EMERGENCE, NATURE AND DECAY OF RUSSELL'S PLATONISM

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Introduction

0.1 Topic and comment

The fame of Bertrand Russell (1872-1970) is partly due to his discovery in 1901 of the paradox named after him. No one who, like me, feels some kind of Platonic love of logic and mathematics, can be wholly indifferent to it. The challenge has been and still is, to lay bare its root and to find as natural a solution as possible.

Indeed, Russell never disqualified the rather complicated way out he proposed together with Whitehead in *Principia Mathematica* (1910-1913), namely the *Ramified Theory of Types*. But, as appears from his book *My Philosophical Development* (1959), that solution could not completely satisfy him, at least not emotionally. In the short, but remarkable chapter *The Retreat from Pythagoras* (which of course is also meant to be a retreat from Plato), the 87 years old Russell says:

> Mathematics has ceased to seem to me non-human in its subject-matter. I have come to believe, though very reluctantly, that it consists of tautologies. I fear that, to a mind of sufficient intellectual power, the whole of mathematics would appear trivial, as trivial as the statement that a four-footed animal is an animal. (…) The solution of the contradictions mentioned in an earlier chapter (i.e. the different version of the paradox, H.B.) seemed to be only possible by adopting theories which might be true but were not beautiful. I felt about the contradictions much as an earnest Catholic must feel about wicked Popes.

An *historical* approach to the paradox may – that is the main point I hope to make clear – be valuable to a *systematic* discussion about its nature. Consequently, my lecture will consist of two parts.

Firstly I'll discuss the two main turning points in Russell's philosophical development: a) the great revolutionary change from neo-Hegelian holistic idealism to unrestricted Platonic realism and pluralism, that took place about 1899 and b) the discovery in 1901, of the contradiction on occasion of an attempt to refute what at first seemed to be a minor point in Cantor's mathematical theory of the infinite. I hope to show the 'internal relation' between these two landmarks. They demarcate two sides of one and the same thing, namely the beginning and the end of the short-lived, but very influential "intellectual honeymoon" of Russell's unrestricted Platonism.

Secondly, on the base of this historical analysis, I'll put forward some suggestions concerning the root and possible solution of the paradox.

0.2 Perspective

Before going into these two parts, I want to say something about the rather peculiar philosophical outlook that is presupposed in my attempt to combine, of course without confusing them, historical exegesis with criticism. It may be summarized in the following two points:

a) *Oppositions* are of general importance. They play a prominent role *everywhere*, both in human life and outside it, both in language, in human relationships, in human conflicts, in personal development, in society, history, art, religion, philosophy, all the sciences (including logic and mathematics) and in their subject matter. Because they are to be found everywhere, it is impossible to define oppositions as such or to reduce them to something else. A vantage point outside their realm cannot be found. For if it existed, it would be, in virtue of that, be opposed to oppositions.

b) We are *acquainted* with them. Every child knows that great is opposed to small, warm to cold, even to odd, inside to outside and yes to no. Nevertheless it is *difficult* to understand oppositions as such, especially those we are involved in. This difficulty is *subjective*. There is nothing problematic about oppositions themselves. The problem is *ours*. And its source is our unwillingness to acknowledge it. That is the main point of my philosophical orientation: I am opposed to the widespread view that as a matter of course we do understand oppositions.

As far as I can see, this illusion is mainly due to ignoring that antipoles are counterparts. *As such*, i.e. in virtue of their being opposed to each other, they must have something in common. Making their opposition possible, this something cannot be outside the antipoles. For example: the journey from A to B is opposed to the journey from B to A; even more so if the very same road is followed. But the different journeys cannot be opposed to each other, unless the road is such, that it *can* be travelled in two opposite directions. Or, to give a similar example, heads and tails are opposite sides of one and the same coin. There must be something, different from the two sides, that *has* them both. Nevertheless, it is impossible to isolate the coin from the two sides it has.

1 Historical interpretation

1.1 Preliminary remarks

As far as the study of the history of philosophy is concerned, the impact of this general outlook is as follows. Paying attention to oppositions may help to see and feel the inspiration in the work of great thinkers. For a philosophy is inspired in so far as it is opposed to a deep-rooted and general illusion. Furthermore, asking what philosophy X must have in common with philosophy Y in order to make it possible to be opposed to it, may help to get a clearer view on the nature of both.

So, let us apply this approach to the most fundamental change in Russell's philosophical development. In what way is his Platonic realism opposed to his former neo-Hegelian idealism and what presuppositions do these philosopies have to share in order to make their opposition possible?

