Some comparisons between incompleteness in arithmetic and set theory

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27 February 2009

The truth or falsity of familiar examples of sentences undecidable in Peano Arithmetic is known. This is immediate for Π_1-sentences such as Gödel and Rosser sentences (whose negations are Σ_1 so if not provable then false, since PA is Σ_1-complete). Other familiar examples of sentences undecidable in PA such as the Paris-Harrington sentence and Goodstein's theorem, which are Π_2 so not known to be true from their undecidability, are nonetheless known to be true. There are, however, Δ_2-sentences of a kind constructed by Kreisel that are undecidable in PA and whose truth value is unknown. The most famous undecidable sentence whose truth value is unknown is Cantor's Continuum Hypothesis. I will consider whether the nature of incompleteness in set theory can be illuminated by comparing these two cases.