Radboud University

"Why the quantum?" - *Bananaworld*

Submitted by

Myrthe Scheepers

Supervisor

Klaas Landsman

Bachelor Physics and Astronomy

Radboud University Nijmegen

2018

Abstract

Bachelor Physics and Astronomy

"Why the quantum?" - *Bananaworld*

by Myrthe Scheepers

In his book *Bananaworld: Quantum Mechanics for Primates*, Jeffrey Bub proposes that quantum mechanics is fundamentally a theory about the structure of information. He explains, by using the correlations between in- and outputs of simulation games with entangled bananas, how indeterminacy follows from nonlocality and relativistic causality. Even though this view seems to be completely different from more traditional views of quantum mechanics, such as a computational view, there are some interesting aspects in the relation between the two views. Discussing this relatively new view of quantum mechanics described by Bub might be really promising for the future of foundations of quantum mechanics.

Contents

Chapter 1

Introduction

There is much to say about quantum mechanics. That is one reason why it is hard to explain what this theory is really about. For example, one could say that quantum mechanics is about wave-particle duality, or about the uncertainty principle, or about quantized energy. All cover some aspects of quantum mechanics and none are completely right or wrong. And no matter what one says that quantum mechanics is about, all descriptions agree that quantum mechanics is, in at least some aspects, counterintuitive.

There are many different theories and interpretations of quantum mechanics, many of which can tell you a different story about its relation to reality or its implications. There are a couple of ways in which quantum mechanics is hard to understand. The most fundamental reason concerns the fact that quantum mechanics has not been understood completely, or that the theory is not 'finished.' You cannot understand something that has not been figured out yet. The most brilliant scientists do not agree on whether the theory is incomplete or limited in another way. They cannot explain why it describes some experiments and phenomena very well, but fails in other experiments or situations.

Another reason that quantum mechanics is hard to understand is that there are many different possible interpretations of quantum mechanics. As long as there is not just one clear answer to the question what quantum mechanics is, it will not get any easier to get a grip on quantum mechanics. Though some interpretations are more popular or easier to understand than others, there is no consensus about which interpretation fits the theory best.

The last reason that nobody understands quantum mechanics, is that some of the theories on its own are hard to understand and not easy to accept. Even if you have the proper knowledge of mathematics and physics to understand quantum mechanics, there will still be aspects of it that you cannot grasp or accept, because they are highly counterintuitive.

Studying physics in Nijmegen has taught me a lot about quantum mechanics. But the thing you mostly learn, or maybe 'do' is more appropriate for the courses that I am talking about, is how to use quantum mechanics to do calculations and make approximations in different situations. However, there is a whole other side of quantum mechanics that does not get much attention during the courses. That is the interpretation and the more conceptual side of it.

It is far too ambitious to try to find one superior characterization or explanation of quantum mechanics. So that is not the aim of this thesis. What I want to do instead is discover a completely different view of quantum mechanics from the minimal one that I was brought up with as a student. The view I am going to look into is the one discussed by Jeffrey Bub in his recent book *Bananaworld*. He claims that quantum mechanics is fundamentally a theory of information. Question I will take into account to discuss this view are:

- What are the main characteristics of this view?
- What are problems and/or unanswered questions of this view?
- In what way(s) is this view different from the minimal one?
- Is it possible to relate this view to the minimal view?

This thesis will therefore consist firstly of a brief introduction on the author of this book, and the book in general. Thereafter I will summarize some of the arguments and explanations important to understand the presented view that are described in *Bananaworld*. After this summary, the questions listed above will be discussed and the view of quantum mechanics as a theory of information will be compared to the minimal view of quantum mechanics. A review of the book is added in the appendix, to make sure the discussion of the presented view of quantum mechanics is not too much influenced by the presentation used in the book by Bub.

Chapter 2

Bananaworld

2.1 About the author and his work

Jeffrey Bub mainly focuses his work on the conceptual foundations of quantum mechanics. He is a philosopher of physics, but started out by studying pure mathematics and physics at the University of Cape Town. After those bachelor degrees, he did another bachelor degree in applied mathematics at the same university. Then he went to the University of London to do a PhD in mathematical physics, which he finished in 1966. His PhD-advisor was David Bohm, who had a great influence on the later work of Bub, and the foundations of quantum mechanics in general.

After a couple of years during which he worked at different universities, he became a full professor at the University of Maryland in 1986, and has worked there ever since. He became a Distinguished University Professor in 2007.

Bub has written three books about the interpretation of quantum mechanics. The first one, *The Interpretation of Quantum Mechanics*, is from 1974. The second book is entitled *Interpreting the Quantum World*, first published in 1997. With that book, Bub won the Lakatos Award of 1998, which is each year awarded to an English-language book that is an outstanding contribution to the philosophy of science.

Around 2000, Bub's interest changed from his work on a version of a dynamical collapse theory, to quantum information theory. One of the reasons for this change was a comment from Gilles Brassard about wanting to derive quantum mechanics from the possibility of secure key distribution and the impossibility of secure bit commitment. This got Bub really interested, and after discussions with Rob Clifton, they eventually wrote an article together with Clifton's student Hans Halvorson titled "Characterizing quantum mechanics in terms of information-theoretic constraints" (Clifton, Bub, and Halvorson, 2003). This article marked the beginning of Bub's work on quantum information theory (Bub, 2018a).

