Radboud University Nijmegen

Faculty of Science

Analysis of the works by Simon Stevin

Analysis of Stevin's contributions to mathematics and physics

Thesis BSc Mathematics

Supervisor:
Riccardo Cristoferi

Author:
Tom Hegeman

Second reader:
Klaas Landsman

July 6, 2023

1 Acknowledgments

Words cannot express my gratitude towards my supervisor Riccardo Cristoferi and second reader Klaas Landsman. Both have greatly supported me in this endeavour by giving feedback, providing expertise and giving overall support. I also want to express my thanks to Fokko Jan Dijksterhuis and Floris Cohen who sent me interesting source material and gave me advice on the topic. Special credit also goes to Eduard J. Dijksterhuis and Dirk Struik, whose work laid the foundation from which this thesis could be constructed. I also want to thank my family and friends for their continuing support.

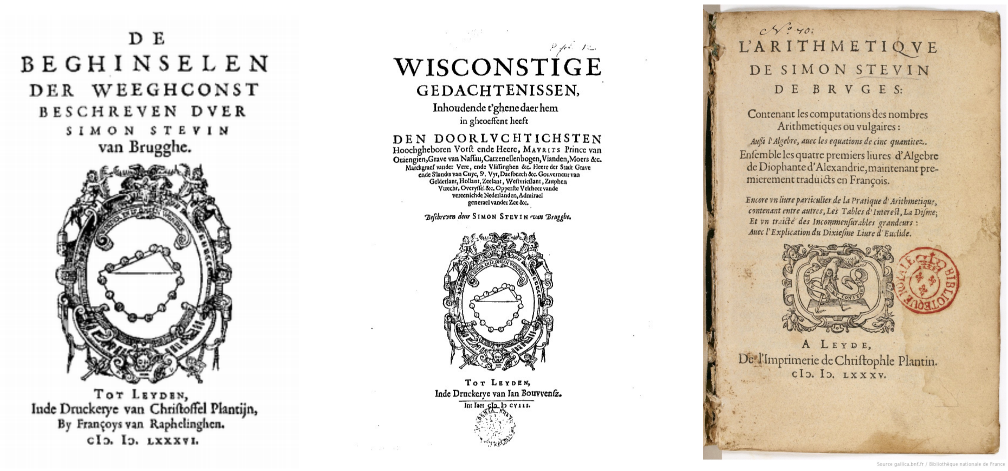
2 Abstract

Simon Stevin (1548-1620) was a Dutch mathematician and engineer, who studied and worked in a large variety of scientific fields. During his lifetime he wrote several significant scientific works on mathematics and physics such as *De Beghinselen der Weeghconst* (*Art of Weighing*), *Wisconstige Gedachtenissen* (*Mathematical Memoirs*) and *L'Arithmetique* (*Arithmetic*). This thesis has conducted a thorough scientific historic analysis of a selection of problems from these three works, which involve the static equilibrium on an inclined plane, the calculation of the circumference of an ellipse and the solutions to quadratic equations. With this research, this thesis sheds light on the contents and value of Stevin's contributions to the wider development of mathematics and physics. This thesis is also an example of the value of interdisciplinary research with regard to the fields of mathematics, physics and history.

Contents

3 Introduction

The topic of this bachelor's thesis is the analysis of the contributions made by Simon Stevin in mathematics and physics. Simon Stevin was a Dutch engineer, who lived at the end of the sixteenth and the beginning of the seventeenth century and spent the majority of his life in the Netherlands. Stevin made significant contributions to the previously mentioned scientific fields. The main works of Stevin include *De Beghinselen der Weeghconst* (*Art of Weighing*) [9], *L'Arithmetique* (*Arithmetic*) [8] and *Wisconstige Gedachtenissen* (*Mathematical Memoirs*) [10], where he included the description of a stable mechanical equilibrium on an inclined plane, the first general solution to the quadratic formula, and the methods to obtain the circumference of an ellipse. Furthermore, it is worth noting that Stevin introduced and popularised the Dutch term for mathematics, 'wiskunde', of which the etymological origin is unknown.



(a) (b) (c)

Figure 1: The covers of the three works which are discussed in this thesis. Cover (a) is the cover of the *Art of Weighing*. Cover (b) is the cover of the *Mathematical Memoirs*. Cover (c) is the cover of the *Arithmetic*.

Although Stevin achieved significant results in several scientific fields, the analysis of his works on mathematics and physics has been relatively scarce, especially when compared to similar noteworthy scientists such as Galileo Galilei in this period. The most in-depth research on the works of Stevin has been conducted in the *Principal Works of Simon Stevin*, of which several volumes have been published between 1955 and 1966. The contents of the first volume on mechanics [11] and the second volume on mathematics [12] will be discussed in this paper. The main contribution to the research surrounding Stevin has been made by Eduard Jan Dijksterhuis, who was one of the authors of the *Principal Works of Simon Stevin*. Dijksterhuis primarily worked on the first volume on the works of Stevin with regard to mechanics. Furthermore, he wrote a biography, *Simon Stevin* [3] (published in Dutch in 1943), and an overview of Stevin's scientific achievements, *Simon Stevin: Science in the Netherlands around 1600* [4] (first published posthumously in 1970).

The main focus of this research is the analysis of Stevin's research and comparing it with modern standards with regard to notation, structure, proofs and results. The analysis of Stevin's work is subdivided into two methods of scientific historical research: historism and presentism. To define these concepts we need to explain the notion of a *paradigm*. In *The Structure of Scientific Revolutions* [6] the philosopher Thomas Kuhn defined a paradigm as the frame of reference by which (physical) reality is described. This frame of reference is a conglomerate of axioms, theories, concepts, methods of proof, syntax and models. A paradigm is therefore the foundation on which the notion of science and scientific truth is defined. These paradigms shift throughout history as these features change over time. For instance a change of axioms or the creation of new concepts by which reality can be modelled results in a new manner in which science is conducted and scientific truth is perceived such that there is a shift in paradigm. This results in a new conceptualisation of science dependent on the paradigm in a particular time frame. Consequently, this raises the difficulty of analysis of results from previous paradigms as their presuppositions on the attainment of truth and knowledge differ from the paradigm in which the contemporary researcher operates. This seeming inability to compare these results is also described as being *incommensurable*. This issue of incommensurability leads to different methods in scientific historical research. This thesis applies the following two methods: historism and presentism.

Historism is the method in which scientific material is analysed from the perspective of the scientific paradigm in which it was first conceived. This method is applied by using the definitions and methods which the original author used to ascertain results. The main objective is to approach the perspective of the original author (and by extent the paradigm in which she/he operated). It should be noted that the historist approach always comes down to an approximation of the original perspective, as a contemporary perspective from which one operates cannot be fully disregarded whilst trying to conceptualise the historical context. This is primarily present in the analysis of the *discourse*. Discourse is the generalisation of verbal communication by which reality is perceived and interpreted. In Stevin's case, it amounts to the interpretation of concepts, the choice of wording, his notation and the structure of his texts. The sections on the historist analysis of Stevin's work also feature discourse analysis.

Presentism is the method in which scientific material is analysed using the contemporary paradigm. This approach translates the original concepts and syntax to the current scientific standards. This method is prone to misconception since several concepts and methods can be misidentified or misinterpreted due to the researcher's interpretation. However, it is useful to connect the work of Simon Stevin with the *status quaestionis* (current state of research) and to analyse his contribution to the development of various scientific fields.

The last crucial concept which applies in this thesis is the concept of *historiography*. It is vital to note that historiography has a rather ambiguous meaning, since its literal definition is 'the writing of history'. However, it is frequently used by historians to refer to the study of writing history, which researches the changing historical paradigms by which history is written. As for the history of science, the historiography is primarily concerned with the perception of scientific contributions by the scientific community in different periods of time (with accompanying paradigms).

To summarise, the following research questions will be discussed in accordance with their respective subjects:

1. Analysis of Stevin's work: What are the main scientific contributions of Simon Stevin in the *Art of Weighing*, *Mathematical Memoirs* and *Arithmetic*?
    - (a) Historist analysis:
        - i. What are the definitions and axioms from which Stevin operated?
        - ii. What kind of methods of proof did Stevin apply to acquire his results?
        - iii. What are the primary features of his syntax and notation?
    - (b) Presentist analysis:
        - i. What presentist methods are used to solve these problems?

ii. Are Stevin's methods still valid from the perspective of the current paradigm? What assumptions are required?

2. Historiography:

   (a) To what degree has Stevin contributed to the development of mathematics and physics?

   (b) How has he been regarded through different periods of time?

   (c) Why has there been relatively little research on Stevin's contribution to mathematics and physics?

A number of significant results have been selected to showcase Stevin's main scientific contributions and to illustrate the paradigm in which he operated. As for the *Art of Weighing*, the static equilibrium on an inclined plane is discussed (the so-called 'Clootcransbewijs') in Chapter 4. In Chapter 5 the different methods to calculate the circumference of an ellipse from the *Mathematical Memoirs* are analysed. From the *Arithmetic* a couple of geometric proofs to solve quadratic equations have been selected which are discussed in Chapter 6. Furthermore, the analysis of Stevin's work can also grant insight into the history of mathematical notation. Lastly, using the analysis of Stevin's work, one can formulate a conclusion on the content and value of his contributions to the fields of mathematics and physics.

It should be mentioned that the content for the presentist analysis in this paper is largely based on the definitions and methods published in the following works: *Physics for scientist and engineers* ([7]), *Calculus: a complete course* ([1]) and *Calculus of Variations I* ([5]).

Lastly, this thesis ends with a chapter on the historiography of Simon Stevin and his contributions in which the herefore mentioned questions are discussed.

3.1 Life of Simon Stevin

Despite the significant contributions of Simon Stevin the source material on his life is fragmentary, which makes it hard to determine undisputed facts on his life. Several authors such as E.J. Dijksterhuis in *Simon Stevin* [3] and A.J.J. van de Velde in *Simon Stevin 1548-1948* [13] have attempted to ascertain as much information as possible from the limited source material.

His date and place of birth have not been recorded, but the consensus is that Stevin was born in Bruges (Flanders) around 1548. This is based on the large number of references which Stevin makes to this city as his place of origin. For example, he used the epithet 'Brugensis' when he enrolled at the university in Leiden. Dijksterhuis posits that this lack of background information might be because Stevin was born out of wedlock. His mother, Cathelijne, was a scion of a prosperous merchant family which enabled Stevin to receive a good education and later study at the university in Leiden.

One aspect of Stevin's life which has been the topic of significant debate is his religion. It is assumed by Van de Velde that Stevin converted to Calvinism in his early life, which prompted him to leave Flanders after the Catholic Spanish recaptured Flanders in the 1570s and 1580s during the Dutch Revolt (1568-1648) and reintroduced the suppression of other Christian denominations.

Apart from his scientific career, the most noteworthy aspect of Stevin's career is his close connection as tutor to Maurice of Orange, the later stadtholder (the highest functionary of the Dutch Republic) and commander of the Dutch army. Stevin's knowledge on the construction of forts was particularly of use.