Anyhow, there is one obvious element they have in common. From the very start of his philosophical career, Russell wants to pay attention to the sciences and especially more attention than most neo-Hegelians are prepared to pay. Before the change, he tries to do so within a dialectical and holistic conceptual framework. The change consists in rejecting this framework as an impediment that prevents him from attaining the very same aim: becoming a philosopher with an *open mind to the sciences*.

But why does this ideal require a change in perspective? What is, according to the self-critical Russell, wrong with his former 'idealism'? Why is it supposed to prevent him from being open-minded to the sciences? His answer is just as concise as it is sweeping: because it

presupposes the doctrine of *internal relations*. This doctrine is to be replaced by the doctrine of *external relations*. Now we have two key words that in ordinary language are used to mark an opposition: *external* versus *internal*. But what do these words mean in this connexion?

Answering this question would be very easy if the notorious axiom of internal relations explicitly occurred in the writings of the idealist Russell. Unfortunately, that is not the case. If it occurs at all, it is presupposed without saying. Nevertheless, I have found a piece of text, in which Russell is very near to actually making use of that doctrine. Before quoting it, I'll introduce it by means of a short sketch of the context in which it belongs.

1.2 Russell's idealism

According to Russell's view at that time, philosophy has not to add new items to scientific knowledge, but it has to take care of the *whole*. The specific sciences are not able to do so, because they cannot understand the limits of their own perspective. Therefore, the first task of the philosopher is: to lay bare the fundamental ideas and principles of each science. These are apriori in a non-subjective sense of that word: i.e. such that without them that science would be impossible. The second task of the philosopher is more important: to show that these ideas and principles *inevitably* contain certain contradictions which only can be solved by another, less 'abstract' and more 'concrete' science. That science may be subjected to the same procedure and so on, until the highest meta-scientific and metaphysical level, the level of philosophy is reached.

It is quite essential to this dialectical approach, that the *most abstract*, i.e. the lowest science has to be treated *first*. According to Russell's view at that time, it happens to be *mathematics*. Consequently, as a neo-Hegelian who was very much disappointed by the three years of mathematical study he just had finished, the young Russell, by the logic of his own philosophical programme, is forced to use and extend his mathematical knowledge. His following the course of James Ward on *Kant and non-Euclidian geometry* also played an important role in that connexion. Out of it grew Russell's doctoral dissertation (1895), the book based on it, entitled *An Essay on the Foundations of Geometry* (1897) and several articles on the same subject.

In one of those articles, namely *The Logic of Geometry*, appeared in *Mind* in 1896, Russell writes near the end of it:

> This is the quality which distinguishes space from any other manifold – in the colour- and tone-systems, every element has an intrinsic nature, sensationally given, from which the relations between the elements are intellectually constructed. In space, on the contrary, the relations are also sensationally given, and the elements (points) are never given except as terms in a relation. (…..) in geometry, we have a space which cannot stand by itself, a thing all relations, without any kernel of thinghood to which the relations can be attached. This forces us to attempt a resolution of the contradiction by abandoning the purely geometrical standpoint; but such an attempt would fall outside the limits of the present paper, and would only be possible on the basis of a general metaphysic.

This then, is the piece of text mentioned before. What is remarkable about it, is Russell's explicitly comparing geometrical space with other, more 'concrete' manifolds, such as the colour- or tone-system. There, in virtue of their *intrinsic nature*, the elements are supposed to stand in certain relations to each other.
Now, this is exactly what the later realist Russell has in mind when he is attacking the 'axiom of internal relations'. The issue at stake is not so much, as in Wittgenstein's *Tractatus*, whether a relation is either contingent or necessary, but rather whether *a relation must be*

based on something non-relational. An affirmative answer is as a matter of course presupposed in the idealist Russell's use of the opposition between 'abstract' and 'concrete'. According to him, the notion of pure 'abstract' geometrical space inevitably leads to a vicious circle and to contradictions. Why so? Because the "kernel of thinghood to which the relation can be attached" is lacking. There is, as Russell in that period used to say, a "conception of diversity without diversity of conception".

In §196 of his book *An Essay on the Foundations of Geometry*, he writes:

> After hypostatizing space, as Geometry is compelled to do, the mind imperatively demands elements, and insists on having them, whether possible or not. Of this demand, all the geometrical applications of the infinitesimal calculus are evidence. But what sort of elements do we thus obtain? Analysis, being unable to find any earlier halting-place, finds its elements in points, that is, in zero quanta of space. Such a conception is a palpable contradiction, only rendered tolerable by its necessity and familiarity.