However, the formal framework of that article was still C*-algebras. After hearing Sandu Popescu talk about PR-boxes, Bub realized that by using that framework, one is looking at quantum mechanics 'from the inside'. The C*-algebraic formalism can only be used to describe classical and quantum correlations, but not all PR-correlations. That is why he started looking at quantum mechanics 'from the outside,' by looking at a broader class of no-signaling correlations. This new approach lead to new questions and eventually led to his new book *Bananaworld: Quantum Mechanics for Primates* (Bub, 2018a).

This book discusses various of the questions raised by the information theoretic approach to quantum mechanics. And with this book, Bub addresses a broader public, not necessarily highly educated in physics and mathematics.

2.2 About the book

Bananaworld: Quantum Mechanics for Primates was first published in 2016. A second edition was published very recently in May 2018. Apart from some minor corrections, the changes in the second edition mainly occur in the part called "The Information-Theoretic Interpretation".

Although the book is written for a public not only consisting of people with a solid background in physics and mathematics, it is not certainly a 'quantum mechanics for dummies'. The main topic of the book is to explain a new concept in the foundations of quantum mechanics. Even though it is written for a wide range of readers, it is just as interesting for the experts as it is for the non-experts.

The idea for *Bananaworld* came from the article 'Nonlocality beyond quantum mechanics' written by Popescu (Popescu, 2014), and the book *Quantum Chance: Nonlocality, Teleportation and Other Quantum Marvels* written by Nicolas Gisin (Gisin, 2014). Both works are about the conceptual revolution of quantum mechanics. This conceptual revolution started when people realized that quantum mechanics is not an incomplete theory with hidden variables, and started accepting it as a nonlocal theory (Bub, 2016, p. ix).

To make *Bananaworld* accessible to a broad public, much of the technical and mathematical details of the arguments are limited to "More"-sections at the end of each chapter and the supplement of the book called "Some mathematical supplement". The regular chapters and paragraphs of the book are as much limited to the conceptual arguments as possible.

The examples and arguments of *Bananaworld* are supported by drawings made by Bub's daughter, Tanya. She used figures inspired by the Tenniel drawings for *Alice in Wonderland* (Bub, 2016, p. ix). Just like in all papers on quantum information theory, Bub uses agents called Alice and Bob, who are in the book drawn as a girl and a rabbit respectively. I also used some drawings from the book for this thesis, for example on the title page.

Chapter 3

Summary

3.1 Bananaworld

> Bananaworld is an imaginary world in which there are classical and quantum correlations, but also superquantum correlations between separated systems that are even more nonclassical than the correlations of entangled quantum states. The conceptual puzzles of quantum correlations arise without the distraction of the mathematical formalism of quantum mechanics, and you can see what is at stake - where the clash lies with the usual presuppositions about the physical world (Bub, 2016, p. 7).

This is how Bub describes the thought experiment he calls 'Bananaworld' We will later discuss what is exactly meant by the different relations and how he uses the imaginary bananas in Bananaworld to explain the theory of quantum mechanics as a theory of the structure of information.

The bananas in Bananaworld can be compared to what is called a 'box' in other, maybe more known, explanations of correlations in quantum mechanics.

Just like the simplest boxes, the bananas in Bananaworld have two possible inputs. The inputs in the case of the bananas correspond to different ways of peeling, either from the top end or from the stem end, which are denoted by T and S respectively. It is important to note that each banana can only be peeled once. The bananas also have two possible outputs describing their taste called ordinary and intense, which correspond to 0 and 1 respectively, like in the explanations with boxes. The taste of a banana is supposed to be an objective fact, and does not depend on one's subjective opinion. The correlations that we will be talking about occur between the different ways of peeling and the tastes of the bananas (Bub, 2016, pp. 8-9).

Bub chose to use bananas instead of boxes because he thinks they make certain properties more concrete. For example, it is easier to think of two bananas to be far apart from each other, than it is to think of two parts of a box to be stretched apart. Another advantage of the bananas with respect to the boxes is that they can be used to describe for example correlations where more than two bananas are correlated (Bub, 2016, p. viii).

Even though the bananas Bub uses are a thought experiment, they can be useful to discover facts about the real world. It can mainly show that certain correlations can be simulated with quantum entangled bananas, but not with classical local resources (Bub, 2016, p. viii).

3.2 Correlations

Physics is all about correlations, or statistical dependence, and is especially about explaining why correlations occur in the circumstances in which they occur. However, we will see that the different kinds of explanations from which we can choose are very limited (Gisin, 2014, p. 7).

We distinguish different kinds of correlations which can appear between different events. The events are always separated from each other either spatially or temporally, or both. Correlations between events can be divided into different categories.

An important category of correlations that is relevant for this discussion is that of classical correlations, appearing in classical physics. Those correlations can either be explained by local resources or a common cause, and are therefore often called 'local correlations' (Bub, 2016, p. 10).

When a correlation can be explained by local resources, a signal can travel between the two events. However, that signal cannot travel faster than the speed of light, so there can be no instantaneous transfer of information between the different events (Bub, 2016, p. 11). This limit of the speed of light is a result of Albert Einstein's theory of special relativity. In this case, the events will be called causally separated, and in Minkowski space one of the events is in the so-called light cone of the other event.

In a correlation characterized by a common cause, the transfer of information seems to be traveling faster than the speed of light, so it seems instantaneous. But because special relativity does not allow this, these correlations in classical physics must be caused by a common cause lying in the past of both events. An example of such a common cause is a flash of lightning, which is the common cause for two events being two persons far away from each other both hearing thunder (Bub, 2016, p. 11). These events then will be called space-like separated, and lie both in the future light cone of the common cause event.

