prove the Local Langlands Correspondence for GL_2 and $p \neq 2$, [1, §33.1 Thm].

State (maybe sketch the proof) of ℓ-adic Langlands, [1, §35.1].

13 Outlook

Speaker: ?

Survey talk about global Langlands. Maybe ask Bas Edixhoven.

References

[1] Colin J. Bushnell and Guy Henniart. *The local Langlands conjecture for* GL(2), volume 335 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 2006.

[2] J. T. Tate. Fourier analysis in number fields, and Hecke's zeta-functions. In *Algebraic Number Theory (Proc. Instructional Conf., Brighton, 1965)*, pages 305–347. Thompson, Washington, D.C., 1967.

[3] Remy van Dobben de Bruyn. *The Local Langlands Correspondences.* 2012. Unpublished MSc-thesis.