1.3   Unrestricted realism versus idealism

In 1899, Russell gives a course on Leibniz. He was asked to do so, because Mc Taggert, who had announced such a course, was, because of 'extraordinary' circumstances, not available. Leibniz discusses the topic of relations much more extensively and consistently than the idealist Russell had ever dared to do. According to Leibniz a, relation cannot really hold *between* two different subjects. According to the axiom of internal relations, what seems to be one fact that concerns two different entities, must eventually be analyzed in two complementary private facts.

This made Russell aware of the *logical* principle involved in *all* kinds of *metaphysical* idealism, not only in the particular one propounded by Leibniz, but also, to mention two important examples, in Bradley's holistic monism and in Russell's former pluralistic holism, namely that *eventually every proposition must have a subject and a predicate*. In his book *A critical Exposition of the Philosophy of Leibniz* (1900), p.15, he writes:

> In the belief that propositions must, in the last analysis, have a subject and a predicate, Leibniz does not differ either from his predecessors or from his successors. Any philosophy which uses either Substance or the Absolute will be found, on inspection, to depend upon this belief. Kant's belief in an unknowable thing-in-itself was largely due to the same theory. It cannot be denied, therefore, that the doctrine is important. Philosophers have differed, not so much in respect of belief in its truth, as in respect of their consistency in carrying it out. In this latter respect, Leibniz deserves credit.

What does the "belief that propositions must, in the last analysis, have a subject and a predicate" have to do with idealism? And why does its rejection lead to realism? Here we meet an obstacle that might prevent us from understanding the nature of Russell's realism. These questions cannot be answered as long as the classical Aristotelian and Kantian view of logic as a kind of formal proto-science which precedes real knowledge and does not involve any metaphysical assumptions, is taken for granted. But there are good reasons not to do so, for Russell never subscribed to such a view, neither as an idealist, nor as a realist. Therefore, the subject-predicate principle is to be construed as logico-metaphysical in character. There is one, and as far as I can see only one, way to meet this requirement: by taking into account the notion of *substance*, mentioned in the text quoted above.

The principle Leibniz made Russell aware of, bears on the connection between *truth and being*. Whatever is true is eventually, "in the last analysis"[i], true about what is supposed to be

the only really real kind of thing, namely a concrete existent, a *substance*. Single substances are, in virtue of their being 'in themselves' and not in something else, the ultimate subjects of propositions. And what is true about them, consists in their being qualified, in their having predicates which are supposed to be their private properties. These predicates or 'accidents' are not in themselves. The only way they are is to be in the substance they belong to. Their *esse* is *in-esse*.

In my opinion, the idealism Russell is opposed to, is not primarily epistemological in character, as has been often supposed, but metaphysical. It involves the exclusion of all kinds of things from the realm of real being. Whatever cannot be construed as a concrete individual or as one of its private properties must be unreal, or at least not fully real. It is relegated to the realm of the merely *ideal*, i.e. the realm of abstract conceptual deviations from reality which may point to it because they are derived from it, but do not properly belong to it.

What kinds of items are deemed to dwell in this limbo? Universals, relations, propositions (especially false ones), space, time, infinity and last but not least: plurality. And these, of course, are exactly the things Russell is anxious to allow unrestricted entrance in the realm of being. Just as idealism is based on *restricted* aboutness of propositions, so the main philosophical principle underlying the logic of *The Principle of Mathematics* (1903) claims their *unrestricted* aboutness.

1.4   Mathematical allies

During the same short period Russell not only starts to see mathematics in a new light, but to take notice of its unsuspected developments as well. It is hardly possible to overrate the intensity of his enthusiasm and delight. Telling is his saying that Kant has not been refuted by German philosophers, but by German mathematicians. New heroes appear on the stage, above all Weierstrass, Cantor and Dedekind. What Russell thinks to see is, that these and similar masters of science have freed themselves from essentially the same fetters he just freed himself from: the bounds of the repressive Aristotelian logic and metaphysics of substance. The new logic, in a mathematical way explored by Boole, Peirce and Peano, is not only welcoming mathematics, but essentially one with it.