The correlations that cannot be explained by either local resources, or a common cause explanation, are called nonlocal correlations. Some of these correlations can be explained by quantum resources, which can either be local or nonlocal. A geometrical representation of what this looks like will be given and discussed in section 3.8. It is good to know that there is no advantage of local quantum resources over local classical resources. So we will often refer to correlations that can be simulated with local quantum resources as classical correlations (Bub, 2016, p. 11).

Our discussion based on *Bananaworld* will be about the correlations represented by the probabilistic correlation between the inputs (peelings) and outputs (tastes) of the bananas (or boxes, experiments, etc.).

3.2.1 Simulating correlation

To simulate different correlations, Bub uses a game played by Alice and Bob. The moderator of the game contacts both Alice and Bob, and gives them both individually a prompt (S or T) at the beginning of each round. They each respond with an answer (1 or 0). They win the round if the responses and the prompts satisfy a certain given correlation. The game is played over many rounds. They cannot have contact with each other during the game, but they are allowed to discuss a strategy before the simulation game starts (Bub, 2016, pp. 52-53).

For example, if the correlation they are looking for is such that they must respond with the same answer when the prompts are the same, and with a different answer if the prompts are different, the winning strategy is to respond with a 0 for S and a 1 for T (or the other way around) (Bub, 2016, p. 52).

In addition to the searched correlation, one can restrict Alice and Bob by demanding that the response of a given prompt has to be random. This would make them satisfy the the no-signaling principle, which is in the relativistic limit equivalent to the assumption of free choice, this means that, for example, the no-signaling principle in a certain inertial frame, can be the assumption of free choice when looked at from another inertial frame. Both will be discussed in section 3.4.2. For now, this means that the responses 0 and 1 should come up with equal probability over many rounds of the game when given the prompt S (and the same for T). Alice and Bob can still win this game by having a shared list of random numbers. They can agree on a list of randomly ordered 0's and 1's before the game, and use the strategy to answer the output on the list, or the opposite output, depending on the prompt. So when they are given the prompt S they respond exactly the bit on the list, but if the prompt is T then they give the opposite bit as an answer. Now the correlations is perfectly simulated, and the new condition of both answers turning up with an equal probability is satisfied (Bub, 2016, pp. 52-53).

In this example, Alice and Bob only used local resources to simulate the correlation, so you can conclude that this is a local correlation. However, this example was very easy, so we will look ate more complicated correlations in the next sections.

3.2.2 Measuring correlations

We discussed an example of how a simple correlation can be simulated by a game. But let's now look better at what the simulation represents and at what the correlation in the real world is. Bub uses his bananas, with inputs S and T, and outputs 0 and 1, to explain the correlations. It is, however, useful to remember that the correlations, even though most of the time the in- and outputs are not as simple as in the examples given, occur in our real world as well.

Bits

A classical 'bit' is a unit of information, best known for its use in computer science. It is two-valued, which means that it can have the value of either 0 or 1, denoting the two different 'states' in which the bit can be (Bub, 2016, p. 30).

Measuring an observable, for example a tossed coin, or the polarization direction of a photon, reveals the state of the measured coin or photon in these cases. The value of the measurement can only be 0 or 1, not any other value. What is important is therefore that the measured observable has only two possible outcomes, which you can call either 0 or 1, as long as you are consistent during the experiment.

The correlation can be seen as the distribution of probabilities for the outputs, given certain inputs. So in the example of the toss of a coin, the correlation, in case of an ideal coin, would be that both heads and tails, called 0 and 1 or the other way around, have an equal probability of $1/2$. As for the coin toss, a correlation will become more obvious when the experiment is repeated many times.

Qubits

There is also a quantum version of the classical 'bit', which we call a 'quantum bit', or 'qubit' in short. The measurement of a qubit gives one of two possible values, just like the measurement of a bit. However, a qubit can be seen as containing an infinite set of noncommuting two-valued observables that cannot all have definite values simultaneously (Bub, 2016, p. 30).

For example, when you measure a polarization direction of a photon, there are different directions in which you can measure, but you have to choose one. Each measurement in a different direction has two different outcomes. Therefore, you end up with one output, which you can call either 0 or 1, for the one measured polarization direction, or observable, of the photon. You cannot have information about any of the other observables at the same time.

Measuring the polarization of a photon in a certain direction forces it to randomly transition into that direction. The transition is detected by the measurement device as one of two possible outcomes in that polarization direction (Bub, 2016, p. 35). Keep in mind that this measuring does not reveal pre-existing properties of the photon; the measurement itself forces the photon to get into a state.

For simplicity, Bub discusses photons with two different observables. The first is measuring the polarization in the z direction, corresponding to observable Z. When this observable is measured, the result will be that the photon is in either one of the states labeled by $|0\rangle$ or $|1\rangle$. The other observable is measuring the polarization of the photon in the x direction, corresponding to an observable X. This results in the photon being in one of the states labeled by $|+\rangle$ or $|-\rangle$. The angle between Z and X is 45° (Bub, 2016, pp. 37-38). A representation of these states is shown in figure 3.1.