As with his birth, the circumstances of Stevin's death are also shrouded. Based on an appeal of his widow it seems that Stevin died in 1620. It is unknown whether he passed away at his home in The Hague or somewhere else.

Figure 2: Engraving of Simon Stevin made in the early 1600s.

3.2 Notation and structure

The main features of Stevin's notation and structure are discussed in this section since these are universal for the majority of his works and are referred to in the upcoming chapters on the *Art of Weighing*, *Mathematical Memoirs* and *Arithmetic*. It should be noted that Stevin conducted his work in a time period long before the standardisation of calculus and the regularisation of mathematics in general. Consequently, there are several features of his notation and structure (compiled by his differing methods of proof) that need to be taken into account when studying his works. These are also discussed in the *Principal Volumes*. Stevin frequently used three basic concepts to structure his work.

The first concept is the term 'TGHEGHEVEN', which roughly translates into 'that which is granted'. This term is used to introduce the presuppositions of a statement that Stevin intends to prove. Secondly, Stevin uses the concept of 'TBEGHEERDE' which literally translates to 'that which is desired'. This generally designates the statement that needs to be proven. This is frequently translated in the *Principal Works* by proposition, although depending on the context it could also be viewed as either a lemma or a theorem on its own. Such a translation will be used in the historist analysis of the various subjects in this thesis. Thirdly, Stevin follows up a 'TBEGHEERDE' with 'TBEWYS', which can literally be translated as 'the proof'.

Stevin also used several other concepts to denote the problems he was working on and the methods of proof he used. These concepts depend on the goal and content of his works and will therefore be discussed in the chapters on the *Art of Weighing*, *Mathematical Memoirs* and *Arithmetic*.

4 Art of Weighing

4.1 Introduction

The *Art of Weighing* was published in 1586 by Plantijn, a prominent publisher in the Low Countries during this period. The first volume was published together with *De Beghinselen des Waterwichts* (*Hydrostatics*) and *De Weeghdaet* (*The Act of Weighing*). The book describes several mechanical systems such as the motion of levers and masses on inclined planes. It serves as a more theoretical elaboration on mechanical systems with regard to the works on engineering by Stevin such as his works on the construction of fortifications.

The *Art of Weighing* was discussed in the first volume of *The Principal Works of Simon Stevin, Volume I Mechanics* by E.J. Dijksterhuis. The analysis by Dijksterhuis will be drawn upon in the historist analysis of this section. It is important to note that Dijksterhuis produced a facsimile combined with an English translation of the text. He comments relatively little on the scientific content of this work apart from an introduction and a relatively small number of footnotes. Furthermore, the translation of the Dutch text into English is ambiguous or grammatically incorrect in several cases. In this chapter, the definitions and postulates of the *Art of Weighing* are introduced and discussed. Modifications have been made to resolve ambiguities in the translation of several concepts as well as fix grammatical errors. Further explanation of Stevin's paradigm and the comparison with the current paradigm accompanies the definitions and postulates.

After having discussed the relevant definitions and postulates, the 'Clootcransbewijs' is discussed, which concerns the mechanical equilibrium of a string of beads on an inclined plane. The historist and presentist analyses which have been discussed in the introduction are conducted on this problem. Lastly, discourse analysis features in all the sections in this chapter.

4.2 Definitions

At the beginning of the *Art of Weighing*, Stevin states the definitions which he uses to construct his proofs. The principal definitions on which Stevin based his theorems and proofs are discussed in this section. The structure of the definitions in the *Art of Weighing* resembles the Aristotelian structure of definitions, which features an enumeration of statements with corresponding explanations.

The definitions are further discussed with regard to the contents of the *Art of Weighing*, the worldview of Stevin and his historical context. Stevin used the Dutch term 'Bepaling' to state a definition. As was previously mentioned, the translation of E.J. Dijksterhuis in his *Principal Works of Simon Stevin, Volume I Mechanics* will be commented on in this section as well.

Before discussing Stevin's definitions one should take a couple of characteristics of his paradigm into account. It is paramount to note that Stevin does not define either force or energy in his work. Physical quantities stemming from these concepts (such as torque) do not feature either. This is a key realisation to have in order to properly discuss Stevin's perspective on his mechanical problems. One of the most important ambiguities in his work is the concept of 'swaerheydt' which E.J. Dijksterhuis translated with the term 'gravity'. It should be stressed that the term 'gravity' does not refer to the force of gravity, but rather the property of an object having weight. This distinction will become clear in the explanation of Stevin's definitions, as some of the properties of his concept of 'swaerheydt' do not align with the same conceptualisation of gravity as an attractive force between two masses.

Definition 4.1. The *art of weighing* is the art which teaches the ratios, proportions and properties of the weights or gravities of solids.

This definition is mainly focused on the distinction between the art of weighing ('weeghconst') and geometry ('meetconst'). He describes that geometry is mainly concerned with the magnitude of physical objects, whilst the art of weighing focuses on the gravity ('swaerheydt') of an object. These physical quantities are discussed with regard to their ratios, proportions and properties. Here one also notices that the key concept of 'swaerheydt' has already been introduced and is defined in the subsequent definition.

Definition 4.2. The *gravity* ('*swaerheydt*') of a solid is the power of its descent in a given place.

The second definition is the crucial definition of 'swaerheydt' in Stevin's work. It should be noted that the notion of 'power' is not defined and does not specifically refer to energy as the work of a force in the current paradigm. The designation of 'swaerheydt' as 'gravity' will be used from now on, except when a clear distinction is made between Stevin's 'swaerheydt' and the contemporary concept of gravity force. When a specific point or object with 'swaerheydt' is mentioned, the translation 'point of gravity' is used. This is done, because 'gravity' (contrary to 'swaerheydt') is not a quantitative noun, which leads to grammatical errors when directly translated (as happened with the translations of Dijksterhuis who used 'a gravity' and 'gravities').

Definition 4.3. A known gravity is one expressed by a known weight.

This notion of gravity resembles the definition of the force of gravity in Newtonian mechanics as being dependent on the mass m of an object. The acceleration of descent g is regularly used for cases on Earth, whilst it can be further elaborated upon as being dependent on the greater mass M, the gravitation constant G and the distance r between the centres of mass. This is demonstrated in the following equation:

$$F_g = mg = G\frac{Mm}{r^2}. \tag{1}$$

Notably, 'swaerheydt' is a quantity which is only dependent on the properties of the solid itself (rather than another mass which exerts an attractive force). Stevin based his perspective on the medium in which an object moves (e.g. air or water). Consequently, a parallel can be drawn with the ambiguous notion of 'weight' in contemporary physics. Weight distinguishes itself from gravity by taking into account all the forces on an object (thereby resulting in a change of value when lifted as the normal force which reacts to the gravity force (by Newton's Third Law of Motion, the law of action and reactions) since the normal force is eliminated when the object is lifted from the surface which exerts the normal force.

Definition 4.4. The *centre of gravity* is the point such that if the solid is conceived to be suspended from it, it remains at rest in any position given to it.

Stevin explains his definition of the centre of gravity by using the example of a sphere on a cord, which is featured in Figure 3. He posits that the sphere has an equal distribution of mass and a midpoint D. He uses a thought experiment to prove this. He posits that the sphere is at rest in this specific constellation of the points ABC around the middle point D with equal distribution of mass. If the sphere is turned such that A, B and C change position the constellation should still be at rest. Stevin then uses the implication that a system not at rest does not have a constant, homogeneous distribution of mass. This implication is not explicitly formulated by Stevin, but could be contrived from Definition 4.2 as well as the notion that gravity is proportional to the notion of weight as stated in Definition 4.3. As a result, if by turning the sphere the system is not at rest, this would imply that there is no equal distribution of mass, which is in contradiction with the presupposition that there is an equal distribution of mass. Therefore, the object is at rest at the middle point D even when it is suspended from its original position. In conclusion, D is the centre of gravity.

Subsequently, Stevin generalises the notion of a centre of gravity to any solid, where he distinguishes between cases with uniform and non-uniform mass distributions and remarks upon corresponding cases when the geometric centre of a solid and the centre of gravity coincide. He states that the centre of gravity is unique, but he does not prove this.

Stevin states in his own explanation that he was inspired by the definition of the centre of gravity given by Pappus. Stevin also gives the following equivalent definition: the *centre of gravity* of a solid is the point through which any plane divides the solid into parts of equal apparent weight. The concept of 'equal apparent weight' ('evenstaltwichtigheid') is not explicitly defined, but it is mentioned in Definition 4.11 and is discussed in detail there.

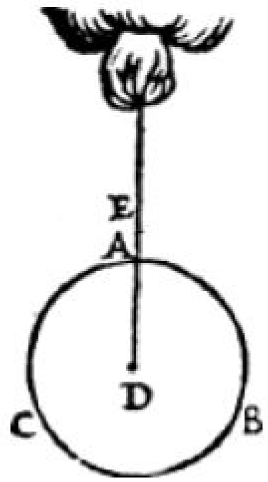


Figure 3: The image of a sphere on a cord, which Stevin uses to explain his notion of the centre of gravity.

Definition 4.5. The *centre line of gravity* of a solid is the infinite vertical through its centre of gravity.

Stevin continues from his definition of the centre of gravity to the definition of the centre line of gravity. He only shortly discusses this definition and refers to the example of the line DE in Figure 3. He does not explicitly mention or prove the non-uniqueness of this concept, but implicitly assumes this.

Definition 4.6. The *centre plane of gravity* of a solid is any plane dividing it through its centre of gravity.

Stevin extrapolates his definition of the centre line of gravity in Definition 4.5 to a three-dimensional space. Once again, he does not mention or prove the non-uniqueness of this concept, but implicitly assumes this.

Definition 4.7. Any straight line contained between two centre lines of gravity is called the *beam* of these points of gravity.

From Definition 4.8 to Definition 4.11 Stevin used the system in Figure 4 to explain the definitions. These definitions mainly serve as designations for components of a mechanical system. The system in Figure 4 consists of two points of gravity A and B in this setup, which are connected by the beam CD. The beam CD is divided by the line FG, which is the centre line of gravity.

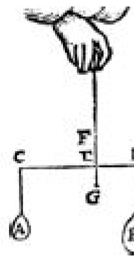


Figure 4: The setup which is used to explain Definitions 4.8 to 4.11.

Definition 4.8. The divided parts of the beam due to the centre line of gravity at which two points of gravity are of equal apparent weight are called the *arms.*

The arms of the beam are EC and ED in Figure 4. It is important to note that the concept of *equal apparent weight* is used here as well and implicitly defined as the circumstance at which the beam is divided along the centre line of gravity. The use of equal apparent weight stems from the second formulation of Definition 4.4.

