Practices of mathematization – Fokko Jan Dijksterhuis - University of Twente

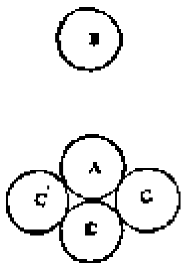
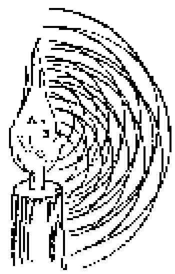
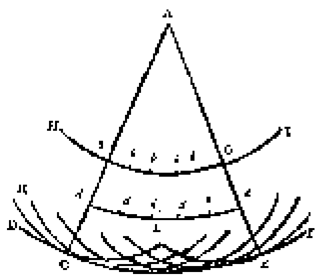
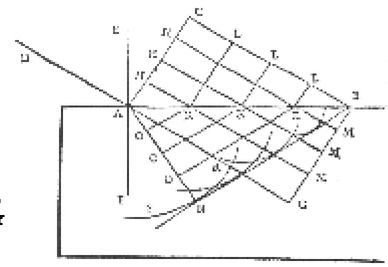
Once upon a time there was a grand narrative. In the sixteenth and seventeenth centuries the Scientific Revolution took place in which Copernicus, Kepler, Galileo, Descartes, Newton and the like mechanized the world-picture. Mathematics figured prominently: the protagoniststs of the Scientific Revolution created a mathematical science of nature. As it goes with myths of creation, the grand narrative of the Scientific Revolution has been superseded by historical scholarship that focusses on practices rather than ideas, on trust rather than truth, on historical context rather than universal wisdom. Timeless heroes are out of vogue; the direct connection between Galileo's and Newton's achievements and modern science is being questioned. With the grand narrative the theme of mathematization also seems to have disappeared from the history of early modern science. This is unfortunate, because mathematization in my view is an epistemic phenomenon of utmost historical and philosophical importance.

The grand narrative of the Scientific Revolution as elaborated by historians like Koyrè and (Eduard Jan) Dijksterhuis is founded upon an essentially platonistic understanding of mathematics and its relationship to reality. In short: a mathematical world-picture was created – an 'universe de la precision' as Koyré called it – from which phenomena were subsequently analyzed and described by means of geometry and, later on, calculus. Mathematics is understood as an abstract entity existing in a autonomous, ideal realm. The use of mathematics in science and (eventually) technology presupposes a mathematical metaphysics and comes down to the application of ideas and insights from this mathematical realm to concrete objects of nature and artifact. As an historian I find this understanding of mathematization troublesome as it does not seems to reflect the historical processes and events in which new mathematical physics (and eventually technology) were created. Moreover, I think that such a platonic understanding of mathematization does not help to understand how mathematization took place and its meaning for modern science and technology.

In this paper I will start with two examples of mathematization in early modern science. Mathematization is understood as the use of mathematics in new domains were mathematics was not used previously. I understand this spreading of mathematical practices quite broadly: it can be the use of mathematics in the inquiry of natural phenomena – as in the cases discussed – or in technologies, in educational, professional, cultural domains. After having discussed my cases, I will ask how the processes of mathematization encountered may be articulated philosophically, and what help philosophy of mathematics may give. This paper is part of a research project, *The Uses of Mathematics in the Dutch Republic*, that studies the cultural history of mathematics between ca. 1580-1750. It is aimed at developing a new historical perspective of mathematization in early modern science and technology. To that end we study the practices that gave rise to the increased use and valuation of mathematics.

Huygens, waves of light

Huygens' wave theory of light was a unique instance of mathematization in seventeenth-century science as it concerned the invisible world of the particles in motion of mechanistic philosophy. Whereas the observable realm of motion was being fully explored mathematically by the likes of Galileo, Torricelli, Huygens himself, Newton, and so on, the underlying realm of microparticles that should explain phenomena like light, gravity, magnetism largely remained a domain of qualitative reasoning. Huygens was virtually the only one up until the late eighteenth century to integrate this domain of natural philosophy with mathematics. His principle of wave propagation stipulates the mathematics of the motions that he saw responsible for light and its properties. Given the characteristics of wave propagation and the effects of varying media, the explanation of reflection, refraction and the so-called strange refraction of Iceland crystal becomes a matter of geometric construction that does not require additional physical considerations.