The states $|+\rangle$ and $|-\rangle$ of the X observable can be expressed in terms of the states of the Z observable. This gives a linear superposition:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \tag{3.1}$$

and

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle. \tag{3.2}$$

In this case the states $|0\rangle$ and $|1\rangle$ are called the 'basis', and the factors $\frac{1}{\sqrt{2}}$ are normalization constants. Analogously, the states of observable Z can also be expressed with the states of observable X as a basis (Bub, 2016, p. 39).

If you measure the observable Z of a photon in the state $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, it has a probability $\cos^2(\frac{\pi}{4}) = \frac{1}{2}$ of finding the state to be $|0\rangle$ and a probability $\sin^2(\frac{\pi}{4}) = \frac{1}{2}$ of finding the state to be $|1\rangle$. This is because of the angles between the different observables. The resulting probability of the output state is also equivalent to the square of the normalization constant directly before that output state (Bub, 2016, p. 40).

To get the analogue with the bananas: peeling a banana from the stem end corresponds to measuring observable Z, the polarization in a certain direction, and peeling from the top end corresponds to measuring observable X, the diagonal polarization at an angle $\frac{\pi}{4}$ to Z. The tastes 0 and 1 correspond to the two outcomes of the different measurements (Bub, 2016, p. 36).

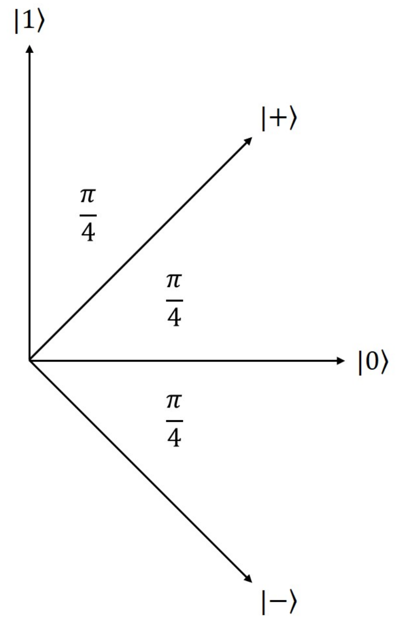

FIGURE 3.1: Directions of the different basis vector states belonging to observables Z and X of a photon.

Entanglement

A pair of qubits can be in a state that is a superposition of product states called an 'entangled state'. For example the entangled state of two photons can be expressed in the Z observable and X observable states as

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle = \frac{1}{\sqrt{2}}|+\rangle|+\rangle + \frac{1}{\sqrt{2}}|-\rangle|-\rangle \tag{3.3}$$

respectively. In this entangled state, each state of the product states can be attributed to one of the two entangled photons (Bub, 2016, p. 41).

Independently of whether you measure Z or X on both these photons, the probability of finding both photons in the state $|0\rangle|0\rangle$ or $|1\rangle|1\rangle$ if you measure Z, or in the state $|+\rangle|+\rangle$ or $|-\rangle|-\rangle$ if you measure X is $\frac{1}{2}$. This value for the probability can be found by the square of the normalization constant in front of the state in the entangled state (Bub, 2016, p. 41).

The possible outputs $|0\rangle|+\rangle$, $|0\rangle|-\rangle$, $|1\rangle|+\rangle$, and $|1\rangle|-\rangle$ all have an equal probability of $\frac{1}{4}$. The explanation for this value is that each photon has a probability $\frac{1}{2}$ of ending up in one of the two possible states of the measured observable. The product $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ gives the probability of the product state (Bub, 2016, pp. 41-42). The reason that this method of calculating the probability for a certain product state, i.e. by multiplying the probabilities of both photons, can only be used when different observables are measured on the photons, is that the observables are commuting. The results for measurements on different observables are uncorrelated, whereas the results for measurements of the same observable are correlated.

3.3 Some useful definitions

In this sections, some concepts that Bub uses throughout the book are defined more explicitly. They are also explained in parts of the summary, but they are briefly listed here so they can be consulted at any time.

Classical means that the correlation can be explained by either local resources, or a common cause. This is explained mainly in section 3.2.

Local resources are resources that communicate by sending a signal that can travel not faster than the speed of light. It is explained mainly in section 3.2.

Common causes are resources that seem to be sending an instantaneous message, but in which case the correlation can be explained by a common event in the past of the correlated events. This is explained mainly in section 3.2.

Nonlocal resources are resources that are not local, but still satisfy the no-signaling principle. These can be used to explain nonlocal correlations as described mainly in section 3.2.

Quantum resources contain all local resources, but include some, not all, nonlocal resources. The boundary between the quantum resources and the nonlocal resources that are not quantum resources is given by the Tsirelson bound. Quantum resources are entangled objects as explained mainly in section 3.2.2.

No-signaling is the condition that if Alice and Bob perform measurements on space-like separated locations, the marginal probabilities for Alice's measurement outcomes are independent of those for Bob's measurement outcomes. This signifies that it is not possible, given relativistic constrains, for Bob's choice of measurement to influence Alice's measurement outcomes. This is explained mainly in section 3.4.2.

Free choice is about the assumption that explains the result of measurement outcomes in local realistic terms. This comes down to the condition that the parameters of an experiment can be chosen freely. A parameter can be considered 'free' if it is statistically independent of all other parameters and observations that do not lie in its causal future. This is explained mainly in section 3.4.2.

3.4 Einstein-Podolsky-Rosen argument

In 1935 Albert Einstein, Boris Podolsky, and Nathan Rosen published their famous article in which they argued that quantum mechanics must be an incomplete theory

(Einstein, Podolsky, and Rosen, 1935). The argument presented in this article is often referred to as the 'EPR-argument'.