Definition 4.9. The centre line of gravity of two points of gravity is called the *handle.*

Definition 4.10. The point at which the handle is attached to the beam is called the *fixed point.*

The handle is the line EF in Figure 4.8 according to Stevin's explanation. He does not mention that the handle is the centre line of gravity up to the point where the handle is attached (i.e. the fixed point E) since he defines EF to be the handle rather than the centre line FG.

Definition 4.11. The two points of gravity are said to be of *equal apparent weight* ('evenstaltwichtig').

It is interesting to note that Stevin does not explicitly define 'evenstaltwichtig' (of equal apparent weight) or 'evenstaltwichtigheid' (equal apparent weight). This definition rather seems to stem from the presupposition of a centre line of gravity and is therefore circular. Based on the equivalent description of Definition 4.4, the notion of equal apparent weight seems to be largely equivalent to the notion of 'equilibrium' in the current paradigm. Dijksterhuis states that equal apparent weight corresponds to our current notion of 'evenwicht', which should be translated as 'equilibrium'. It is paramount to note that equal apparent weight does not merely convey force equilibrium, but also the equilibrium of torque as is shown in the case of the system in Figure 4. Torque τ is the rotational analogue of force which is derived from the cross-product of the force vector $\vec{F}$ and the position $\vec{r}$ such that

$$\vec{\tau} = \vec{F} \times \vec{r}, \tag{2}$$

with magnitude

$$\tau = Fr\sin(\theta),$$

where θ is the angle between the force vector and the position vector.

Definition 4.12. *Lifting weight* causes the ascent of a point of gravity and *lowering weight* causes the descent of a point of gravity.

This definition merely serves as a denotation for different points of gravity in the mechanical systems which are discussed by Stevin. There are two more definitions, which merely serve as denotations corresponding with *lifting weight* and *lowering weight.* These denotations are not used in the subsequent theorem and corollaries and are therefore not included.

4.3 Postulates

The postulates in the *Art of Weighing* are described as 'Begheerten' which can be interpreted as 'desirable statements'. These postulates are used to state axioms and assumptions on the theoretical systems that Stevin discusses. A couple of postulates have been selected which are relevant to the historist and presentist analysis.

Postulate 4.1. *We postulate that it is granted that equal weights at equal arms are also of equal apparent weight.*

From a presentist perspective, this postulate corresponds with torque equilibrium. Equal weights $m_1 = m_2$ (resulting in equal gravitational forces $F_1 = F_2$) at equal arms $r_1 = r_2$ grant an equilibrium by Equation (2).

Postulate 4.2. *We postulate that any weight can hang or rest at the mathematical line without breaking or bending.*

This postulate can be viewed as the assumption that the connections between the masses in a system are unbreakable and inelastic such that elasticity can be ignored.

Postulate 4.3. *We postulate that gravity always keeps the same mass, no matter whether it hangs higher or lower.*

This postulate corresponds with the current notion of gravity as expressed in Equation (1). Furthermore, Stevin uses the implicit postulate that perpetual motion is impossible, which seems to be based on an empiricist observation coming from his work on engineering. This notion will be discussed in the analyses on the 'Clootcransbewijs'.

4.4 Static equilibrium on an inclined plane

One of the best-known results in the *Art of Weighing* is the static equilibrium of a string of beads around an inclined plane. The wreath of beads curled around a triangle is Stevin's trademark, which was accompanied by his motto 'a miracle, but not a miracle' ('wonder en gheen wonder' in archaic Dutch). This trademark featured on the covers of his primary works among which is the *Art of Weighing* and the *Mathematical Memoirs* as can be seen in Figure 1. The 'Clootcransbewijs' is also known as the 'Epitaph of Stevinus' with the Latinized version of Stevin's name.

The historist analysis in this section is conducted on the theorem with regard to the static equilibrium of a string of beads around an inclined plane as well as some additional systems which Stevin discussed. These additional systems are described by a 'Vervolgh' which can literally be translated as 'continuation' and has been translated by E.J. Dijksterhuis as 'corollary'. Notably, these texts do not have a specific structure of statement with corresponding proof such that they do not correspond to the current notion of corollaries. In the historist analysis a subdivision has been made between the corollary and its proof. This is already slightly presentist, but serves to grant better insight in Stevin's work. The presentist analysis draws upon the theorem and the corollaries and contains the equivalent methods and proofs using the contemporary paradigm.

Lastly, the following remarks with regard to the translations need to be taken into account. The translations which are used in the historist analysis are partially based on the translations by E.J. Dijksterhuis in his *Principal Works, Volume 1 Mechanics*, but have been significantly altered to remove grammatical errors. Repetitions of earlier made statements have also been omitted when they can be deemed unnecessary. At several points, the work of Stevin has been paraphrased, but not literally translated in order to clarify the content and grant better insight.

4.4.1 Historist analysis

Firstly, Stevin describes the composition of his system as consisting of a right-angled triangle ABC (as is shown in Figure 5). It is worth noting that Stevin defines this triangle by describing the relation between the different angles and the sides of the triangle ('rechthoukig op den sichteinder'). Furthermore, he describes that there is a bead ('cloot') D on the side of AB and a bead E on the side of BC. The beads D and E have equal mass m and equal volume V ('euewichtich ende euegroot'). Moreover, he states that the side AB is double the side BC.

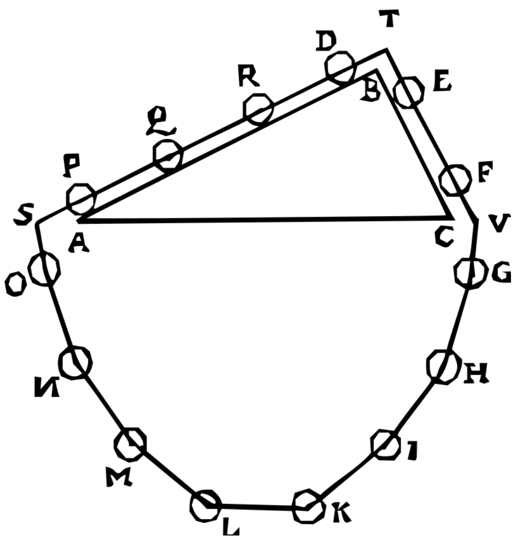


Figure 5: The image of the string of beads around an inclined plane in the *Art of Weighing*.

Theorem 4.13. *The apparent mass ('staltwicht') of the beads is proportional to the relation between the sides AB and BC, which is in this case* $AB : BC = 2 : 1$.

Proof. Critically, Stevin uses the concept of 'staltwicht', which he has not previously defined and can be approximately translated as *apparent weight* (as was done by Dijksterhuis), since this concept has no precise translation in the current paradigm. From the inferred definition of equal apparent weight after Definition 4.11 and Postulate 4.1 it can be inferred that this is the component of gravity along the side of the triangle. This leads to a self-evident equivalent definition of *equal apparent weight*.

If the apparent weight of the four spheres D, Q, R and P is not equal to the apparent weight of the two spheres E and F, then one of them is heavier. Suppose this is one of the four spheres D, R, O or P. But the four spheres O, N, M and L are of equal weight to the four spheres G, H, I and K. Consequently, the side of the eight spheres D, R, Q, P, O, N, M and L is heavier in appearance than the side of the six spheres E, F, G, H, I and K. Then the eight spheres will roll downwards and the other six will rise. If this is so with D on the position where O is now, then E, F, G and H will be where P, Q, R and D are now, and I and K where E and F are now. But if this is the case, the wreath of the spheres will have a similar composition as before such that the eight spheres on the left side will again have greater apparent weight than the six spheres on the right side. Consequently, the eight spheres will again roll down and the other six will rise. This descent on the one and ascent on the other side will continue forever because the composition always has a similar difference in apparent weight. The spheres will automatically perform a perpetual motion, which is absurd. Therefore, the part of the wreath D, R, Q, P, O, N, M and L must be of equal apparent weight to the part E, F, G, H, I and K. But if from such equal apparent weights, there are weights subtracted, the remainders will have equal apparent weight. Let us, therefore, subtract from the former part the four spheres O, N, M and L and from the latter part the four spheres G, H, I and K (which are equal to the aforesaid O, N, M and L); then the remainders D, R, Q, P and E, F will be of equal apparent weight. But the two latter being of equal apparent weight to the four formers, E will have twice the apparent weight of D. As therefore the line AB is to the line BC so is the apparent weight of the sphere E to the apparent weight of the sphere D. □

The first point of interest is the fact that Stevin does not mention any resistance by either air or friction. Dijksterhuis claims that Stevin did not take such a resistance into account. He also seems to assume the weightlessness of the string as well. The inelasticity of the string is assumed by Postulate 4.2. Furthermore, Stevin uses the supposition that the subtraction of weight from equal apparent weights results in equal apparent weights. If we identify apparent weight as a combination of equilibrium of force and torque this is the case with this system, but in general it is not true. Stevin assumes this, but does not give any arguments. Stevin also made an error (which has already been rectified by Dijksterhuis in the previously given proof) when he confused weight with apparent weight.

Stevin proceeds with corollaries from Theorem 4.13. As previously mentioned, a distinction between statement and proof has been made in order to clarify the structure.

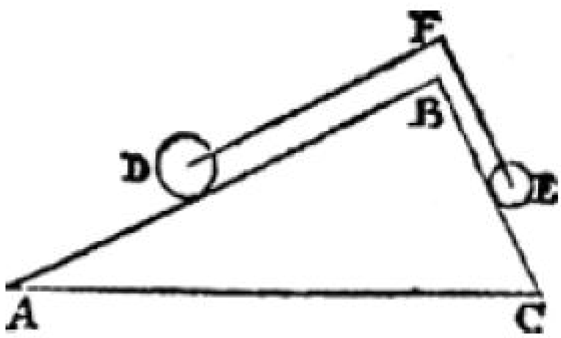


Figure 6: The setup of the system which was described in Corollary 4.13.1.

Corollary 4.13.1. *Let ABC be a triangle, as before, whose side AB shall be double of BC, and on AB let there lie a sphere D and on the side BC a sphere E is of equal weight to half of D. This is shown in Figure 6. And in F let there be a fixed point, over* *which the line DFE (from the centre of the sphere D via F to the centre of the sphere E) can slide, in such a way that DF shall remain parallel to AB, and FE to BC.*

Proof. Since the four spheres P, Q, R and D in the preceding case were of equal apparent weight to the two spheres E and F, this sphere D will be of equal apparent weight to the sphere E. P, Q, R and D are similarly to E and F as D to E. Therefore, as the line AB is to BC, so is the sphere D to the sphere E. □

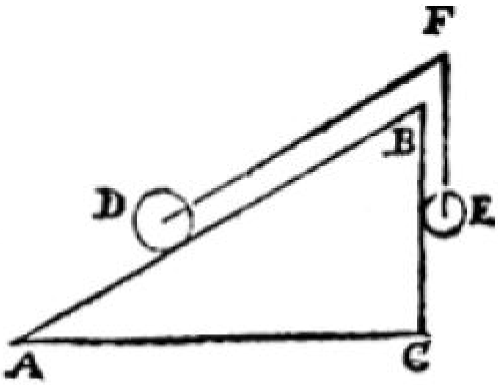


Figure 7: The setup of the system which was described in Corollary 4.13.2.