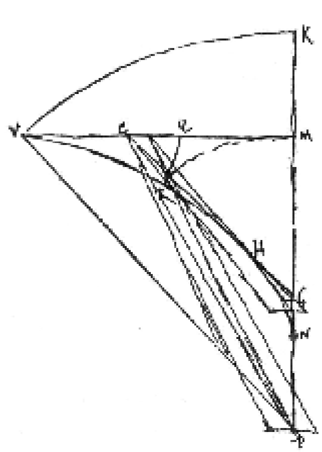
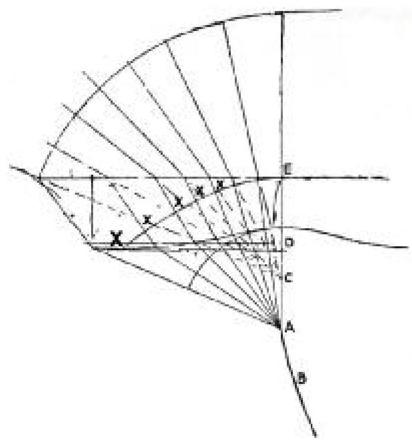
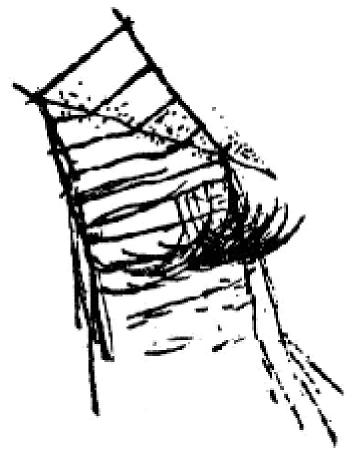





Huygens published his wave theory in 1690 in *Traité de la Lumière*, but the theory and the manuscript had been largely finished in 1678 when he read it to the Académie Royale in Paris. He had elaborated the core of the theory – the principle of wave propagation and the explanation of strange refraction – in a short period in 1677, preceded by an initial exploration and problem definition in 1672. In order to find out how Huygens developed his wave theory one should not turn to *Traité de la Lumière*.

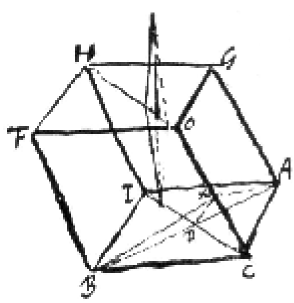
In his published theory Huygens started from the natural philosophical basis, explaining that light must be the propagation of some kind of action through a material ether rather than an actual transport of particles. (the pictures above) This action is the impact of violently moving particles of a luminous source transferred to the particles of the ether. (first picture) Huygens then explained, on the basis of his knowledge of colliding (billiard)balls, that this action propagates spherically through the ether as waves with a constant speed depending upon the nature of the medium. (second picture) The crux is the idea (third picture) that each part of the wave B*bbbb*G is the source of a wavelet KCL and that the propagated wave DCEF is created where wavelets from a multitude of sources coincide. Having established these natural philosophical foundation, Huygens went on to explain reflection, refraction and strange refraction by geometrical constructing propagated waves when waves hit a impenetrable medium, a medium of different density, or an inhomogeneous medium like Iceland crystal. For refraction (fourth picture), Huygens supposed that waves travels somewhat slower in glass, distance OO instead of LL in the air. Drawing the tangent of wavelets SNR successively produced when a plane wave hits the surface of the glass at AKKKB, the propagated wave NB is found. He then proves that the sine law derives when rays DA and AN are regarded.