This argument is pretty controversial and thousands of papers have been written about it. But in short, and in modern language, the EPR-argument claims that assuming a strong version of locality entangled states in quantum mechanics must only involve classical correlations that can be simulated with local resources. They took this as a signal that quantum mechanics is incomplete, because they could, in theory, explain quantum phenomena with classical resources. They only did not know exactly which classical resources.

Later, Einstein published in an article in 1948 explaining that the EPR-argument rests on two important assumptions about locality (Einstein, 1948). The first assumption is called the 'separability assumption', and means that two space-like separated objects have their own 'being-thus', which is an independent state of existence. The second assumption is called the 'locality assumption', and means that if objects A and B are separated, an event in A cannot instantaneously affect the 'being-thus' of B, and conversely (Bub, 2016, p. 46). These assumptions are the basis of Einstein's concept of local realism (Gisin, 2014, p. v).

3.4.1 Correlation

As Bub describes in *Bananaworld*, one could view the EPR-argument in terms of a correlation. He describes the EPR-correlation as follows (Bub, 2016, p. 49):

- if the same observable is measured on A and on B, the outcomes are the same, with equal probability of $\frac{1}{2}$ for 00 and 11;
- if different observables are measured on A and on B, the outcomes are uncorrelated, with equal probability of $\frac{1}{4}$ for 00, 01, 10, 11;
- the marginal probabilities for each possible outcome, 0 or 1, when either observable is measured on A, or when either observable is measured on B, are $\frac{1}{2}$, irrespective of what observable is measured on the paired particle, or whether any observable is measured at all.

Here 00 means that both outputs for A and B are 0, and 01 means that the output for A is 0 and the output for B is 1, etc.

Translated into terms of Bananaworld, this correlation becomes (Bub, 2016, p. 50):

- if the peelings are the same, SS or TT, the tastes are the same, with equal probability of $\frac{1}{2}$ for 00 and 11;
- if the peelings are different, ST or TS, the tastes are uncorrelated, with equal probability of $\frac{1}{4}$ for 00, 01, 10, 11;
- the marginal probabilities for the tastes 0 or 1 if a banana is peeled S or T are $\frac{1}{2}$, irrespective of how the paired banana is peeled, or whether or not the paired banana is peeled.

This time, the input SS means that both Alice and Bob peel their entangled banana from the stem end, and ST means that Alice peels her entangled banana from the stem end and Bob peels his entangled banana from the top end, etc. Analogously for the outputs.

3.4.2 Local simulation

To successfully simulate this correlation using local resources only means that the no-signaling principle is satisfied. This principle is contained in the last condition of the correlation, namely that the probability for the two possible outputs must be equal to $\frac{1}{2}$, irrespective of which observable is measured on the other photon or banana, or whether an observable is measured at all on the other photon or banana (Bub, 2016, p. 51).

In the relativistic limit, the no-signaling principle is equivalent to what is called the 'assumption of free choice'. In Bananaworld, this assumption means that Alice's choice of peeling her banana from the stem or from the top end is independent of Bob's choice of the peeling of his banana (Bub, 2016, p. 95).

A strategy to simulate the EPR-correlation, without violating the no-signaling principle and the assumption of free will would be to give Alice and Bob each two lists of classical bits. The lists must contain at least as many bits as rounds of the game they are going to play. They both get the same two lists, one for the S-peeling, called the S-list, and one for the T-peeling, called the T-list. They then use the following strategy (Bub, 2016, p. 53):

- if the prompt is S, the response is a bit from the S-list;
- if the prompt is T, the response is a bit from the T-list.

This strategy has a success rate of 100%. For if they get the same prompt, they respond with the same bit, so the output is the same. But when they both get different prompts, they respond with a bit from different lists and the chance of responding with the same bit is $\frac{1}{2}$, because of the randomness of the lists.

Figure 3.2 shows an illustration from *Bananaworld* of the local simulation of the EPR-correlation (Bub, 2016, p. 54).

In this simulation, the shared random variable is the intrinsic property, or 'being-thus', of the bananas used in the game. Each pair of random bits from one of the lists Alice and Bob have, is a shared random variable of the bananas (Bub, 2016, p. 53). Like the lists that Alice and Bob have were generated before the simulation game and are the same, this is a common cause explanation for the successful simulation of the EPR-correlation.

3.4.3 Consequences

The conclusion Einstein, Podolsky, and Rosen draw from their argument is derived from the possibility to simulate the classical correlations, which are involved in entangled states of quantum mechanics that local resources (Bub, 2016, p. 49). In the simulation of the EPR-argument, the local resources are the 'being-thus' of the systems, which tells them there is a common cause explanation of the correlation. Therefore, a causal relationship between the two systems can be excluded and the no-signaling principle is satisfied (Bub, 2016, pp. 54-55).

The 'being-thus' of the system is an element of reality which quantum mechanics does not describe. There is no common cause variable in the description, like $|\phi^+\rangle$, of a quantum state. This is the argument Einstein, Podolsky, and Rosen used to claim that quantum mechanics must be an incomplete theory (Bub, 2016, pp. 54-55).