Corollary 4.13.2. *Now let us put one side of the triangle, as BC (AB being double of it) at right angles to AC, as in the annexed Figure 7. Then, the weight of D is to the weight of E as the side AB is to the side of BC.*

Proof. The sphere D, which has double the weight of E, will still be of equal apparent weight to E, for as AB is to BC, so is the sphere D to the sphere E. □

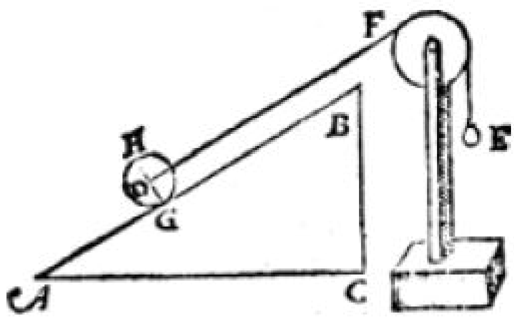


Figure 8: The setup of the system which was described in Corollary 4.13.3.

Corollary 4.13.3. *Let us now put in the place of the point F a pulley, as shown in Figure 8, in such a way that the oblique lifting line from D to F shall remain parallel to AB. And in the place of the sphere E let there be some arbitrary weight, but which is of equal apparent weight to the sphere E. Then, the weight of D is to the weight of E as the side AB is to the side of BC.*

Proof. This weight E is still of equal apparent weight to D. Therefore, as AB is to BC, so is the weight of D to the weight E. □

Corollary 4.13.4. *Since the sphere of Corollary 4.13.3 touches the line AB in the point G as a fixed point, the axis GH will be at right angles to AB. Therefore, let us take away the sphere, and put in its place the prism D, of equal weight to the sphere, in such a way that the axis GH (its fixed point being G) shall be at right angles to AB, and the oblique lifting line between D, F still parallel to AB and meeting the side of the prism in I, as shown in Figure 9.*

Proof. The axis GH is perpendicular to the axis AB and the weight of the prism is equal to the weight of the sphere in Corollary 4.13.3. Then the prism D and weight E are still of equal apparent weight. Therefore, AB is to BC as the prism D is to the weight E. □

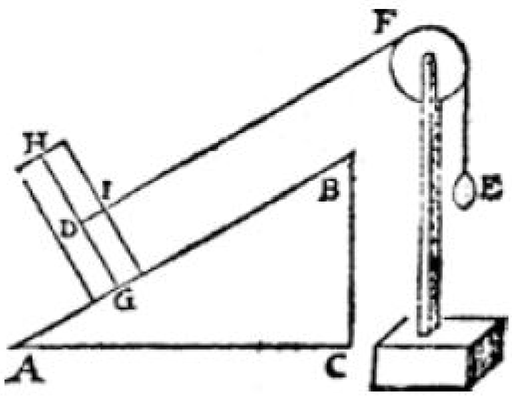


Figure 9: The setup of the system which was described in Corollary 4.13.4.

4.4.2 Presentist analysis

In the contemporary paradigm, Newtonian mechanics can be used to prove the theorem and corresponding corollaries. A key feature is the use of force vectors $\vec{F}$. The notion of 'staltwicht' (the component of the bead's mass along the side) can be interpreted as the parallel component of the gravitational force F_g with the side. The notation $\vec{F}_{\parallel}$ denotes the component of the force vector parallel to the side of the triangle on which the point of mass is positioned. We denote the magnitude of this vector with $F_{\parallel} \equiv |\vec{F}_{\parallel}|$. The notation $\vec{F}_g^A$ represents the force vector of gravity on mass A in the system with magnitude $F_g^A \equiv |\vec{F}_g^A|$.

In the upcoming Proposition 4.14 and Corollaries 4.14.1 and 4.14.2 we assume that there is no friction on the inclined plane, no friction with air, the rope is inelastic and massless and the mass of the beads is homogeneously distributed. Based on Theorem 4.13 a presentist perspective results in the following proposition.

Proposition 4.14. *The relation between the parallel components of the gravitational force on the beads P, Q, R, D on the side AB and E and F on the side BC is equal to the relation between the sides of the triangle:*

$$\frac{BC}{AB} = \frac{F^i_{g,\parallel}}{F^j_{g,\parallel}}, \text{ for } i \in \{P, Q, R, D\} \text{ and } j \in \{E, F\}.$$

Proof. From Stevin's description, we know that $\triangle ABC$ is a right triangle with $\angle B = \beta = \frac{\pi}{2}$. We denote $\angle A = \alpha$ and $\angle C = \gamma$ such that the parallel components of the force of the beads in Figure 5 have the following value:

$$F_{g,\parallel} = m_i g \sin\alpha, \tag{3}$$
$$F_{g,\parallel} = m_j g \sin\gamma. \tag{4}$$

By the sum of the angles of the triangle, it is evident that the following equalities hold:

$$\alpha + \beta + \gamma = \pi,$$
$$\alpha + \gamma = \frac{\pi}{2}.$$

We can now apply this to show that we can subdivide $\triangle ABC$ into two similar triangles. If we draw a new line from B to the side AC such that the line fragment BZ is perpendicular with AC (as is shown in Figure 10) we can conclude that $\triangle ABZ \sim \triangle BCZ$ since the following equalities hold:

$$\angle BZA = \angle BZC = \frac{\pi}{2},$$
$$\angle CBZ = \beta - \gamma = \frac{\pi}{2} - \gamma = \alpha = \angle BAD.$$

Using trigonometric identities we derive our required equality:

$$\sin\gamma = \sin\angle BCZ = \frac{CZ}{BC} = \cos\angle CBZ = \cos\alpha.$$

Consequently, the parallel components of the gravity force in Equations (3) and (4) can both be expressed with angle α:

$$F^i_{g,\parallel} = m_i g \sin\alpha, \tag{5}$$
$$F^j_{g,\parallel} = m_j g \cos\alpha. \tag{6}$$

All beads have the same mass which we denote with m. Since the parallel components of beads on one side of the triangle have the same angle, they can be added. This yields the following result:

$$\sum_{i\in\{P,Q,R,D\}} F^i_{g,\parallel} = g \sum_{i\in\{P,Q,R,D\}} m_i \sin\alpha = 4mg\sin\alpha,$$
$$\sum_{j\in\{E,F\}} F^j_{g,\parallel} = g \sum_{j\in\{E,F\}} m_j \cos\alpha = 2mg\cos\alpha.$$

We can choose two methods to derive the required answer. First, by the trigonometric identities, we know:

$$AB\sin\alpha = BC\cos\alpha \iff \frac{BC}{AB} = \frac{\sin\alpha}{\cos\alpha}. \tag{7}$$

Using this and the fact that all masses are equal, we can derive the following result:

$$\frac{F_{g,i}}{F_{g,j}} = \frac{mg\sin\alpha}{mg\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \frac{BC}{AB} \text{ for } i \in \{P,Q,R,D\} \text{ and } j \in \{E,F\}.$$

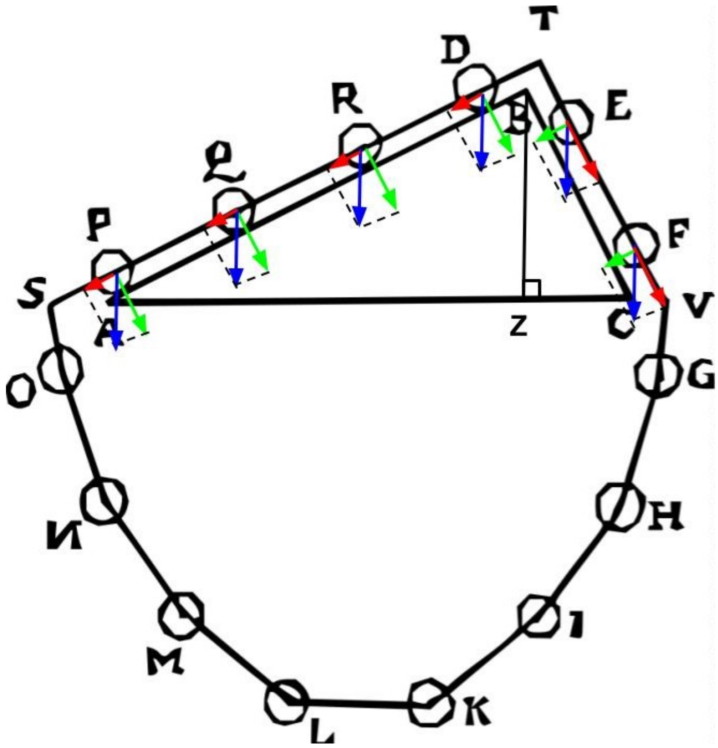


Figure 10: The set-up as was described in Theorem 4.13 featuring the components of the force of gravity and the point Z. The blue vectors denote the gravity F_g on the beads of mass on the triangle. The red vectors are the parallel components of the gravity force $F_{g,\parallel}$. The green vectors are the perpendicular components $F_{g,\perp}$ which are compensated for by the normal force exerted by the surface of the triangle in the opposite direction.

Alternatively, we assumed there to be an equilibrium such that we derive the following result:

$$\begin{aligned}\sum_{i\in\{P,Q,R,D\}} F^i_{g,\parallel} &= \sum_{j\in\{E,F\}} F^j_{g,\parallel},\\ 4F^i_{g,\parallel} &= 2F^j_{g,\parallel} \text{ for } i\in\{P,Q,R,D\} \text{ and } j\in\{E,F\},\\ \frac{F^i_{g,\parallel}}{F^j_{g,\parallel}} &= \frac{1}{2} = \frac{BC}{AB}.\end{aligned}$$

□

The following corollary is the presentist version of Corollary 4.13.1. The system is depicted in Figure 6.

Corollary 4.14.1. *Let $\triangle ABC$ be a right triangle with a right angle B. The system is in static equilibrium. Therefore, the relation between the masses of the beads D and E is equal to the relation between the sides of the triangle:*

$$\frac{AB}{BC} = \frac{m_D}{m_E}.$$

Proof. We can use an equivalent method of proof in this case. Using Equations (5) and

(6) we derive:

$$F_{g,\|}^{D} = m_D g \sin\alpha,$$
$$F_{g,\|}^{E} = m_E g \cos\alpha.$$

A static equilibrium is required such that the following holds:

$$\frac{F_{g,\|}^{D}}{F_{g,\|}^{E}} = 1.$$

Consequently, we find the following relation:

$$1 = \frac{F_{g,\|}^{D}}{F_{g,\|}^{E}} = \frac{m_D g \sin\alpha}{m_E g \cos\alpha} = \frac{m_D}{m_E} \cdot \frac{\sin\alpha}{\cos\alpha}.$$

Using Equation (7) we conclude

$$\frac{m_D}{m_E} \cdot \frac{BC}{AB} = 1 \iff \frac{AB}{BC} = \frac{m_D}{m_E}.$$

This finishes the proof. □

It should be noted that Stevin described the specific relation between AB and BC as being 2 to 1. It is not necessary to include this in Corollary 4.14.1 since the result does not depend on the value for the relation between AB and BC. Now we continue with the presentist version of Corollary 4.13.2 with annexed Figure 7.

