Feel like writing a review for The Mathematical Intelligencer? You are welcome to submit an unsolicited review of a book of your choice; or, if you would welcome being assigned a book to review, please write us, telling us your expertise and your predilections.

➢ Column Editor: **Osmo Pekonen**,
Agora Centre, 40014
University of Jyväskylä, Finland
e-mail: osmo.pekonen@jyu.fi

Einstein's Unification

by Jeroen van Dongen

CAMBRIDGE: CAMBRIDGE UNIVERSITY PRESS, 2010, X + 213 PP., £ 50, US $85, ISBN: 9780521883467 (HARDBACK)

REVIEWED BY N. P. LANDSMAN

This book is about the interaction between mathematics and physics in the work and thought of Albert Einstein, with emphasis on his later years. In his early work, culminating in his *annus mirabilis* 1905, Einstein's use of mathematics was elementary; indeed, the picture of space-time as a four-dimensional manifold that has become standard was introduced in 1907 by the mathematician Hermann Minkowski, rather than by Einstein himself (though based on the special theory of relativity formulated by the latter in 1905 in terms of clocks, rods, and light signals). This changed in 1912, when Einstein's friend and (ETH Zürich) colleague Marcel Grossmann introduced him to Riemannian geometry and the closely associated tensor calculus of Gregorio Ricci-Curbastro and his pupil Tullio Levi-Civita. This turned out to be a decisive event in the history of science, as Einstein's subsequent application of this area of mathematics to the physics of gravity climaxed in his General Theory of Relativity of November 1915, surely a high point in human thought comparable with (and to some extent superseding) Isaac Newton's *Principia* of 1687.

But here the controversy starts. First, there seems to be a significant discrepancy between Einstein's later recollection of his creation of General Relativity – for example, in his autobiographical notes (Einstein, 1949) – and the careful and detailed recent reconstruction of this process by historians of science based at the Max Planck Institute for the History of Science at Berlin (Renn, *et al.*, 2007). In Einstein's own view (Einstein, 1949), mathematical intuition and deduction had played the essential creative role:

> I have learned something else from the theory of gravitation: no ever so inclusive collection of empirical facts can ever lead to the setting up of such complicated equations [i.e., Einstein's field equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$]. A theory can be tested by experience, but there is no way from experience to the setting up of a theory. Equations of such complexity as the equations of the gravitational field can be found only through the discovery of a logically simple mathematical condition …

On the other hand, a study of his notebooks and other sources displays a constant interplay between physical and mathematical arguments, where only at the very end Einstein

DOI 10.1007/s00283-011-9202-y

Published online: 01 April 2011

indeed used the mathematical requirement of general covariance (i.e., the condition of invariance under general coordinate transformations) to clinch the issue. And even so, this requirement was (mistakenly) felt by Einstein to be a reflection of two *physical* ideas that had guided him toward his theory of gravity right from the beginning, namely the equivalence principle (between free fall in a homogeneous gravitational field and inertial motion in the absence of gravity) and the idea of general relativity of motion (cf. Norton, 1993).

So it seems that Einstein's memory was colored by the course his post-1915 work had taken: sidestepping the quantum theory, which initially he had pioneered himself (but whose probabilistic character he famously rejected from 1926 onward), he put most of his effort into attempts to create a unified field theory of classical gravity and electromagnetism, hoping to recover quantum phenomena (such as elementary particles) in the guise of singularity-free solutions of the classical field equations he sought.

Jeroen van Dongen's remarkable book – an updated and revised version of his Ph.D. Thesis of 2002, meanwhile matured through the author's research in the Berlin group just mentioned and subsequently at the Einstein Papers Project based at CalTech – offers a delightful tour through Einstein's efforts in that direction, constantly analyzed from the dialectics of Einstein's use of (pure) mathematics versus actual physics. The author thereby establishes – and this may be said to be the main point of the book – a gradual shift from the latter (which formed the strength of Einstein's youth) to the former (which, some would say, marked his decline; see below).