According to the realist Russell, the so-called 'inevitable contradictions' in the principles of mathematics are only due to the philosophical limitations of the traditional point of view. As soon as its fetters are broken, the purported necessity of contradictory notions completely disappears. In reading Weierstrass, who proves that the notion of the 'infinitely little' is not only contradictory, bur needless as well, Russell must have blushed with shame. The same must have happened in his reading Cantor, who makes clear that what seems strange and paradoxical about the infinitely great, is only in conflict with the dogmatic, narrow-minded expectation that the infinite must, as much as possible be like the finite.

1.5   The master's 'very subtle fallacy'

As appears from the article *Mathematics and the Metaphysicians* (1900), there is one exception, one little cloud in the blue sky of total fraternization. After having honoured Cantor as the very first master of thought who, after more than two thousand years of inflated philosophical ego's, was able to answer Zeno, Russell writes:

> There is a greatest of all infinite numbers, which is the number of things altogether, of every sort and kind. It is obvious that there cannot be a greater number than this, because, if

everything has been taken, there is nothing left to add. Cantor has a proof that there is no greatest number, and if this proof were valid, the contradiction of infinity would reappear in a sublimated form. But in this one point, the master has been guilty of a very subtle fallacy, which I hope to explain in some future work.

Cantor's proof may be summarized as follows. The number of elements of any set V is smaller than the number of subsets of V. For, if you assign in some way to each element of V one subset of V, then, in virtue of that procedure, all the elements of V are divided in two subsets: the *reflexive* elements i.e. those that are themselves a member of the subset assigned to them and the *un-reflexive* elements for which this does not hold. This latter subset cannot be assigned to any element. For if this were the case, that supposed element would either be reflexive or un-reflexive. The first alternative is impossible. For by definition the set of un-reflexive elements excludes reflexive ones. The second alternative is impossible as well. For if the set of all un-reflexive elements were assigned to an un-reflexive element, that element would in virtue of that be reflexive. Therefore, the supposed element does not exist.

The following optical illustration may be helpful. Suppose V to consist of points on the floor of a room. Different mirrors in different positions are fixed at the ceiling and the walls of the room. Then, according to that arrangement, to each point on the floor is assigned one subset of points on the floor, namely the set consisting of all points that can be seen from that point of view in some of the mirrors. Some points are situated in such a way, that they can see themselves mirrored. The set of all the other, un-reflexive points cannot be seen in the mirrors from any point on the floor. For if there were such a point, it would have to be either reflexive or un-reflexive. Both alternatives lead to a contradiction. Therefore, such a point does not exist.

As seen from Russell's perspective, this proof *must* be wrong. Why so? Because the set of "things altogether", i.e. the set of anything that is fully accepted in his unrestricted realist ontology, must be the greatest of all possible sets.

This asks for some explanation. Russell makes use of the notion of *entity*. An entity is "whatever can be counted of one" and can occur as subject of a true or false proposition. In other words: entity is whatever is such, that some things (propositions) about it are true or false. Nothing can be excluded by this criterion. For if there were something about which nothing is true or false, then at least something would be true of it, namely that nothing is true or false about it.

Although each proposition *is* one entity (for different propositions are true or false about it), it needs not be *about* one single entity. For example, the proposition that Brown and Jones are two, is neither exclusively about Brown, nor exclusively about Jones, but about Brown and Jones. Nevertheless, although they do not occur together as one entity in *this* proposition, they can do so in *another* one, such as "The set consisting of Brown and Jones *is* one and *has* two members".

Consequently, sets, or 'collections' as Russell calls them, play an important role in his unrestricted realism. Although they need not occur as one in propositions about them, they *can* and consequently they are to be ranked among entities. Therefore, the set of all entities must, although it is itself one single entity, contain among other things all possible sets as its members. And any set must, of course, be a set consisting of entities and, in virtue of that, be a subset of the set of all entities.

1.6   Discovery of the paradox

At least half a year later, in June 1901, Russell makes an attempt to elaborate the proposed refutation of Cantor's proof. In order to do so, he makes the following arrangement. To each entity that is not a set, one set of entities is assigned, namely the set having that very same entity as its sole member. To each entity that is itself a set, that very same set is allocated. For example, to the colour sepia, which is an entity but not a set, the set having the colour sepia as its only member is assigned. To the set of all prime numbers, which is an entity, the very same set of all prime numbers is assigned.

Although this mapping is not one-one, it must be surjective. No set can be left out. Nevertheless, according to Cantor's proof, there *must* be such a set. Which one would it be? The set of all the entities that are un-reflexive in the setting of Russell's arrangement. The colour sepia for example does not belong to it, for it is reflexive. Neither does the set of all abstract entities, for it is reflexive as well. But the set of all prime numbers does, for, not being itself a prime number, it is un-reflexive. In short, the set that according to Cantor Russell must have forgotten, is the set of all un-reflexive sets, i.e. the set of all sets that are not members of themselves.