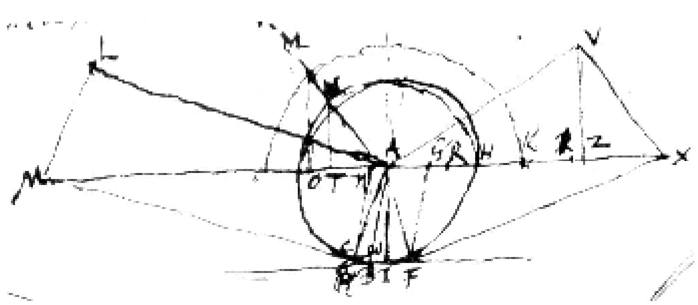
The line of reasoning in *Traité de la Lumière* perfectly fits the idea of mathematical physics as a form of mathematized natural philosophy. For this EJ Dijksterhuis has hailed Huygens as the first true Cartesian, who managed to turn mechanistic philosophy into mathematical physics whereas Descartes had stuck with a mathematical ontology. Wonderful, but as said: to find out how Huygens developed his wave theory one should not look at *Traité de la Lumière*. One should not even look at the *Oeuvres Complètes*, in which Huygens' writings and manuscripts have been published. The manuscripts related to the wave theory have been edited so heavily that Huygens' original line of reasoning is virtually unretraceable. To find out how Huygens developed his wave theory one has to go to the original manuscripts. These consists of a series of some ten pages of loose sketches between the analysis of a sophisticated problem of rays traversing lenses and the explanation – seemingly out of the blue – of strange refraction.[i]

The problem Huygens was working on in the summer of 1677 regarded so-called caustics, the bright curves arising when for example sunlight falls on a glass of water. Huygens could not reconcile such caustics with the theory of light waves he had adopted five years earlier. His Paris acquaintance, the jesuit father Pardies, had explained the phenomena of light in terms of waves and the cornerstone of this theory was that rays of light are the direction of propagation of these waves. Consquently, rays should always be normal to waves. However, the intersection of rays refracted by a curved surface that produced caustics was problematic to Huygens. (the left two pictures below) He could not figure out what kind of wave might be produced after refraction. It was a sophicasted mathematical problem and Huygens struggled with it accordingly. He meticulously constructed refracted rays and propagated waves in several lenses and then found out that one should not try to follow the wave as it traverses various media; that is: as if the wave were some kind of coherent whole. Instead, one should from one instance to the next, and from one point to the other, examine how the action giving rise to the wave behaves. (XxxxxE in the second picture) In order to do so he constructed refracted rays, the distance over which the action propagated, and then the ensuing wave. This is done by drawing the ‘common tangent curve of all particular waves’ as Huygens noted, making clear that he used an insight he had only noted in a tiny sketch (the picture on the right, terribly out of proportion in comparison to the other two).

This is the principle of wave propagation and at this moment it was a purely mathematical artifact aimed at penetrating the seemingly paradoxal phenomenon of caustics. Which in its turn was a problem of mathematics, of rays and waves being normal or not. The next manuscript pages slowly progress through sketches of ellipses and refracted rays to the solution of strange refraction: an elliptical wave IFH formed after light is refracted at the surface of Iceland crystal. (right picture below) Refraction in Iceland crystal is called strange because it does no follow Snel’s law of refraction: a perpendicularly incident ray is refracted and some oblique rays pass unrefracted. (left picture below) Originally, it had also been a problem with Pardies’ theory: how can a perpendicular ray be refracted if a wave falls flat on the surface? (second picture, originally a tiny sketch) The elliptical wave solved the problem. It arose because in Iceland crystal light propagates with varying speeds in different directions. With the principle of wave propagation – still nothing more than a sketch – the propagated wave was constructed and turned out to be elliptical. And indeed rays AF are not normal to the wave FX, but this was no problem anymore because normality had been replaced as a constructing principle by the wavelet of the principle of wave propagation.