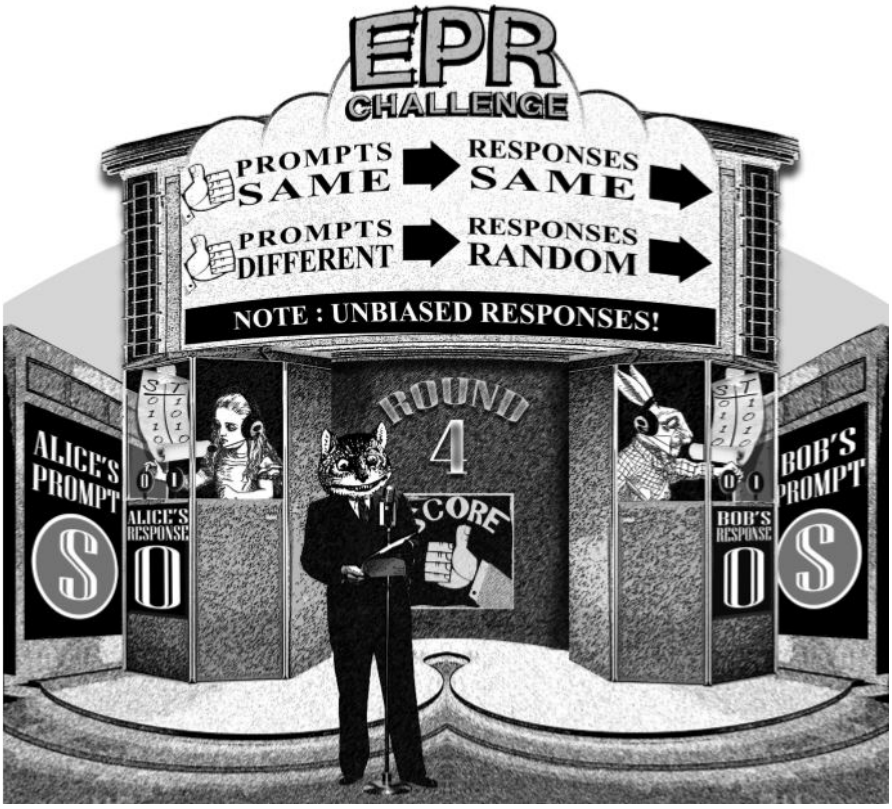

FIGURE 3.2: The Einstein-Podolsky-Rosen simulation game. Alice and Bob can win the game if they respond on the basis of two shared random lists of bits that they consult in order for each round of the game, an S-list for S prompts and a T-list for T prompts. This is Round 4, the prompts are both S, so according to the shared S-list, Alice and Bob both respond with a 0 (Bub, 2016, p. 54).

3.5 Bell's inequality and Bell's theorem

A little less than thirty years after the EPR-argument, John Bell introduced what is now known as 'Bell's inequality'. Every common cause explanation of a probabilistic correlation between outcomes of measurements on two separate systems would have to satisfy this inequality (Bub, 2016, p. 56).

Bub's expression for Bell's inequality in the case of the simulation game is given by

$$\frac{1}{4} \leq p_L(\text{successful simulation}) \leq \frac{3}{4} \tag{3.4}$$

where p_L is the probability of success with local resources (Bub, 2016, p. 62). The boundaries of p_L will be discussed, based on the Popescu-Rohrlich correlation, together with the proof of the inequality, in section 3.6.3.

Bell also showed that, when the inequality is violated, there are possible correlations between two quantum systems without a common cause explanation and causal influence between the two systems. This result is known as 'Bell's theorem' (Bub, 2016, p. 56).

Bell's original paper in 1964 tries to prove that not all correlations can be explained by common causes. Bell wrote

> In a theory in which parameter are added to quantum mechanics to determine the results of individual measurements [...] there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously (Bell, 1964, p. 199).

He claims that even if a hidden variable would be introduced to quantum mechanics, it should be nonlocal. Bell's paper was intended to show that correlations explained by a hidden variable, satisfy a certain inequality, and was not directly about other possible explanations of the correlations. However, the paper is highly relevant because violation of the inequality is realized by the correlations of entangled quantum states (Bub, 2016, p. 60). This violation will be discussed in section 3.6.5.

Bell's inequality can be violated by measurements of certain two-valued observables of a pair of quantum systems in an entangled state. Bell looked at the correlation between outcomes of measuring different observables on two separated entangled systems, whereas Einstein, Podolsky, and Rosen based their argument on the correlation between outcomes of measuring the same observable on two separated entangled systems (Bub, 2016, pp. 56-57). This proves that Einstein's intuition about the correlations of entangled quantum states, and the EPR-argument that depends on this intuition, is wrong.

It took a number of years before Bell's theorem and its implications became known in the physics community. But after Alain Aspect and his colleagues did an experiment on entangled photons and successfully confirmed the violation of Bell's inequality in the 1980s, more people became familiar with Bell's theorem. In the 1990s this even lead to a revolution in quantum information, which made foundational questions of quantum mechanics more popular (Bub, 2016, p. 56).

3.6 Popescu-Rohrlich argument

Bub does not follow the exact argumentation of Bell, instead he uses a correlation introduced by Sandu Popescu and Daniel Rohrlich to derive Bell's theorem. This method fits the other argumentation in the book, based on the simulation game played by Alice and Bob, much better (Bub, 2016, p. 57).