Now, according to Russell, this set cannot be forgotten. The all-inclusive set of all entities contains all possible sets and consequently this particular set as well. The entity to which this set is allocated, is actually there. It happens to be the very same set of all sets that do not contain themselves. Therefore, Russell seems to be right.

So, what is Cantor's argument in favour of Russell's having forgotten something? According to that argument, the supposed entity to which the said set is assigned, cannot exist. For both the assumption that such an entity is reflexive and the assumption that it is not, leads to a contradiction. But in this case, the so-called forgotten set, that even according Cantor exists (otherwise it could not be forgotten), and the non-existing entity to which it might be supposed to be assigned, coincide. The dilemma that was meant to prove non-existence seems now to be applied to something that actually belongs to Russell's unrestricted universe: the set of all un-reflexive sets. Consequently it takes the form of a paradox. Is that set itself reflexive or un-reflexive? If it were reflexive, it would be, in virtue of that, un-reflexive. If it were un-reflexive, it would, in virtue of that, be reflexive.

2 Critical remarks

2.1 Two general presuppositions in Russell's way out

Russell's way out of the paradox is very complicated. Nevertheless it is based on two rather natural assumptions. Firstly that his ontology is *too wide*, the set of all entities is too great. It has in some way or other to be pruned without violating too much the main principles of Russell's realism. Secondly, that *reflexivity is suspicious*. It involves a vicious circle that is to be avoided, of course in as natural a way as possible.

Russell combines these two points of view in introducing different types of objects that cannot be counted as one. Furthermore he reduces the realm of entities (that are all of the same type!) in banishing all those that may involve reflexivity, such as sets, propositional functions and propositions. A lot of philosophical work was needed to justify this reduction. Especially the claim that propositions are not entities, asks for a rather radical change in the theory of truth.

Although Russell's final *Ramified Theory of Types* is based on the said two assumptions, it cannot be deduced from them. It is quite remarkable, that the young Wittgenstein, who disliked Russell's solution very much, proposed a radically different way out that is nevertheless based on the same presuppositions. According to the *Tractatus*, sets are non-

entities and all propositions about propositions are impossible. All reflexivity is banished to the realm of the unsayable.

2.2 Weakness of the first assumption

Although I am not able to provide an elaborate alternative solution, I'll give at least some hints that may possibly lead to it. Anyhow, my view is rather unconventional. It is based on not accepting the presuppositions mentioned before. In this section, I'll call in question the first one.

As expounded in 1.3, Russell's unrestricted realism is opposed to the rather repressive logic and metaphysics of substance. The key concept of that logic is being *in*. According to it, everything is either a substance or *in* a substance. Universals are only 'ideal' i.e. conceptual. In reality everything is private and individual. Red *only* exists as this red of this red thing or that red of that red thing. As far as the realist Russell is opposed to this exclusive, Aristotelian notion of being-in, I admire him. Its exclusiveness deserves to be excluded.

But in fact, Russell excludes more, namely the Platonic notion of being-in as well. This is the reason why his so-called Platonism is quite different from Plato's; it is *external*. As applied to what seem to be different red things, this means that the colour they share is only *outside* them. In other words, there is no participation, no exemplification and,eventually, no sharing at all. Russell is proud of wholeheartedly welcoming the entities that might be supposed to be universals. But he forgets to welcome their being universals. The same holds in my opinion of relations. How can they relate if they only exist outside related things?

It is true, Russell does actually accept a certain form of being-in, namely occurring in a proposition. If A, B and C are red, the very same entity, say the (extra-mental) concept 'red' or the propositional function 'x is red', occurs in different propositions, namely in 'A is red', 'B is red' and 'C is red'. But as long as red is supposed to be both outside A and B and C, it hardly deserves to be called universal. Russell's Platonism is essentially anti-Aristotelian. It consists in replacing the substantial *only in* by *only out*. This opposition seems to be contradictory, but is in fact contrary.

The failure to see this, is in my opinion mainly due to the failure to see being universal as one pole of an opposition. Universal and particular cannot be opposed to each other, unless they are so as forms of one and the same something (see 0.2). *The* universal 'red' may be opposed to *an* individual red thing. Of course the said universal is not a particular red thing. Nevertheless it is something particular, namely a particular colour, or a particular universal. The opposition only holds as far as the universal is universal *in* the very same something in which the individuals are individuals. Otherwise there could not be any universality at all.