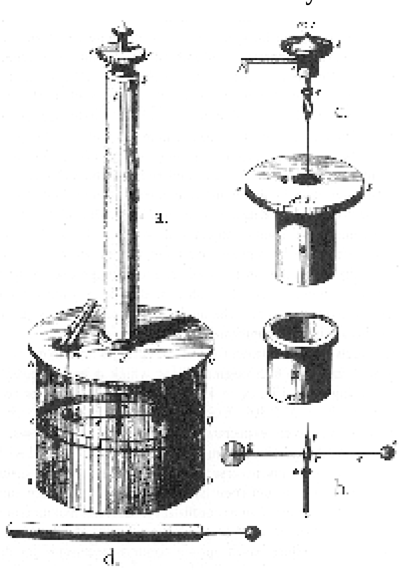




Without going further into the details of Huygens' brilliantly ingenious elaboration of the mathematics of light waves, I would like to make my point. The mathematization of the nature of light that Huygens brought about here was a matter of formulating and solving mathematical problems. More specifically, problems in geometrical optics as Huygens had pursued since his debut twenty-five years earlier. Problems of rays, lenses, etcetera had occupied him and he was well-versed in the science of optics. His analysis of strange refraction and the propagation of waves through lenses was an extension of this, with the only difference that it did not only concern visible rays but also invisible, hypothetical waves. But it were problems of mathematics, rather than physics. Huygens would spell out the physics of light waves only later on in his presentation to the Académie Royale and in *Traité de la Lumière*. And even then his natural philosophical considerations were highly functional and pragmatic instead of systematic. In other words, Huygens did not even prepare the physical ground before elaborating the mathematics of his wave theory. Not to mention the metaphysical ground, which he largely ignored throughout his life. Not your 'true Cartesian', I would say. Rather than saying that Huygens integrated mathematics and natural philosophy, it would be more correct to say that he extended mathematics – the mathematical science of optics – to the natural philosophical realm of invisible matter in motion.

Coulomb, measures of force

My second example is Coulomb's 1780s measurements of magnetic and electrostatic force. It is an intriguing episode as scholars like Musschenbroek and Lambert had been expressly looking for such a law since Newton's time. In his history of electricity in the eighteenth century, John Heilbron argues that Coulomb was the first to apply the newtonian doctrine to electricity and magnetism, which can be summarized by saying that he understood that the effects of forces ought to be reduced to elementary forces.[ii] I agree with Heilbron, yet there is a conspicuous blind spot in his account. Heilbron does not explain how Coulomb's successful approach came about. By looking at the origins of Coulomb's experimental set-up, I think we can understand how he achieved this particular instance of mathematization. Crucial is the way Coulomb elaborated his command of the newtonian doctrine. Rather than in theory, in the form of a mathematical model like Sir Isaac had done in the case of gravity, Coulomb translated his understanding of elementary force points into a material set up: the torsion balance that almost directly measured the effects of elementary forces.



Coulomb's experimental set-up was highly sophisticated. (left picture above) In the case of magnetism it featured two thin, two-feet needles of excellent steel, carefully magnetized so that the poles were precisely located (at 5/6 inch from the extrimity). One needle was suspended on a silk wire horizontally in its magnetic meridian, the other put vertically in the meridian repelling the other one. Twisting the wire turn after turn, Coulomb could determine the force of repulsion (taking into account the effect of the earth's magnetic field). In the case of electrostatic force, Coulomb used a hair stiffened with wax and a gilded elder seed. The measurements confirmed the inverse square law Coulomb had supposed.

Coulomb was able to create this set-up and execute the measurements because he already had a thorough knowledge of the earth's magnetism as well as torsion. He determined the torsion of the wire and knew the relationship between the angle of twist and the force exerted. In fact, the torsion balance experiment was a reversal of an instrument Coulomb had deviced some ten years earlier and that embodied his knowledge of magnetism and torsion. In 1777 he had competed for a competition of the Académie to determine 'the beste way to make magnetic needles, mounting them, ascertain they are in the true meridian and finally explain their daily variations'. Coulomb won (shared) with a proposal for suspending magnetic needles from a thread rather than mounting them on a pivot. He explained that this yields a more precise instrument, less subject to friction and air resistance. He explained the effect of the torsion of the thread and reported on his measurements of magnetic variation.

Coulomb had executed his work on the torsion compass in Cherbourg – far away from Paris – where he was stationed as engineer. His new compasses were installed at the Paris observatory in 1780 and Coulomb himself followed the next year. He had moved from engineer to savant: he was promoted, decorated and had become a member of the Académie. In the 1780s he continued his research of torsion in order to overcome difficulties when the compass becomes magnetically charged. Then he turned his perspective around and built his torsion balance to measure electricostatic and magnetic forces. Coulomb read his memoirs on electrostatic and magnetic measurements to the Académie in 1785 and 1787.

So, Coulomb was well-equiped to mathematize electrostatic and magnetic forces. He had exact knowledge of magnetism and torsion and he had an instrument capable of performing the measurements. Yet, and now come to my central point, Coulomb was equiped in a very particular way. The instrument literally embodied his understanding of magnetism and torsion, in the set-up, the choice of materials, the measuring process. And his knowledge of torsion and magnetism was closely connected to these same materials and artifacts, the needles, wires, as well as the surrounding magnetic field. Coulomb had a specific approach to the problems of compasses, magnetic forces, and so on, in which the materiality of the matter was both the starting point and result. The result was an artifact expressing the so-called newtionan understanding of elementary forces; the starting point was the exact investigation of the properties and effects of concrete objects like wires, needles and so on.