The Popescu-Rorhlich argument, called PR-argument from now on, shows restrictions of simulating correlations when one is limited to using local resources only. It also shows the possibility to have nonlocal correlations that are stronger than the correlation of entangled quantum states, without violating the no-signaling principle (Bub, 2016, p. vii). The original PR-argument is based on the 'PR-box', this gives the PR-correlation that can be used for the bananas in Bananaworld too.

In the article in which they introduced the PR-box, Popescu and Rohrlich changed the usual order in quantum mechanics, which is to derive nonlocality as a theorem from indeterminacy as an axiom. They took nonlocality and relativistic causality as axioms to derive indeterminacy as a theorem. Their goal was to get a more fundamental description of quantum mechanics from this new approach. Bub uses some of the same conclusions in *Bananaworld* as Popescu and Rohrlich, such as, the so-called Tsirelson bound, and nonlocal correlations violating a version of Bell's inequality: the CSHS-inequality (Popescu and Rohrlich, 1994). These conclusions will be discussed in sections 3.6.6 and 3.6.7 respectively.

3.6.1 Correlation

The correlation proposed by Popescu and Rohrlich is as follows (Bub, 2016, p. 57):

- if the inputs are 00, 01, 10, the outputs are the same, but if the inputs are 11 the outputs are different;
- the marginal probabilities of the outputs 0 or 1 for any input separately are $\frac{1}{2}$.

In this case the two possible inputs of the PR-box are, just like the outputs, given by 0 and 1. The second condition of this correlation makes sure that the no-signaling principle is satisfied.

Translated into terms of Bananaworld, this correlation becomes (Bub, 2016, p. 59):

- if the peelings are SS, ST, TS, the tastes are the same, 00 or 11;
- if the peelings are TT, the tastes are different, 01 or 10;
- the marginal probabilities for the tastes 0 or 1 if a banana is peeled S or T are $\frac{1}{2}$, irrespective of the taste of the paired banana and whether or not the paired banana is peeled.

Again, the last condition guarantees that the no-signaling principle is satisfied.

3.6.2 Local simulation

For this correlation there is a successful strategy, which gives Alica and Bob a probability $\frac{3}{4}$ to win the game. This strategy would be to use a sufficiently long list of random bits of which Alice and Bob each have a copy. They should respond to a prompt with the bit on the shared list, in same order as on the list. Due to the random bits on the list, they will satisfy the no-signaling condition in each round. They will win all rounds of the simulation game for which the prompts are SS, ST, or TS, because they give the same response. This means that they will ignore the prompt in each round, and always respond with the same bit as the other. Therefore, they lose all rounds for which the prompts are TT (Bub, 2016, p. 59).

This will be shown to be the optimal strategy for the simulation game of the PR-correlation when limited to local resources in the next section. The local resource that Alice and Bob used was the shared list of random bits playing the role of the common cause of successfully simulating the correlation. Just as Alice and Bob only had their common cause as a local resource, there do not exist any local resources in our real world which can give a higher probability for success either. Therefore, in the real world the optimal result of simulating PR-correlations with local resources is $\frac{3}{4}$, too (Bub, 2016, p. 60).

Given that this is the optimal strategy, this simulation shows that the $\frac{3}{4}$ probability of success is the highest possible and therefore is the upper bound of success with local resources only, just like stated in Bell's inequality, seen in equation 3.4.

3.6.3 Proof of Bell's inequality

Bub uses a proof of Bell's inequality as given in equation 3.4 from Gisin's book *Quantum Chance: nonlocality, teleportation, and other quantum marvels*. The proof is very straightforward and easy to understand when you use the Bananaworld simulation game for the PR-correlation.

There are four different local strategies available to Alice and Bob individually. They are responding with 0 to every prompt, responding with 1 to every prompt, responding with the same value as the prompt (0 for S and 1 for T), and responding with a different value as the prompt (1 for S and 0 for T). The combination of the strategies of Alice and Bob gives 16 possible local strategies in total. They are all represented in table 3.1 (Bub, 2016, p. 61). Each of these 16 strategies represents a different common cause, because each combined strategy gives Alice and Bob a different local instruction set to respond (Bub, 2016, p. 63).

Alice's strategy	**Bob's strategy**	**Response for input *SS***	**Response for input *ST***	**Response for input *TS***	**Response for input *TT***	**Score**
0	0	**00**	**00**	**00**	00	3
0	1	01	01	01	**01**	1
0	same	**00**	01	**00**	**01**	3
0	different	01	**00**	01	00	1
1	0	10	10	10	**10**	1
1	1	**11**	**11**	**11**	11	3
1	same	10	**11**	10	11	1
1	different	**11**	10	**11**	**10**	3
same	0	**00**	**00**	10	**10**	3
same	1	01	01	**11**	11	1
same	same	**00**	01	10	11	1
same	different	01	**00**	**11**	**10**	3
different	0	10	10	**00**	00	1
different	1	**11**	**11**	01	**01**	3
different	same	10	**11**	**00**	**01**	3
different	different	**11**	10	01	00	1

TABLE 3.1: All possible response strategies for simulating the Popescu-Rohrlich correlation. The correct responses are indicated in bold (Bub, 2016, p. 62).

As can be seen in the last column of table 3.1, no strategy is more successful than a score of $\frac{3}{4}$, and no strategy less successful than a score of $\frac{1}{4}$. If all prompts are random, the probability of a successful simulation can be calculated with

$$\begin{aligned} p(success) = \frac{1}{4}[&p(\text{same output}|SS) + p(\text{same output}|ST) + \\ &p(\text{same output}|TS) + p(\text{different output}|TT)] \end{aligned} \tag{3.5}$$

because each possible pair of prompts SS, ST, TS, and TT occurs with the same probability of $\frac{1}{4}$ (Bub, 2016, p. 62).