Corollary 4.14.2. *Let $\triangle ABC$ be a right triangle with a right angle C. The system is in equilibrium if and only if the following relation holds:*

$$\frac{m_E}{m_D} = \frac{BC}{AB}. \tag{8}$$

Proof. For the first implication, we need to show that the system is in equilibrium i.e. $F_{g,\|}^{D} = F_{g,\|}^{E}$ with the use of Equation (8). In this case, the gravity force of mass E is entirely parallel to BC. The parallel components of F_g^D and F_g^E yield the following equations:

$$F_{g,\|}^{D} = m_D g \sin\alpha = m_D g \sin\alpha, \tag{9}$$
$$F_{g,\|}^{E} = F_g^E = m_E g. \tag{10}$$

By the definition of $\sin\alpha$ it is evident that:

$$\sin\alpha = \frac{BC}{AB} = \frac{m_E}{m_D}.$$

Substitution in Equation (9) yields:

$$F_{g,\|}^{D} = m_E g = F_{g,\|}^{E}.$$

For the second implication, $F_{g,\|}^{D} = F_{g,\|}^{E}$ such that by Equations (9) and (10) the following holds:

$$m_D g \sin\alpha = m_E g.$$

Then, we can derive the required relation:

$$\frac{BC}{AB} = \sin\alpha = \frac{m_E}{m_D},$$

which gives the desired result. □

Once again, we did not specify the relation between either m_D and m_E or AB and BC since the result of Corollary 4.14.2 only depends on the equality of these relations and not their value. The addition of a pully removes friction with the surface of mass E in Figure 8 and the change of shape of the mass D in Figure 9 changes friction with both the surface and the air. Since it was assumed that there is no friction with either air or the surface of the plane, Corollary 4.14.2 is sufficient to give a presentist result on Corollaries 4.13.3 and 4.13.4.

5 Mathematical Memoirs

5.1 Introduction

The *Mathematical Memoirs* was published in several volumes in the period between 1605 and 1608. The book contains three separate works by Stevin: *Van 't Weereltschrift* (*Trigonometry*), *De Weeghdaet* (*Practice of Measuring*) and *De Deursichtighe* (*Perspective*). Stevin mainly describes the properties of geometric objects in his *Trigonometry* and *Practice of Measuring*, whilst his *Perspective* focuses on optics. The current analysis of the *Mathematical Memoirs* is primarily restricted to the *Principal Works of Stevin, Volume IIB* by Dirk Struik and Ernst Crone, which also contained the research on the *Arithmetic*. The relevant parts of that volume for this chapter have been written by Struik. It should be mentioned that Struik only selected the book *de Driehouckhandel* from the *Mathematical Memoirs* and discussed this work only partially. This chapter discusses the methods to calculate the circumference of an ellipse from the second book of the *Mathematical Memoirs*: the *Practice of Weighing*.

5.2 Procedures

Contrary to Stevin's methods in the *Art of Weighing* and *Arithmetic*, the methods of proof in *Practice of Weighing* are based on a procedure of drawing the geometric figures. Stevin denotes such a procedure as a 'Voorstel' by which he instructs the reader on how to draw the geometric figures in his setup. These didactic explanations can be viewed as constructivist definitions from which results can be construed which features in the historist analysis of the methods to obtain the circumference of an ellipse. This method of geometric proof is clearly inspired by the works of Euclid and shows the influence of Greek Mathematics on Stevin's work, although Stevin does not explicitly mention Euclid in the *Mathematical Memoirs*. This is also a sharp contrast to the Aristotelian structure which was discussed in Chapter 4.

An example of the instructions Stevin gives to acquire a geometric proof can be found at the beginning of the section on the circumference of an ellipse. Stevin describes the method by which the reader ought to draw an ellipse and the subsequent method of obtaining the circumference by deduction using added geometric figures, which will be further discussed in Section 5.3. The instrument which Stevin referenced, is shown in Figure 12.

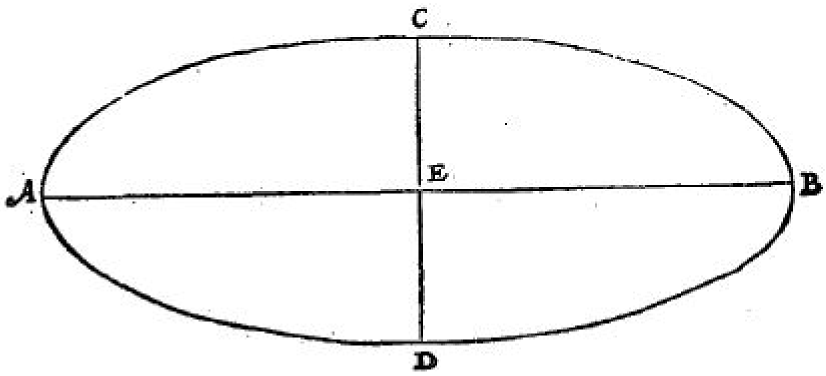


Figure 11: The ellipse $ABCD$ with midpoint E which Stevin uses throughout his sections on the different methods of calculating the circumference of an ellipse.

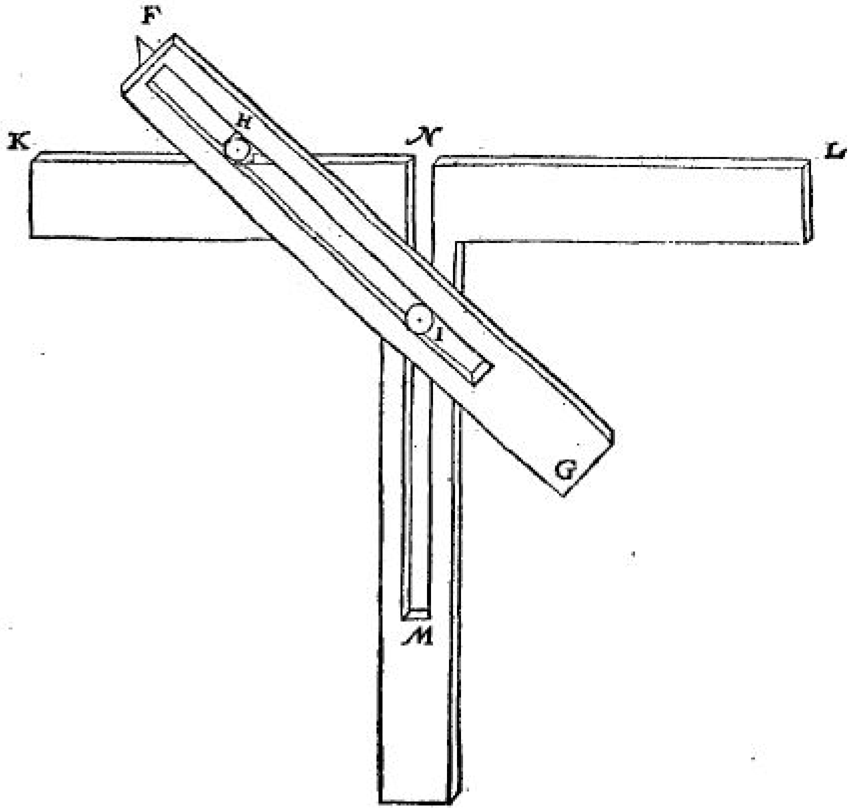


Figure 12: The instrument which is used to draw the ellipse and indicates the circumference of the ellipse.

Proposition 5.1. *FG has a movable ruler with a slit in the middle, in which two small pegs H and I are screwed. At the endpoint F there is a point with which the circumference is drawn. KL is a handle with a slit MN. The drawing of the required circumference with this instrument is done in the following way. The point of the peg H is fastened as far from the same point F as the distance EA. Then the pin F is put in the point C and the peg H in the point N such that the ruler FG is positioned in the middle of the handle and the line KL fits on AB. After this, the peg H is moved along the edge KL, whilst the peg I runs its course in the slit MN. This being so, the pin F describes half the required circumference. Having done the same on the other side, the entire circumference is obtained.*

Notably, Stevin does not state the proof for this proposition. He only mentions that he came to know this method through the work of Guidobaldo del Monte. He does not specify the work itself (since he seems 'to have lost it'). This book is probably either the *Mechanicorum Liber* or the *Planisphaeriorum universalium theorica*.

5.3 Circumference of an Ellipse

One of the main subjects Stevin discusses in his *Practice of Measuring* is the methods to determine the circumference of an ellipse. Since the circumference of an ellipse has no analytical solution in the current paradigm, one needs to use numerical approximations.

5.3.1 Historist Analysis

This section on the historist analysis contains the methods which Stevin used to determine the circumference of the ellipse apart from using the instrument in Figure 12. These methods will be denoted as 'propositions' rather than 'procedures' since Stevin also added proofs to his procedures. The propositions in this section are shortened paraphrases from Stevin's work and partially inspired by the work of Struik. However, mistakes in the translation of Struik have been omitted or corrected.

Proposition 5.2. *By drawing CD to F in such a way that CF equals EA and thereafter taking between the compasses the distance EF and putting one leg on EF in any place, we assume point EF to come in point G as EA comes into point H. Subsequently, we produce GH to I so that HI equals EC. In that case, I is a point on the circumference of the ellipse. Therefore, when a sufficient amount of points have been found such that by drawing straight lines between them these lines do not significantly differ from the circumference we have what was required (i.e. the circumference $AICBD$).*

Proof. Since in Proposition 5.1 the length FH of the instrument in Figure 12 was equal to CE and FI equal to AE and F was on the circumference of the ellipse in Figure 11, the point I also has to be on the circumference of the same ellipse. In this case, IH is the equivalent of FH in Proposition 5.1 to CE. Similarly, HG is equal to the difference between the height and the width as was HI in the first method. □

The resulting figure by this procedure is shown in Figure 13. The method of drawing straight lines between the points on the ellipse resembles the method by which the circumference of the circle can be obtained by partitioning the circle in small triangular sections to such a point that the curvature of the sections of the circumference is negligible.