I would argue that this approach arose from his training and experience as an engineer. The theory of torsion is particularly significant. Torsion was a hitherto unexplored phenomenon and is a typical kind of subject for an engineer as it concerns the properties of concrete materials. In his engineering work Coulomb had already built up ample experience with these kinds of questions – the supporting power of constructions, the optimal incline of a bridge, etcetera. He had practiced the typical engineering way of investigating concrete materials, experimentally and mathematically. Thus equipped he mathematized electrostatic and magnetic forces, in a way that even surpassed Sir Isaac's methods.

Reflection upon ‘mathematics’

These two examples from the rise of mathematics in science – examples that I believe to be exemplary – make clear that mathematization did not start out from some mathematical world-view elaborated in advance. For both Huygens and Coulomb their mathematical understanding of the ‘world’ was implicit. Philosophical reflection was a justification afterwards at the most – apology as Levinas characterizes philosophy in general. With Huygens and Coulomb it remained so; they never cared to go into the metaphysics of mathematical physics. In this regard they were different from their contemporaries, like Newton, Leibniz, Laplace, who went on to inquire into the (natural) philosophical, epistemic and metaphysical consequences of their findings.

The examples of Huygens and Coulomb show that, rather than working up from some mathematically prepared ground, mathematization consisted of transferring mathematical practices to new domains of inquiry. Such a transfer is not a mere translation, but a transformation of both the original practices and the new domain. Huygens transposed his learning in geometrical optics to the mechanistic nature of light and the causes of reflection and refraction; transforming it into the construction of hypothetical models of ether interactions. Coulomb applied his engineering skills of materially analyzing, designing and experimenting to, first, instrument making and, then, the measurment of natural forces; transforming, on the one hand, his inventive practices into inquisitive ons and, on the other hand, a theoretical understanding of nature into a material, instrumental model.

Transfers of practices also entail transfers of knowledge claims. Early modern mathematics, natural philosophy, engineering, each had their own conceptions of the truth, range, foundation and purpose of knowledge and a transfer may also imply the introduction of foreign conceptions. In his wave theory, Huygens developed quite a novel conception of natural philosophical truth, privileging comprehensibility rather than certainty. Coulomb’s example may illustrate the intermediate laws typical of modern, robust mathematical physics.[iii]

The mathematical analysis of light, electricity and magnetism are generally considered applied mathematics. Yet, the concept of ‘applied mathematics’ does little to clarify the transformations realized by Huygens and Coulomb. ‘Applied mathematics’ presupposes a ‘pure mathematics’ to be a given in a separate, abstract realm, that is ‘applied’ to natural and artificial phenomena, thus yielding an specific interpretation of the mathematical structure. Even if application is re-interpreted as a transformation of some body of mathematics into a mathematics of concrete objects, the notion of a ‘pure’ mathematics to be essential – and to be had in an a priori pure form – stands in the way of grasping what mathematical inquiry may be. Huygens and Coulomb did not start ‘out of the blue’, creating mathematical structures to grasp the kinematics of ether particles or the dynamics of forceful attraction. Rather, they used mathematical structures from adjoining domains (of mechanics, construction, compasses) to explore and establish mathematical regularities. Two points are important here: the practical nature of the use of mathematics (with the conceptual understanding suspended) and the concrete nature of the domains between which transfers happen. Elsewhere, I have argued that in mathematics in particular the distinction between contemplation and action is difficult and that in fact construction (by artifact, by hand, by mind) *is* mathematical inquiry.[iv] As regards the other point it is important that the notion of an abstract mathematical entity hovering above or behind light, electricity and magnetism is hard to be found with Huygens and Coulomb. A propagating wave and an attracting magnet are mathematical objects; the principle of wave propagation and the torsion of magnetized hairs mathematical structures. Huygens and Coulomb created new mathematical objects while inquiring into the mechanics of light and the measurement of forces respectively.