Following the opening chapter reviewing Einstein's road to general relativity, as a second starter toward the main course we find a chapter on Einstein's method of theoretical physics. Helpfully, Einstein occasionally commented on his own methodology, as well as on the relationship between mathematics and physics in general: for example, his famous aphorism "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" is actually part of a very insightful essay (Einstein, 1921), in which he analyzes the said relationship in the light of the "modernist" tendency (initiated by David Hilbert) to strip mathematics of its traditional meaning in the physical world (cf. Gray, 2008). Elsewhere – notably in a letter to his friend Maurice Solovine from 1952, originally unearthed by Gerald Holton – Einstein explained how the uncertain application of mathematics to reality ought to proceed: from experiments E we infer mathematical axioms A for some physical theory, which (by a purely "logical" deductive path) implies certain assertions S about E. The inference from E to A is neither "logical" nor "inductive" – Einstein's criticism of induction predated Karl Popper's, and likewise for the falsification criterion – but is only "intuitive" or "psychological." But, as Einstein's own practice (or, more precisely, his own recollection thereof) shows, it is precisely at this stage that arguments related to mathematical simplicity, symmetry, beauty, etc., play a decisive role. The relationship between S and E is not "logical" either, because E is itself not of a logical nature. On the other hand, it is "far less uncertain than the relation of A to E."

The irony, then, is that in his later years Einstein increasingly failed to pay attention to the experiences E; apart from bypassing quantum mechanics, his unified field theories never took the strong and the weak nuclear forces into account. More generally, as van Dongen points out, the mathematically oriented top-down style of Einstein's later years sharply contrasted with the general trend of theoretical physics at the time, which was bottom-up, phenomenological, and relied on as little mathematics as possible – as a case in point, throughout his career Niels Bohr only used high-school level mathematics. To make things worse, even within the style he had chosen for himself, Einstein was outclassed by the mathematician Hermann Weyl, whose attempts to unify gravity and electromagnetism – though as unsuccessful as Einstein's *per se* – led him to the unprecedented "gauge principle" that forms the basis of modern quantum field theory and elementary particle physics.

More generally, though rightly recognized as one of the supreme geniuses humanity has produced, Einstein – unlike Newton within the same category – simply lacked the mathematical talent and creativity that would have been necessary to bring his program forward. This is particularly clear in an episode discussed in detail by van Dongen, namely Einstein's adventures with *semivectors* in the early 1930s. During his work on general relativity Einstein had become used to vectors and tensors, but the spinors introduced by Paul Dirac in 1928 (subsequently analyzed by mathematicians such as Weyl, Bartel Leendert van der Waerden, and Élie Cartan) were new and alien to him. Thus Einstein looked for analogous objects that did behave like vectors, coming up with the notion of a semivector. In the style of Dirac, he guessed a field equation for semivectors, which Einstein initially interpreted as a sensational prediction of *two* elementary particles, identified with the electron and the proton! Unfortunately, a straightforward group-theoretical analysis carried out by Valentin Bargmann (at the time a doctoral student at Zürich of Wolfgang Pauli's, one of Einstein's sharpest critics) showed that semivectors were just direct sums of Dirac spinors, so that Einstein's prediction had simply (though implicitly) been put in by hand. Einstein's subsequent work on Kaluza–Klein theory (a five-dimensional generalization of general relativity intending to unify gravity and electromagnetism, dating back to 1919) in the late 1930s and early 1940s was less ridiculous, but despite persistent effort and the presence of excellent collaborators such as Bargmann (who had moved to Princeton) and Peter Bergmann, it led to absolutely nothing, neither in physics nor in mathematics.

This sounds like a sad story, though van Dongen livens it with entertaining side information (e.g., on Einstein's collaborators), and also provides a psychological explanation, in that "an emotionally defining moment [i.e., the discovery of general relativity] was instrumental in locking him, eventually, in a belief in his idealized method and the pursuit of unified field theories …, validated by a one-sided recollection of the experience of [general] relativity." But where does this leave us in our perception of Einstein?

His scientific biographer Abraham Pais left no room for doubt in a TV-documentary on Einstein (Kroehling, 1991):

> In fact, in his first part of his life when he did his really important work, his notion of simplicity were [sic] the guide to the 20th century insofar as science is concerned. Later on I think he was just completely off base. I mean if Einstein had stopped doing physics in the year 1925 and had gone fishing, he would be just as beloved, just as great. It would not have made a damn bit of difference.

