STUDY GROUP "HARMONIC ANALYSIS ON QUANTUM GROUPS AND HECKE ALGEBRAS"

SPRING 2023

Plan and preliminary schedule (tentative). In the study group we study the various techniques that have been developed over the past 25 years to relate spherical functions on higher rank quantum symmetric spaces and Hecke algebras to special functions, and we aim to understand how they are connected. In addition we learn from each-others research activities in these directions. We meet four afternoons in the spring of 2023. In principle each meeting will consist of a QG-talk, AH-talk and a RT-talk,

- QG: explaining results on quantum symmetric pairs and related spherical functions
- AH: explaining results on affine Hecke algebras and Macdonald polynomials
- RT: research talk.

We also occasionally might have a guest speaker.

Schedule (provisional).

1. February 16, 12:30-16:30 (Amsterdam).
   QG-talk: Valentin
   AH-talk: Stein
   RT-talk: Maarten
2. March 30, 13:00-17:00 (Nijmegen).
   QG-talk: Jasper
   AH-talk: Philip
   RT-talk: Carel
3. April 25, 13:00-17:00 (Amsterdam).
   QG-talk: Erik
   AH-talk: Edward
   QG-talk: Stein (crystal bases)
4. May 23, 13:00-17:00 (Nijmegen).
   AH-talk: Jasper
   RT-talk: Valentin (on the connection of the fourth AH-talk with representation theory of metaplectic covers of p-adic groups).
   RT-talk: Stefan Kolb (Newcastle, UK)

QG-theme. A first proposal for the topics of the four talks on the QG-theme is

1. Harmonic analysis on higher rank quantum symmetric spaces, following Noumi [8].
2. Letzter-Kolb quantum symmetric pairs [3].
3. Radial part computations for quantum symmetric pairs [8, 5].
4. Many possibilities: Cartan subalgebras in quantum symmetric coideal subalgebras [6], quantum symmetric pairs and reflection equations [1, 4], analytic aspects, relation to dynamical quantum groups, vector-valued quantum spherical functions,....

Remarks:

a. Noumi's [8] "case (SO)" might be the symmetric space to focus on in the first talk. It corresponds to $\mathrm{SU}(n)/\mathrm{SO}(n)$ as compact symmetric space, and hence it corresponds to a split symmetric pair (I ignore here the GL_n versus SL_n issues, Noumi actually considers $\mathrm{U}(n)/\mathrm{SO}(n)$). The corresponding Macdonald polynomials are the of type A (the GL_n-versions, which are already in Macdonald's book on symmetric functions and Hall polynomials).

   In talks 2-4 we could then focus on the Letzter-Kolb theory for the split case. For type A it should then be easy to see the explicit connection between Noumi's case (SO) and the Letzter-Kolb setup. Furthermore, in the split case various technical structural complexities for quantum symmetric pairs can be avoided (for instance, in the split case root system=restricted root system, and the associated Letzter coideal subalgebra has a simple explicit presentation in terms of "Koorwinder-type" twisted primitive elements, which are naturally attached to the simple roots).

b. Noumi [8] actually explains in detail the radial component computations for the two symmetric spaces studied in his paper, and he also explains the conceptual idea. Noumi works with L-operators, i.e. with quantum root vectors for all roots (not only the simple ones), and his techniques are closest to the classical Harish-Chandra & Casselman-Milicic techniques I believe. It should be interesting to compare Noumi's techniques to compute radial components to the ones by Letzter in [5] (which uses rank one computations, Weyl-group equivariance and leading term computations).

AH-theme. A first proposal for the topics of the four talks on the AH-theme is

1. Cherednik's polynomial representation of the affine Hecke algebra [7] (Chapter 4 up to an including (4.3.10)).
2. Cherednik operators and nonsymmetric Macdonald polynomials [7] (continuation of Chapter 4 involving material up to subsection 4.7, and material from sections 5.1, 5.2 and 5.4).
3. Macdonald operators and the symmetric Macdonald polynomials [7] (Sections 4.4, 5.3).
4. Vector-valued and quasi-polynomial generalisations [2, 9].

Remarks:

a. Again there are issues here about which case to discuss (Macdonald divides the theory in three cases). I believe the best choice now is the case with formula number (1.4.2) in Macdonald's book [7]. It is tempting here to take for the two lattices L and L' the co-root lattice $Q^\vee$ instead of the co-weight lattice $P^\vee$ (this kills the role of the abelian group $\Omega = P^\vee/Q^\vee$ in the story). If one would like to recover the GL_n Macdonald polynomials from this perspective, one in principle does need the theory with Ω, but actually with lattices $L = L' = \mathbb{Z}^n$ that are of higher rank than $Q^\vee$. Although for GL_n this extension can be easily explained directly (these are again typical GL_n versus SL_n issues, now at the AH-side).

b. "Radial component computations" do not exist here, since the polynomial representation is already "radial". But there is the intricate explicit computation obtaining the Macdonald operators from symmetrised Cherednik operators. The important technique here is show that the Cherednik operators are triangular operators relative to the basis of monomials, properly ordered by a refinement of the dominance order on the exponents of the monomials, and to compute the leading term explicity. This is discussed in sections 4.5 and 4.6 of Macdonald's book. Understanding the triangularity of the Cherednik operators also allows one to directly define nonsymmetric Macdonald polynomials as simultaneous eigenfunctions of the Cherednik operators (Macdonald defines them in Chapter 5 using orthogonality relations). But more importantly, these triangularity and leading term type of arguments bear similarities with the radial component computations of quantum Casimirs by Letzter [5].

References

[1] M. Balagovic, S. Kolb, *Universal K-matrix for quantum symmetric pairs*, J. Reine Angew. Math. **747** (2019), 299–353.

[2] I. Cherednik, *Induced representations of double affine Hecke algebras and applications*, Math. Res. Lett. **1** (1994), 319–337.

[3] S. Kolb, *Quantum symmetric Kac-Moody pairs*, Adv. Math. **267** (2014), 395–469.

[4] K. De Commer, M. Matassa, *Quantum flag manifolds, quantum symmetric spaces and their associated universal K-matrices*, Adv. Math. **366** (2020), 107029, 100pp.

[5] G. Letzter, *Quantum zonal spherical functions and Macdonald polynomials*, Adv. Math. **189** (2004), 88–147.

[6] G. Letzter, *Cartan subalgebras for quantum symmetric pair coideals*, Rep. Th. **23** (2019).

[7] I. Macdonald, *Affine Hecke algebras and orthogonal polynomials*, Cambridge Tracts in Math. **157**, Cambridge Univ. Press, 2003.

[8] M. Noumi, *Macdonald's symmetric polynomials as zonal spherical functions on some quantum homogeneous spaces*, Adv. Math. **123** (1996), 16–77.

[9] S. Sahi, J.V. Stokman, V. Venkateswaran, *Quasi-polynomial representations of double affine Hecke algebras*, arXiv:2204.13729 (2022).