This point of the materiality of mathematical objects is elucidated by turning to early modern conceptions of mathematics. Besides being conceptually problematic, the concept ‘applied mathematics’ is historically questionable. The term ‘applied mathematics’ was invented in the nineteenth century, in the wake of the establishment of ‘pure mathematics’ and in an expressly subordinated relationship to the latter.[v] Huygens, Coulomb, and their contemporaries did not know the idea of applying mathematics and our modern concept of ‘pure’ mathematics was foreign to them. Prior to Lagrange’s and Cauchy’s rationalist purification of mathematics around 1800, the whole range of mathematical sciences – geometry, arithmetic, astronomy, music, optics, and so on – were seen as *parts* of mathematics. Eighteenth-century encyclopedias formulated it as follows:

> “Mathematics are distinghuish’d with regard to their End, into *Speculative*, which rest in the bare Contemplation of the Properties of Things; and *Practical*, which apply the Knowledge of those Properties to some Uses in Life. ... With regard to their Object, *Mathematics* are divided into *pure* or *abstract*; and *mix’d*. Pure *Mathematics* consider Quantity, abstractedly; and without any relation to Matter: Mix’d *Mathematics* consider Quantity as subsisting in material Beings, and as continually interwove.”[vi]

Notice the plural of mathematics, a field of various mathematical sciences ranging from geometry and arithmetic, to astronomy, music, optics, and to surveying, navigation, book keeping, to name a few.

Two aspects of this stratification of mathematics are important. In the first place they denoted a division of mathematics, not a dichotomy between various levels of mathematicalness as the modern conception of ‘pure mathematics’ implies. All branches where considered as parts of a coherent field of mathematics. Secondly, branches of mathematics where identified on the basis of their ends as well as their objects: speculative versus practical, and pure versus mixed, respectively. Note that ‘pure’ was a signifier irrelevant to issues of utility. Only in the nineteenth century were practical and mixed conflated into applied and was purity (abstractness) contrasted with utility.[vii] Afterwards the concept of ‘pure mathematics’ – the essence of mathematics lies in the rational study of abstract structures – was projected backwards onto preceding mathematics, up until ancient times.

For my argument the concept of ‘mixed mathematics’ is most interesting. The concept of ‘mixed mathematics’ goes back on ancient divisions of mathematics by Pythagoras, Geminus (as discussed by Proclus), although it was coined as such only during the Renaissance by, in the first place, Jesuit mathematicians. It was a division on the basis of the object of study. Geometry and arithmetic studied quantity as subsisting in itself; astronomy, musica, optics studied quantity as subsisting in matter. The mixed parts were not separated, let alone subordinate to the pure parts of geometry and arithmetic: mathematics consisted of the study of reality in its quantitative aspect. Mixed mathematics was the study of concrete quantity, that is: the investigation of the quantitative aspects of natural phenomena. In optics for example, the geometrical aspects of light were studied – its rectilinearity, its regularities – leaving its physical nature out of consideration. The accompanying concept was ‘mathematica pura’, which should not be equated with pure mathematics because it just as well dealt with real quantity, the difference being quantity existing in the mind relating to thought objects, quantity per se.

From this viewpoint mathematics consists of the investigation of quantity in various domains that can be more or less material (or natural, or practical), but is not necessarily structured hierarchically according to levels of abstractedness. Mathematization then is the production of mathematical objects and practices in new domains of inquiry, whereby existing skills and insights are freely borrowed, be it from construction practices, physico-mathematical results or – and this is the point of my argument – abstract mathematics. I would even suggest that pure mathematics is nothing more than such a domain of inquiry, a specific domain invented around 1800 when Lagrange, Cauchy c.s. cut the ties with observable – be it by the body's or by the mind's eye – reality and resurrected mathematics from purely rational foundations. When viewed as another domain of mathematical inquiry, in modern pure mathematics mathematical objects are considered in a specific way, namely as formal, intellectual constructs.