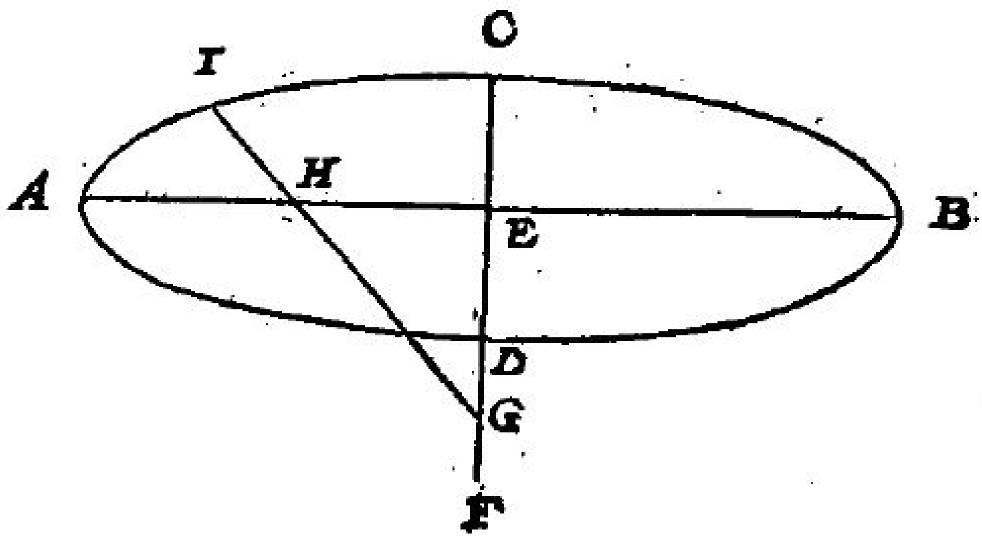


Figure 13: Drawing of the ellipse with tangent by the procedure in Proposition 5.2.

Proposition 5.3. *The distance AE from D to F is marked as well as the line AB from D to G. After having drawn the two extremities F and G, a string of the length AB can be taken with its extremities fastened at F and G. Thereafter, a pin or awl (adapted* *for this purpose) is put against the string at the right angles to the plane at the point* H *such that the string* GH *and* HF *are pulled tight. When the awl is drawn from* A *via* C *to* B *(with the string* GHF *uniformly stretched), half the circumference* ACB *is described. Having done this for the other half of the circumference,* DBA*, we have what is required.*

The proposition shows, once again, clearly Stevin's notion that the readers ought to be able to construct the system themselves. The notion of having the strings 'pulled tight' is a mechanical method to obtain the smallest distance (i.e. straight lines). Again, Stevin refers to Guidobaldo del Monte who (in his view) learned this from 'some old manuscripts'.

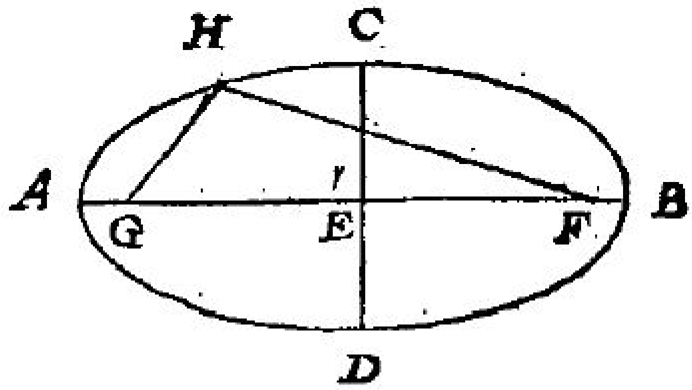


Figure 14: Drawing of the ellipse with the triangle FGH by the procedure in Proposition 5.3.

Proposition 5.4. *The line* BF *at right angles to* AB *and equal to* EC *is drawn. The line* AB *to* G *is also drawn such that a quarter of a circle,* BFG*, is found.* BG *is then divided into a number of equal parts (presumably in four), in the points* H*,* I *and* K *with the lines* HL*,* IM *and* KN *parallel to* BF *in such a way that the extremities* L*,* M *and* N *are on the arc* FG*.* EB *is then divided into the same amount of parts as has been done with* BG*. This partition results in the points* O*,* P *and* Q*. Furthermore, the lines* OR *equal to* HL*,* PS *equal to* IM *and* QT *equal to* KN *are drawn such that all three are parallel to* EC*. Consequently, the three points* R*,* S *and* T *are on the required circumference. Therefore, if* BG *and* EB *The other three quarters can be obtained with the same method.*

Stevin introduces a 'Bereytsel' to discuss his method. Struik translates this as a 'Preparation'. Since it is explicitly used to prove Proposition 5.4 I have described it as a lemma.

Lemma 5.5. *Let* $ABCD$ *be a cylinder with diameter* DC*. Intersect this cylinder by a plane* EF *at oblique angles to the outer line* AD*, which plane is an ellipse with width* EF *and height* CD *(as explained in the first book of Serenus). Let the cylinder once more be intersected by a plane* GH *parallel to the base, then this section will be a circle which, when viewed transversely shall be the line* GH*, intersecting* EF *in* I *such that* IF *is one-fourth* EF *and* GI *the diameter of the previously mentioned circle. With this diameter* GH *the circle* $GKHL$ *is described, which is perpendicular to the base* DC *and the ellipse* EF*. Thereafter,* MN *is a plane which, when viewed transversely, passes* *through the point I perpendicular to the circle $GKHL$. The resulting figure is shown in Figure 16.*

Proof. GF is parallel to EH such that the triangle GIF is similar to the triangle HIE. Therefore, as FI is to IE, so is GI to IH. But FI is one-third of IE or one-fourth of FE. Therefore, GI is also one-third of IH or one-fourth of GH. Furthermore, the line IL is equal to the line in the plane of the ellipse from I to the circumference of the ellipse (for when the diameter GH remains in its place and the circle on it is revolved until it is parallel to the base of the cylinder, IL and the aforesaid line are one and the same). Therefore, if a circle is described on the line equal to the height of an ellipse and a line is drawn at right angles at one-fourth of the latter up to the circumference and thereafter exactly such a line is drawn perpendicular at one-fourth of the latter up to the circumference and thereafter exactly such a line is drawn perpendicular to the width of the ellipse, the extremity of this line must come on the circumference of the ellipse. This result can be applied to any interval in Figure 15. □

Proof. By using the result of Lemma 5.5 we can determine the circumference of the ellipse by the width and the height of the ellipse. □

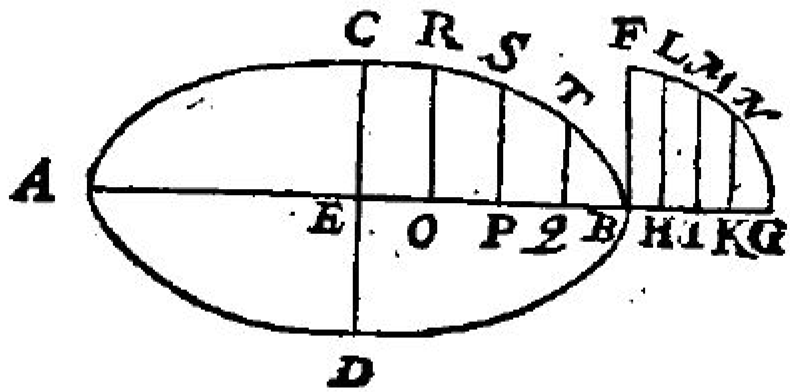


Figure 15: Drawing of the ellipse with the partitions by the procedure in Proposition 5.4.

All three methods concern the discretization of the circumference of an ellipse to such an extent that the intervals can be perceived as straight lines.

5.3.2 Presentist Analysis

After having concluded the historist analysis of the problem of the circumference of an ellipse, the presentist analysis will connect Stevin's methodology to the contemporary paradigm and delve into the method of elliptic functions. It will also be shown that there exists no analytical solution for the circumference of an ellipse.

In all his methods it is clear that Stevin subdivides the arc of an ellipse into segments of which the length can be calculated with the different methods that Stevin presented. Using Carthesian coordinates such a segment can be calculated by using the equation for the arc length of a function $f(x)$ with boundary points x_1 and x_2 with $x_2 > x_1$:

$$L = \int_{x_1}^{x_2} \sqrt{1 + (f'(x)^2)} \mathrm{d}x.$$

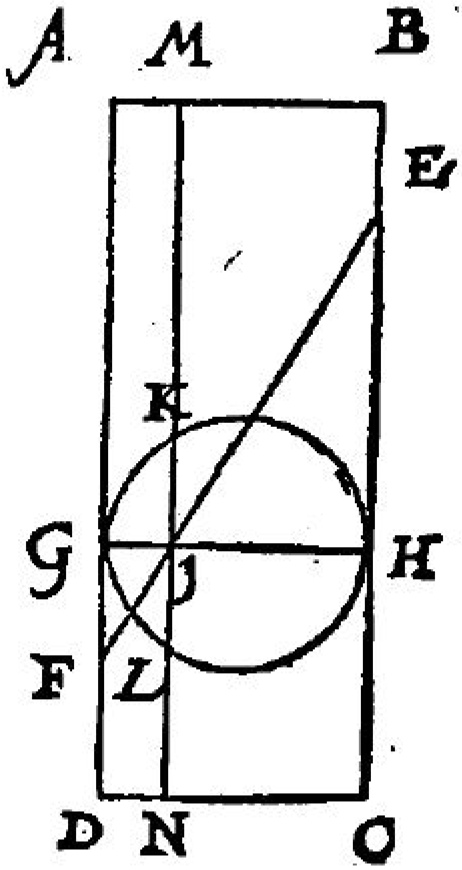


Figure 16: Drawing of the cylinder which Stevin describes in Lemma 5.5.

The standard method to determine the arc length of one half of the circumference of an ellipse is to start with the equation of an ellipse in Cartesian coordinates. In this case, a is taken as the width and b as the height such that $a \geq b$. Consequently, the following equation describes the ellipse:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1.$$

By isolating the y-component of this equation, one acquires the function required to describe the arc length of the ellipse with

$$f(x) = y = \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)},$$
$$f'(x) = -\frac{bx}{a^2\sqrt{1 - \frac{x^2}{a^2}}}.$$

The arc length L of this function can be used to determine the upper half of the circumference of an ellipse with boundaries $-a \leq x \leq a$:

$$L = \int_{-a}^{a} \sqrt{1 + (f'(x)^2)} \mathrm{d}x,$$

$$L = \int_{-a}^{a} \sqrt{1 + \frac{b^2 x^2}{a^2(a^2 - x^2)}} \mathrm{d}x.$$

Applying the substitution $c = \frac{b}{a}$ and $u = \frac{x}{a}$ with $du = \frac{dx}{a}$ (with resulting boundaries $-1 \leq u \leq 1$) we find the following result:

$$L = \int_{-a}^{a} \sqrt{1 + \frac{c^2 x^2}{a^2 - x^2}} \mathrm{d}x,$$

$$L = \int_{-a}^{a} \sqrt{\frac{a^2 + (c^2 - 1)x^2}{a^2 - x^2}} \mathrm{d}x,$$

$$L = a \int_{-1}^{1} \sqrt{\frac{1 + (c^2 - 1)u^2}{1 - u^2}} \mathrm{d}u.$$

Since this integral is symmetric we can rewrite this in the following way:

$$L = 2a \int_{0}^{1} \sqrt{\frac{1 + (c^2 - 1)u^2}{1 - u^2}} \mathrm{d}u = 2aE(c). \tag{11}$$

We conclude that the circumference of the ellipse is $4aE(c)$. The integral $E(c)$ in Equation (11) is the complete elliptic integral of the second kind which does not have an analytical solution. The integral can be approximated by the following power series [14]:

$$E(c) = \frac{\pi}{2} \sum_{n=0}^{\infty} \left(\frac{(2n)!}{2^{2n}(n!)^2} \right)^2 \frac{c^{2n}}{1 - 2n}.$$

This concludes the presentist analysis of Stevin's methods to determine the circumference of an ellipse.