2.3 Weakness of the second assumption

What is just said about universal and particular, namely that they are to be acknowledged as counterparts of one and the same something, also applies to reflexive and un-reflexive. These key words, occurring both in Cantor's proof and in Russell's paradox, mark an opposition. And in my opinion, as such, i.e. as counterparts, they constitute the very heart of the problem. The paradox cannot be adequately solved by playing off un-reflexivity as virtuous against reflexivity as vicious.

There is nothing vicious about reflexivity itself. Running around the vicious circle begins as soon as (un-)reflexivity is *squared*. Something cannot be reflexive or un-reflexive unless it is so in something which is different from both and is liable to assume each form. Being

reflexive or un-reflexive in reflexivity or un-reflexivity is vacuous. It is like writing a book about nothing but myself as being or not being the same person as the one my book is about.

2.4 Grelling's paradox as an example

Most conspicuously, this is revealed by the variant of the paradox put forward by Grelling. It is quite significant that this one fails in the list of contradictions mentioned by Russell in *Principia*. Grelling's paradox runs as follows:

Some words, especially those that may be used to classify words, are such that they *are* what they *say*, others are not. The first group may be called reflexive or 'autological', as Grelling prefers, the second group un-reflexive or 'heterological'. In order to start the game, we may choose two standard examples, say "noun" which actually is a noun and "monosyllable" which evidently is not itself a monosyllable. After having mentioned dozens of other examples and probably also some doubtful cases, such as the word "interesting", we get used to the words "autological" and "heterological". Then the game takes an unexpected turn. One of the players brings in a quite interesting word, namely "heterological". How is it to be classified? It cannot be autological, for if it were, it would be, in virtue of that, heterological. It cannot be heterological either, for if it were, it would be, in virtue of that, autological.

The conclusion seems to be quite simple. Just as the word "interesting", the word "heterological" is a dubious case. It neither belongs to the high society of impeccable autological words such as "noun", nor to its counterpart, the high society of impeccable heterological words such as "monosyllable". But evidently, the word "heterological" is more interesting than words such as "interesting". For we may highlight the problem by introducing the following definition: a word is heterological if and only if for some reason or other it does not belong to the high society of impeccable autological words. Then, we are forced to conclude that "heterological" is heterological. Indeed, but in my opinion, we are *not* forced to rank it, on that account, among the members of the high autological society. Not passing its ballot, cannot be a reason for accepting "heterological" as one of its members. Consequently there can be no reason either to forbid or avoid such a ballot.

2.5 Reflexivity as an internal opposition

In ordinary language, reflexivity appears as a predicate containing words like "re", "auto", "sui", or "self". In mathematical and logical symbolism, reflexivity appears in a different way: as a repetition of the same symbol, such as "a", in "aRa" or "F" in "F({xFx})". That the same symbol occurs twice, is made possible by its relation to one or more other symbols that are not repeated. The symbol "a" stands to the left and to the right of "R", the symbol "F" outside and inside the brackets and the bound variable.

This spatial difference in position vis-à-vis the symbol that itself is not repeated, also *means* something. It means an internal opposition, a formal difference that is not annihilated by numerical identity. When someone writes a book about someone else's life, there is an evident difference between the describer and the described. But when someone writes an autobiography, this difference remains, although the describer and the described happen to be one and the same person. Reflexivity can only be fully acknowledged , if it is accepted that, although it is impossible for something or someone to be something or someone else, it is possible for something or someone to be *different from itself.* Russell's final refusal to accept reflexivity is intimately connected with his refusal to accept such an internal difference.

In connection with the paradox, it is quite important to acknowledge that in symbolism reflexivity is expressed by one and the same symbol, occurring twice in different positions relative to a symbol symbolizing that *in* which the reflexive is reflexive. It is important because it forbids imitating ordinary language in trying to express reflexivity (or its opposite) by means of a one-place predicate. As soon as this is accepted, it is, as far as I can see, impossible to symbolically express the paradox.

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[i] Such an analysis is based on the Aristotelian principle that the more universal only exists in its less universal specifications. A 'generic' universal such as 'coloured' is supposed to have no other being than being embodied in specific ways of being coloured, such as being red. This principle is denied by Russell. See Russell 1903 a, p.138: "Redness, in fact, appears to be (when taken to mean one particular shade) a simple concept, which, although it implies colours, does not contain colour as a constituent."