Mathematization and the philosophy of mathematics

How can such a conceptualization of mathematization – and of mathematics by the way – as the production of mathematical practices, be articulated philosophically? Philosophical literature on mathematics is vast, but analyses of mathematization are rather scarse.[viii] Husserl shows that platonic and cartesian metaphysics make a concept of mathematization superfluous, as they regard reality as essentially mathematical. He offers a subtle analysis in which he distinguishes between Gestalt and Fülle, where the former can be the starting point of mathematization. However, as Fleischhacker has shown, in the end Husserl too maintains a mathematical ontology (reality supplying mathematical objects) assuming time, space and causality to be mathematical. Fleischhacker argues that quantity needs always realization. Reality offers starting points for abstraction but the act and the result of it do not derive from reality. Building upon Fleischhacker, Alberts has articulated mathematization, emphasizing that the product of this act (the mathematical model) cannot express (the possibility of) mathematization.[ix]

Alberts' account of mathematization is searching and illuminating. He rightly points out that historians of the Scientific Revolution tended to adopt a platonic/cartesian conception of mathematics, thereby eluding the question how the mathematization of nature took place. Alberts makes clear that mathematization is an act and that the approach to reality that is presupposed in it, needs explication: how is the ground on which mathematics is pursued prepared? However, Alberts' philosophical account conceives mathematization as an ultimately intellectual act and thus we are back with the understanding of mathematization in the grand narrative of the Scientific Revolution: mathematization is the creation of a mathematical world-view.

My understanding of mathematics and mathematization turns out to be rather oblique regarding current philosophical literature. Philosophy of mathematics is primarily aimed at articulating the nature, content and reach of mathematics with regard to reality.[x] Even Husserl and Fleischacker/Alberts are ultimately interested in this question, defining mathematization as the preparing of knowledge domains for the use of mathematics. I, on the other hand, consider the relationship between knowledge domains and human practices, rather than between ideas and reality. That is, what happens when mathematics is introduced in practices of inquiry and invention. Actually, I find the discussion over the relationship between mathematical structures and nature/reality rather sterile with regard to understanding the meaning and import of the use of mathematics in science and technology. The main problem is that philosophy of mathematics aims at unravelling the conditions of justified true belief. The questions that are central to my historical inquiry that focuses on the production of reasoning and practices, and transfers of practices within contexts of constructions, are thus not addressed.

Central in my conception are notions of creativity (rather than justification) and human activity. A promising source for articulating these conceptually is the literature on didactics of mathematics. Here the learning of mathematics is studied which entails transfers of mathematical knowledge. The theory of 'realistic mathematics education' in particular draws attention to the creation of mathematical structures by practical activities and transfers between domains.[xi] It emphasizes the common sense basis of producing new mathematical understanding and mathematization as the mathematical organization of unmathematical matter. Crucial to creating mathematical understanding in their view is the use of established mathematical understanding in new domains. This is precisely what we have seen happening above. Huygens' and Coulomb's achievements are thus comparable to the process of learning mathematics, which Freudenthal not accidentally saw as the 'guided re-invention' of mathematics.

Didactics of mathematics also draws our attention to the polymorphicity of mathematization. I may have suggested that mathematization is a homogeneous process of which numerous examples throughout history can be given. Realists in mathematics education are convinced that learning of mathematics can only take place on the basis of known, imaginable problem situations. However, these are not only real life situations, but can also be formal worlds of symbols and even fantasy worlds. Furthermore, mathematization can take place on different levels, ranging from informal context-connected solutions, to schematizations and generalizations on more abstract levels, whereby mathematization on one level can be subject of inquiry on another level. Treffers has specified this, by distinguishing two dimensions of mathematization, horizontal and vertical. Horizontal is the use of mathematical knowledge and skills in context situations; vertical is aimed at symbollically formalizing and structuring. The examples given, and the historical process of mathematization I have focused on, can thus be regarded instances of horizontal mathematization.

Lenhard and Otte have also discussed historical instances of mathematization and they too emphasize its heterogeneity. They identify two types of mathematization: ontological and methodological.[xii] The first is top-down, and derives its certainty from earlier principles of hypotheses; the second is bottom-up, and derives its certainty from the methods and constructions of mathematics. Or, using Leibniz' phrasing: characteristica universalis being a symbolic structure that reflects the world of our concepts and calculus ratiocinator being the symbolic manipulation mirroring human reasoning. They follow this dichotomy through history, showing that these two conceptions of mathematization informed debates on the use and status of mathematics from Leibniz-Newton, to Grassmann-Ampère and Wiener-Neumann. This is an enlightening analysis, although their focus eventually is on the nature and epistemic status of mathematical models.