6 Arithmetic

The *Arithmetic* was published in 1585 and primarily concerns the solutions to a large variety of algebraic equations. Therefore, it can be viewed as part of the advancement of the theory of equations which was one of the main developments of mathematics in this period. It was published together with *The Tenth*, which concerned the applications of arithmetic in bookkeeping. The *Arithmetic* was published in French contrary to the previously discussed works in this thesis. The choice of language is indicative of the audience that Stevin intended to reach with this work. Whilst the *Art of Weighing* and *Mathematical Memoirs* primarily contextualise and elaborate on practical works such as the *Problemata Geometrica*, the *Arithmetic* is aimed at a specialist audience with regard to the theoretical advancement of mathematics.

The *Arithmetic* is divided into two books: *Definitions* and *Operations*. The first book is mainly concerned with the definitions of numbers and the explanations of his notation. The second book can be subdivided into three parts. The first two parts define the operations on integers, fractions and radicals. The third part, *Pratique*, describes a large variety of algebraic equations with the solutions of several second, third and fourth other polynomials. Stevin draws upon the work of his contemporaries such as the *Ars Magna* [2] by Gerolamo Cardano (published in 1545). The *Ars Magna* was the first work to systematically use the concept of negative numbers in order to solve cubic and quartic equations. Furthermore, the existence of square roots of a negative number was acknowledged, which preceded the contemporary notion of complex numbers.

The primary analysis of the *Arithmetic* was conducted by Dirk Struik in *Principal Volumes, IIB* [12]. It is important to note that Struik's research is mainly limited to an overview of the work's content. Contrary to his work on the *Mathematical Memoirs* and Dijksterhuis' research on the *Art of Weighing*, he often does not give complete translations of the *Arithmetic*. Furthermore, Struik omitted large parts of the second and third part of the *Operations* (in the same manner as he omitted the lion's share of the *Mathematical Memoirs*). He generally picked a couple of problems to illustrate Stevin's methods. Moreover, on the problems Struik did discuss, there is little discourse analysis and only partial explanations of the presentist notation he applies to his descriptions of Stevin's methods.

6.1 Definitions and Notation

The *Definitions* contains 103 definitions in total (of which half is not included in the *Principal Works, Volume IIB*). This section discusses the most important definitions which are relevant to the analysis of the solutions of quadratic equations in Section 6.2. Furthermore, the most important features of Stevin's notation also feature in this section.

Definition 6.1. Arithmetic is the science of numbers.

Definition 6.2. A number is that by which the quantity of each thing is explained.

In the first two definitions of the *Arithmetic* Stevin defines arithmetic as the science of numbers. Stevin argues in Definition 6.2 that the concept of a number can be explained by the amount which is ascribed to a unit of measurement, which is important to explain the philosophical concept of a number. However, he links the denomination of amount and unit by his phrase "QUE L'UNITE EST NOMBRE" which can be translated as "That the unity is the number".

Since Stevin only uses integers, fractions and radicals in his work on the quadratic equation these definitions suffice. Furthermore, Stevin does not state a definition for real numbers and does not use the notion of imaginary numbers (despite works of his contemporaries such as Cardano already using square roots of negative numbers).

It is important to note that Stevin uses the exponential notation from the *Algebra* (published in 1572) by his contemporary Rafael Bombelli. Stevin uses this notation to describe the unknown quantities in his algebraic notation. If we denote this quantity with x the following presentist equality holds:

$$\begin{aligned} ① &= x, \\ ② &= x^2, \\ ③ &= x^3, \\ ④ &= x^4. \end{aligned}$$

In the *Arithmetic*, Stevin does not discuss exponentials beyond the fourth power. Notably, he also uses the symbol ⓪ to denote the coefficient which is not multiplied by a power of the unknown variable. For instance, in case of the following equation

$$4x^2 + 3x + 2$$

the following notation was used by Stevin (in some cases the + sign was omitted if it did not cause confusion)

$$4 ② + 3 ① + 2$$

where the symbol ⓪ could be used to reference the 2 in this equation.

6.2 Quadratic equations

In the *Arithmetic*, Stevin describes a large variety of equations as problems ('problèmes') which he solves by using constructions ('constructions'). The *Arithmetic* covers a large variety of problems which are related to the quadratic formula. Since Stevin tries to avoid the concept of negative numbers many different versions of the following standard quadratic equation feature in his work:

$$ax^2 + bx + c = 0,\ a \neq 0. \tag{12}$$

Although Stevin applies various methods to solve different versions of the quadratic equation, this chapter will focus on Stevin's geometirc methods to solve the following two equations:

$$x^2 = bx + c,\ b > 0 \text{ and } c > 0, \tag{13}$$

$$x^2 + bx = c,\ b > 0 \text{ and } c > 0. \tag{14}$$

Stevin referenced these equations in the following way (including the dots) leaving out the coefficients:

$$① + ⓪. \tag{15}$$

$$-① + ⓪. \tag{16}$$

The upcoming subsections feature the solutions of Equation (15) and (16) since these are indicative of the geometric methods which Stevin included in the *Arithmetic*.

6.2.1 Historist Analysis

In order to derive the solutions of the Equations (15) and (16) Stevin used both a construction by which the unknown ('fourth') quantity was reached as well as a geometric method to visualise and determine the solution. This section contains the geometrical methods which were used. According to Struik, these methods already featured in Greek and Arab mathematics and were used as the primary verifications for solutions to these problems. Crucially, geometric methods only allow for positive, real solutions of quadratic equations, which needs to be taken into account.

There are a number of remarks that need to be made prior to the translations. Firstly, Stevin regularly does not properly introduce geometric concepts such as points, lines or squares in his explanation. For instance, he mentions a line segment to a point before actually specifying this point. Depending on the accompanying information in the sentence additional information is given or omitted.

Secondly, Stevin often repeats statements on the value of line segments or the area of squares. These have been largely omitted from the translations in favour of clarity and conciseness.

Thirdly, Stevin frequently phrases his sentences as enumerations in such a way that they cannot be translated directly in a grammatically correct manner. This has been taken into account and the translation is done in such a way that it is representative of the explanation of Stevin's explanations.

Fourthly, Stevin does not use terms for either length or area but merely states lines and figures to be equal to a certain value. The denotations of 'length' and 'area' have been added for clarity.

In the case of the first proposition, Stevin demonstrates his method with values $b = 4$ and $c = 12$ in the presentist Equation (13). The visual representation of his approach is given in Figure 17.

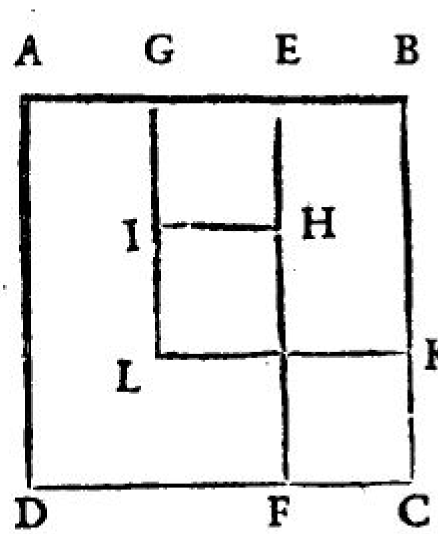


Figure 17: The square used in the geometrical method to solve Equation (15).

Proposition 6.3. *The solution to the equality of* 1 ② *with* 4 ① + 12 *is* ① = 6.

Proof. Let $ABCD$ be a square, with area denoted by 1 ② such that its side AB (which will have the value 6 at the end of the demonstration) has a length of 1 ① because multiplying this with 1 (1) grants 1 (2). Then let a line EF be parallel to AD such that AE has value 4. Consequently, the rectangle $AEFD$ (with AD having value (1)) will then have area 4 (1). Since the entire square $ABCD$ has area 1 (2) which equals 4 (1) + 12 and the rectangle $AEFD$ has area 4 (1) the rectangle $EBCF$ has area 12. Now we can construct the required quantity. Firstly, we take half of the side AE and call it GE with a length of 2. The square $GEHI$ has an area of 4. Then we add the quantity (0) (which is the square $EBCF$ with area 12) such that the surface $HIGBCF$ has area 16. In that case, we can draw a square $GBKL$ with side BK with a length of 4 such that the area of the square equals that of $HIGBCF$. Then add the side BK to the side KC (which equals GE with a length of 2) to get BC with a length of 6. This is what we needed to demonstrate. □

Notably, Stevin does not specify the reason why $GBKL$ should be square in such a way that KC equals GE. This could have been demonstrated by extending the line HI to the side BC with intersection point H'. In a similar way as AE was defined, we could state that $H'I$ is drawn in such a way that CH' has a length of 4. In that case, K is at the midpoint of CH' such that $KH' = KC = 2$ (in a similar way as $AG = GE = 2$ by the definition of G). Then, it can be stated that $KC = GE = 2$.

Stevin demonstrates his second method with values $b = 6$ and $c = 16$ in the presentist Equation (14). The visual representation of his approach is given in Figure 18.

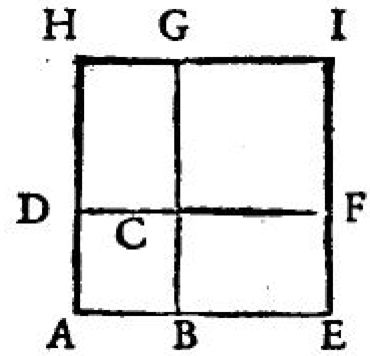


Figure 18: The square used in the geometrical method to solve the second version of the quadratic equation (16).

Proposition 6.4. *The solution to the equality of* 1 (*2*) *with* -6 (*1*) $+ 16$ *is* (*1*) $= 2$.