In addition, Pais all but ridicules Einstein's well-known criticism of quantum mechanics in his biographies of both Einstein (Pais, 1982) and Bohr (Pais, 1991), portraying the latter as the clear victor in the Bohr–Einstein debate on the foundations and validity of quantum theory. Similarly, van Dongen – without taking sides himself – quotes J. Robert Oppenheimer as saying that in his last 25 years Einstein had been "completely cuckoo," his work "a failure," and his attempt at unification a "hopelessly limited and historically rather accidentally conditioned approach."

However, over the past few decades a gradual reappraisal of the later Einstein has emerged. As to the main topic of his own book, van Dongen quotes string-theorist and popular science writer Brian Greene as saying that "Einstein was simply ahead of his time" with his unification program; indeed, Edward Witten – who has led the string theory program for the last 25 years – is sometimes portrayed as Einstein's successor. Similarly, literature on the foundations of quantum theory that has not been written by those under the personal spell of Bohr typically acknowledges the depth of Einstein's critique – even as late as 1935! – and shows its profound influence on the current debate (cf. Landsman, 2006, and references therein). Even Einstein's attempt to find particle-like solutions of classical field theories has been revived from the 1970s onward, notably in theories of solitons, magnetic monopoles, instantons, skyrmions, and the like (Rajaraman, 1982).

What remains is the fact that these days practically no one shares Einstein's rejection of quantum theory: the vast difference between his and current attempts at unification (such as string theory) is that the latter incorporate quantum (field) theory. What van Dongen has now shown is that this rejection was by no means a consequence of senility, but of a post-1915 research style that became increasingly dissonant with Einstein's contemporaries, such as Bohr. Let me add that if Einstein had absorbed as much as the Preface of Dirac's renowned book on quantum mechanics (Dirac, 1930), he would have been hooked:

> The formulation of these laws [of nature] requires the use of the mathematics of transformations. The important things in the world appear as the invariants … of these transformations. … The growth of the use of transformation theory, as applied first to relativity and later to quantum theory, is the essence of the new method in theoretical physics. Further progress lies in making our equations invariant under wider and still wider transformations.

Dirac here refers to the invariance of quantum theory under unitary transformations, clearly suggesting the analogy with the invariance of Einstein's theory of general relativity under general coordinate transformations – which invariance had been so dear to its creator! Indeed, although (in a different context) van Dongen mentions (and documents) the fact that Einstein "knew" Dirac's book, he rightly adds the crucial qualifying remark that Einstein "never seems to have internalized its perspective." Pity him!

REFERENCES

Dirac, P. A. M. (1930). *The Principles of Quantum Mechanics*. Oxford: Clarendon Press.

Einstein, A. (1921). Geometry and Experience. Translation by Sonja Bargmann of an expanded form of an Address to the Prussian Academy of Sciences in Berlin on January 27th, 1921. In: Albert Einstein, *Ideas and Opinions*, pp. 232–245. New York: Bonanza Books (1954).

Einstein, A. (1949). Autobiographical notes. In P. A. Schilpp (ed.) *Albert Einstein: Philosopher-Scientist*, pp. 1–94. La Salle: Open Court.

Gray, J. (2008). *Plato's Ghost: The Modernist Transformation of Mathematics*. Princeton: Princeton University Press.

Kroehling, R. (1991). *Albert Einstein: How I see the world*. PBS Home Video.

Landsman, N. P. (2006). When champions meet: Rethinking the Bohr-Einstein debate. *Studies in History and Philosophy of Modern Physics, 37*, 212–242.

Norton, J. (1993). General covariance and the foundations of general relativity: Eight decades of dispute. *Reports on Progress in Physics, 56*, 791–858.

Pais, A. (1982). *Subtle is the Lord: The Science and Life of Albert Einstein*. Oxford: Oxford University Press.

Pais, A. (1991). *Niels Bohr's Times: In Physics, Philosophy, and Polity*. Oxford: Oxford University Press.

Rajaraman, R. (1982). *Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory*. Amsterdam: North-Holland.

Renn, J., *et al*. (2007). *The Genesis of General Relativity*. Vols. 1–4. Boston Studies in the Philosophy of Science No. 250. Dordrecht: Springer.

Institute for Mathematics, Astrophysics, and Particle Physics
Radboud Universiteit Nijmegen
Heyendaalseweg 135
6525 AJ Nijmegen
The Netherlands
e-mail: landsman@math.ru.nl