Many mathematics

When I speak of transfers between knowledge domains, I loosely use Kuhn's idea of traditions in the history of science. A knowledge tradition can be said to be made up of shared ideas of goals, methods, criteria of knowledge production, as well as institutions and other social structures carrying them. Kuhn showed how the existence of such traditions, and in particular persistent gaps between them may explain some conspicious features of seventeenth-century science, in particular the rarity of quantitative experimentation. However, Kuhn only discerned two traditions: mathematical and experimental. Hakfoort has argued that the development of optics can only be properly understood by introducing a third, and historically notable, tradition, that of natural philosophy. I in my turn, would add at least another tradition, namely that of engineering (or the crafts), but I won't go into the details of argument. Suffice to say that Huygens' wave theory can be understood as a bridge between natural philosophy and mathematics, and Coulomb's measurements as a synthesis of experimental philophy and engineering. Mathematization then is the introduction of mathematical practices in traditions that previously did not include the use of mathematical, like experimental philosophy prior to Newton. In the case of Coulomb there is also the transfer of the actor himself, as engineer into the domain of science. The prize competition of 1775 was his ticket to the Paris Académie. Such social and cultural aspects of mathematization – the circulation of people, objects, etc – is an integral part of my understanding. In my project I study the cultural foundation of mathematization: what made mathematical practices valuable to adopt them in various circles. How did numbers become trusted? This, however, goes beyond the (philosophical) context of this paper.

Centre for Studies of Science, Technology and Society, University of Twente, P.O.Box 217, 7500 AE Enschede, Netherlands. f.j.dijksterhuis@utwente.nl +31 53 489 3318

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[i] I have elaborated this in detail in chapter 5 of my *Lenses and Waves* (Dordrecht, 2004)

[ii] Heilbron, *Elements of Early Modern Physics* (Berkeley, 1982), 65-89.

[iii] Chang, *Inventing Temperature*, 48-52.

[iv] Dijksterhuis, "Constructive thinking", 59-82 in Roberts, *The Mindful Hand* (Amsterdam, 2007)

[v] Alberts, *Jaren van Berekening*, 65-80.

[vi] Chambers, Ephraim, *Cyclopædia, or, An universal dictionary of arts and sciences : containing the definitions of the terms, and accounts of the things signify'd thereby, in the several arts, both liberal and mechanical, and the several sciences, human and divine : the figures, kinds, properties, productions, preparations, and uses, of things natural and artificial : the rise, progress, and state of things ecclesiastical, civil, military, and commercial : with the several systems, sects, opinions, &c : among philosophers, divines, mathematicians, physicians, antiquaries, criticks, &c : the whole intended as a course of antient and modern learning* The Second Volume (1728), 509. Alembert's *Encyclopédie* has a similar definition.

[vii] Mulder, "Pure, mixed and applied mathematics", *Nieuw Archief voor de Wiskunde 4de serie*, 8-1 (1990), 27-41. By the way: this went via chemistry where the term 'mathematica applicata' that arose in Enlightenment Germany was borrowed to distinguish (and promote) the newly invented *science* of chemistry from traditional chemical arts.

[viii] I do not discuss Steiner ("Mathematics – Application and Applicability" in Stewart, *Oxford Handbook of Mathematics and Logic* (Oxford, 2005); *The Applicability of Mathematics as a Philosophical Problem* (Cambridge, 1998)), because his analysis of applicability – however important - is not directly relevant to my argument.

[ix] Husserl, *Krise*; Fleischhacker, *Grenzen van Kwantiteit*; *Beyond Structure*; Alberts, *Jaren van berekening*, 25-28.

[x] In this respect I could have started with the classical contrast of Plato's conception of mathematical entities as autonomous ideas and Aristotle's conception of them as products of abstraction.

[xi] De Lange, *Mathematics, Insight and Meaning* (Utrecht, 1987); Treffers, *Three Dimensions* (Dordrecht, 1987); Freudenthal, *Revisiting Mathematics Education* (Dordrecht, 1991); Van den Heuvel, "Didactical use of models", *Educational Studies in Mathematics*, 54 (2003), 9-35.

[xii] Lenhard and Otte, "Two types of mathematization" (unpublished paper).