Proof. Let $ABCD$ be a square, with area denoted by 1 (2) such that its side AB (which will have the value 2 at the end of the demonstration) has a length of 1 (1). Then draw the line AB to E, which creates BE and from BE and BC the rectangle $BEFC$ can be created. Similarly, draw a line from BC to G and let CG be 3. From CG and CD, we can create the rectangle $DCGH$. The (area of the) rectangle $BEFC$ is 3 (1) (since EB is 3 and BC is (1)). Then the square $ABCD$ with area 1 (2) is equal to -6 (1) $+ 16$ and the two rectangles $BEFC$ and $DCGH$ are equal to -6 (1). Furthermore, the area of $CFEAHG$ will be 16. We now conduct the construction for the quantities. FC and CG are 3. The square $CFIG$ is 9. Similarly, add the quantity (0) (i.e. the $CFEAHG$ with area 16). Then the square $AEIH$ is 25 with root 5 for the side AE. In the same manner, subtract BC (which is 3) from AE to get AB (with $5 - 3 = 2 = 1$). This is what we wanted to demonstrate. $\square$

6.2.2 Presentist Analysis

In the current paradigm, the solution for the roots of Equation (12) is of the following form:

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{17}$$

Using the geometric methods which have been described in Propositions 4.14 and 6.4, the generalised versions of these propositions will be given in the current paradigm. This means that no specific value is ascribed to the coefficients b and c such that a version of Equation (17) is derived. In the presentist analysis, the surface of a figure $ABCD$ is denoted with $O(ABCD)$. The presentist, generalised version of Proposition 4.14 is the following proposition.

Proposition 6.5. *The positive real solution to $x^2 = bx + c$ is:*

$$x = \frac{b + \sqrt{b^2 + 4c}}{2}.$$

Proof. Starting with the square in Figure 17, we take the side $AB = BC = x$ such that the area of the square $ABCD$ is x^2. Subsequently, the side EF is drawn parallel to AD in such a way that $AE = b$. Therefore, $O(AEFD) = bx$. By the presupposition, we derive that:

$$O(EBCF) = O(ABCD) - O(AEFD) = x^2 - bx = c. \tag{18}$$

Now we take the point G on the line AE in such a way that the following holds:

$$GE = \frac{1}{2}AE = \frac{b}{2}.$$

Hereafter, we take the points H, I, K and L in such a way that $GEHI$ and $GBKL$ are squares with the following surface area:

$$\begin{aligned} O(GEHI) &= \frac{1}{4}b^2, \\ O(GBKL) &= O(EBCF) + O(GEHI) = \frac{1}{4}b^2 + c. \end{aligned}$$

We can now conclude that

$$BC = \frac{b}{2} + \sqrt{\frac{b^2}{4} + c} = \frac{b + \sqrt{b^2 + 4c}}{2},$$

which gives the desired formula. $\square$

The presentist, generalised version of Proposition 6.4 is the following proposition.

Proposition 6.6. *The positive real solution to $x^2 + bx = c$ is:*

$$x = \frac{-b + \sqrt{b^2 + 4c}}{2}.$$

Proof. Starting with the square in Figure 18, we take the side $AB = BC = x$ such that the area of the square $ABCD$ is x^2. Then we draw the line BE in such a way that it has the following length:

$$BE = \frac{b}{2}. \tag{19}$$

Similarly, the line CG is drawn with a length of $\frac{b}{2}$. Subsequently, the rectangles $DCGH$ and $BEFC$ can be drawn with:

$$\begin{aligned} O(DCGH) &= \frac{b}{2} \cdot x = \frac{bx}{2}, \\ O(BEFC) &= \frac{b}{2} \cdot x = \frac{bx}{2}. \end{aligned}$$

The area of $CFEAHG$ is in that case:

$$\begin{aligned} O(CFEAHG) &= O(ABCD) + O(DCGH) + O(BEFC), \\ O(CFEAHG) &= x^2 + \frac{bx}{2} + \frac{bx}{2}, \\ O(CFEAHG) &= x^2 + bx = c. \end{aligned}$$

Now we can draw the square $CFIG$ with the following are:

$$O(CFIG) = \left(\frac{b}{2}\right)^2 = \frac{b^2}{4}.$$

We can then derive an equation for the area $O(AEIH)$:

$$O(AEIH) = O(CFIG) + O(CFEAHG) = \frac{b^2}{4} + c.$$

Since $AEIH$ is a square the side AE has the following length:

$$AE = \sqrt{\frac{b^2}{4} + c} = \frac{1}{2}\sqrt{b^2 + 4c}$$

In conclusion, by using Equation (19) the following value for x is derived:

$$x = AB = AE - BE = \frac{1}{2}\sqrt{b^2 + 4c} - \frac{b}{2} = \frac{-b + \sqrt{b^2 + 4c}}{2},$$

which is the positive solution we were looking for. □

7 Historiography

This chapter gives a short overview of the historiography of the works by Simon Stevin. The influence of other mathematicians on Stevin's work is discussed as well as the reasons that relatively little research has been conducted on his contributions.

Stevin's work on the problems in this thesis clearly shows the influence of several paradigms. The definitions in the *Art of Weighing* suggest the influence of Aristotle on the structure of his definitions. Furthermore, Dijksterhuis noted the influence of Archimedes on the contents of the *Art of Weighing*, as Stevin seems to have drawn upon his work on mechanical systems. In the *Mathematical Memoirs* the didactic procedures resemble the methods which Euclid employed. In the *Arithmetic* the geometric methods are also present in works by Ancient Greek and Islamic authors (such as al-Khwarizmi). This is indicative of the influence of Greek and Islamic mathematics on Stevin's work.

Stevin's work is also influenced by the work of his contemporaries. The *Mathematical Memoirs* contains references to the works of Del Monte, whilst the *Ars Magna* by Cardano and the *Algebra* by Bombelli have contributed to the *Arithmetic*. The fact that the *Arithmetic* was written in French also suggests a more specialised, transnational audience of mathematicians.

After considering all of this, one is also curious about the relatively small amount of research on Stevin's work in physics and mathematics. The scientific analysis of Stevin's work is primarily limited to the *Principal Works of Simon Stevin*. Several reasons can be given for this.

Firstly, the paradigm in which Stevin operated was largely superseded by the work of his successors. For instance, Stevin rejected the concept of imaginary numbers, whilst contemporaries such as Cardano already used similar notions. This hampered progress on the methods by which the solutions of polynomial equations could be established. Furthermore, the geometric methods only yielded positive solutions to polynomial equations. Although Stevin does acknowledge the notion of negative numbers, the geometric methods limited any progress on the attainment of negative solutions to polynomial equations. As for the *Art of Weighing*, his notions of 'evenstaltwichtigheid' and 'staltwicht' did not gain much traction with the rest of the scientific community and this conceptualisation would be replaced by Newtonian mechanics. This played a part in Stevin being relatively overshadowed compared to his contemporaries and direct successors.

Secondly, the large variety of topics which Stevin worked on during his lifetime seems to have been to the detriment of his remembrance. Stevin was not forgotten, but his contributions to various fields were often discussed separately by later authors, which obscured the extent and larger significance of his life's work. In *Simon Stevin 1548-1948* a list was compiled of references to Stevin in publications. However, these publications cover a vast quantity of different topics and these references are largely cursory or part of a wider analysis of the development of the respective field through the ages. Dijksterhuis observed that this was also the case for works on mathematics and physics in which Stevin was referenced.

Thirdly, the Dutch language probably also hampered academic research on an international scale. The *Arithmetic* was published in French and there would be a later volume of the *Mathematical Memoirs* in Latin, but the majority of Stevin's works (like the *Art of Weighing*) were published in Dutch and would never be published in another language.

Fourthly, the scant source material on Stevin apart from his works does not serve his cause either. Apart from the supposition by Dijksterhuis and other biographers of Stevin's illegitimate birth, we can point to the unstable societal context in which Stevin was born and operated as the Dutch Revolt (1568-1648) raged during the majority of his lifetime.

These factors have probably caused the relative lack of attention Stevin has received in scientific historical research.

8 Conclusion

It is evident that Stevin's work contains a great variety of scientific knowledge with significant contributions to the fields of mathematics and physics. His methods are a fine example of a synthesis of expertise from Greek mathematics, Islamic mathematics and the work of his contemporaries and can be viewed as part of the Scientific Revolution with regard to the mathematization and the mechanisation of the scientific perspective on the physical world. These methods also resemble several characteristics of current methods in both mathematics and physics.

The example of the 'Clootcransbewijs' is a significant result in the field of mechanics and Stevin's arguments already foreshadow characteristics of later Newtonian mechanics. His largely theoretical approach to mechanical problems is also indicative of his mathematical approach towards physical problems instead of a largely empirical approach.

The *Mathematical Memoirs* delves into several important problems relating to geometry and other physical fields such as optics. With regard to the calculation of the circumference of an ellipse, Stevin deals with a highly interesting problem as no analytical solution exists to calculate the circumference of an ellipse. His division of the upper half of an ellipse into segments (with different methods to obtain their lengths) is already indicative of the later formula for the calculation of the length of a graph.

The *Arithmetic* contains a large variety of solutions on quadratic, cubic and quartic equations. The evaluated problems in this thesis are an accurate representation of the methods by which Stevin obtained the solutions to quadratic equations. The geometric methods which Stevin implemented can still be used to determine the positive solutions of quadratic equations and show the influence of earlier periods of the development of mathematics on his work.

Although Stevin does feature in works on the history of science, it is only modestly and often gives a limited overview of the extent of his work. A combination of the large diversity of contributions, a lack of source material on the man himself and overshadowing successors have probably influenced this. A lot of research still needs to be done on Stevin's contributions to mathematics and physics, but it is certainly recommendable. The research on Stevin is also a fine opportunity to demonstrate the value of interdisciplinary research with regard to the fields of mathematics, physics and history.

References

[1] Robert A Adams and Christopher Essex. *Calculus: a complete course*, volume 10. Pearson, 2022.

[2] Girolamo Cardano. *Ars magna*. 1545.

[3] Eduard J. Dijksterhuis. *Simon Stevin*. M. Nijhoff, 1943.

[4] Eduard J. Dijksterhuis. *Simon Stevin: science in the Netherlands around 1600*. Springer Science & Business Media, 2012.

[5] Mariano Giaquinta and Stefan Hildebrandt. *Calculus of variations I*, volume 2. Springer Science & Business Media, 2004.

[6] Thomas S. Kuhn. *The structure of scientific revolutions*. University of Chicago press, 2012.

[7] Raymond A Serway and John W Jewett. *Physics for scientists and engineers*. Cengage learning, 2018.

[8] Simon Stevin. *L'Arithmétique*. Christoffel Plantijn, 1585.

[9] Simon Stevin. *De Beghinselen der weeghconst*. Christoffel Plantijn, 1586.

[10] Simon Stevin. *Wisconstige gedachtenissen*, volume 1. 1608.

[11] Simon Stevin and Eduard J. Dijksterhuis. *The Principal Works of Simon Stevin: Mechanics*, volume 1 of *The Principal Works of Simon Stevin*. C.V. Swets & Zeitlinger, 1955.

[12] Simon Stevin, Dirk. J. Struik, and Ernst Crone. *The Principal Works of Simon Stevin: Mathematics*, volume 2B of *The Principal Works of Simon Stevin*. C.V. Swets & Zeitlinger, 1958.

[13] Albert van de velde. *Simon Stevin 1548-1948*. 1948.

[14] Zhen-Hang Yang, Yu-Ming Chu, and Wen Zhang. High accuracy asymptotic bounds for the complete elliptic integral of the second kind. *Applied Mathematics and Computation*, 348:552–564, 2019.