When you would look at all possible local strategies, this would give you that

$$\frac{1}{4} \leq p_L(\text{successful simulation}) \leq \frac{3}{4}, \tag{3.6}$$

which is the expression of Bell's inequality as given in equation 3.4. And therefore, the optimal strategy for simulating the PR-correlation with local resources is $\frac{3}{4}$ (Bub, 2016, p. 62).

If Alice and Bob both would not use a local strategy as mentioned above, but instead give random responses to the given prompts, their probability for success would be $\frac{1}{2}$. Their responses would be the same half of the rounds, and be different half of the rounds, and therefore they would have a probability of $\frac{1}{2}$ to successfully simulate the PR-correlation (Bub, 2016, p. 62).

The fact that it is not possible to simulate the PR-correlation perfectly using local resources, implies that the correlation cannot be explained by a common cause (Bub, 2016, p. 64). It calls for another explanation, which might be nonlocal, such as the entangled states of quantum mechanics.

3.6.4 Clauser-Horne-Shimony-Holt version of Bell's theorem

A number of alternative versions of Bell's original inequality have been derived for different sets of observables. One, introduced by John Clauser, Michael Horne, Abner Shimony, and Richard Holt in their article 'Proposed Experiment to Test Local Hidden-Variable Theories' (Clauser et al., 1969), is particularly useful in the argumentation of *Bananaworld* and therefore also discussed by Bub.

Clauser et al. derived their inequality in 1969, five years after the paper which introduced Bell's original inequality. The practical use of the Clauser-Horne-Shimony-Holt inequality, or CHSH-inequality, is mainly that it is easier to test experimentally than Bell's inequality (Bub, 2016, p. 64).

Bub derives the CHSH-inequality based on the simulation of the PR-correlation. Only, instead of using 0 and 1 as outputs, he now writes ± 1. The notation of the conditional probabilities, such as $p(\text{responses same}|A, B)$, is similar. Note that observables of Alice and Bob, A and B, are separated by a comma to avoid confusion with the product of the observables (Bub, 2016, p. 64).

Consider the expectation value of the product of the responses for a pair of Alice's and Bob's observables, A and B, denoted by $\langle AB \rangle$. The expectation value is the weighted sum of the products of the possible pairs of responses, with the probabilities for that pair of responses, it is given by:

$$\begin{aligned}\langle AB\rangle &= p(-1,-1|A,B) - p(-1,1|A,B) - p(1,-1|A,B) + p(1,1|A,B)\\ &= p(\text{responses same}|A,B) - p(\text{responses different}|A,B).\end{aligned} \tag{3.7}$$

This expectation value depends on the choice of units for the output and is the reason for choosing ± 1 instead of 0 and 1 (Bub, 2016, pp. 64-65).

Because the sum of probabilities must be equal to 1, and thus

$$p(\text{responses same}|A,B) + p(\text{responses different}|A,B) = 1. \tag{3.8}$$

The expectation value can now be expressed as (Bub, 2016, p. 65):

$$\langle AB\rangle = 2p(\text{responses same}|A,B) - 1 = 2p(\text{responses different}|A,B), \tag{3.9}$$

and the probabilities can be written in terms of the expectation value (Bub, 2016, p. 65):

$$p(\text{responses same}|A,B) = \frac{1+\langle AB\rangle}{2}, \tag{3.10}$$

$$p(\text{responses different}|A,B) = \frac{1-\langle AB\rangle}{2}. \tag{3.11}$$

Use this notation notation to rewrite the probability of successfully simulating the PR-correlation as given in equation 3.5 yields:

$$\begin{aligned}p(\text{success}) &= \frac{1}{4}[p(responses\ same|A,B) + p(responses\ same|A,B')\\ &+ p(responses\ same|A',B) + p(responses\ different|A',B')]\\ &= \frac{1}{4}\left(\frac{1+\langle AB\rangle}{2} + \frac{1+\langle AB'\rangle}{2} + \frac{1+\langle A'B\rangle}{2} + \frac{1-\langle A'B'\rangle}{2}\right)\\ &= \frac{1}{2}\left(1+\frac{K}{4}\right),\end{aligned} \tag{3.12}$$

where A and A', and B and B' are the two possible observables for Alice and Bob respectively, and

$$K = \langle AB\rangle + \langle AB'\rangle + \langle A'B\rangle - \langle A'B'\rangle \tag{3.13}$$

is a variable (Bub, 2016, p. 65).

The value for K is determined by the expectation values, and therefore depends of the kind of resources used to simulate the PR-correlation. In the next section the value of K for local resources will be discusses. In the following sections, we will try to increase the value for K to acquire a higher probability of successful simulation.

Local simulation

For the local simulation, Alice and Bob use a shared local variable, which we will denote by λ. This variable takes values from a finite set, for simplicity. So the value for λ can be anything, with any probability, as long as the probabilities of λ's add up to 1. The only condition is that the probability corresponding to λ is independent of the probability of the prompt. The probability of the prompt is determined by the moderator giving the prompts, and should, in the ideal case, be equal for all prompts (Bub, 2016, p. 65).

Bub introduces the notation $p_\lambda(a|A)$ and $p_\lambda(b|B)$ for the probability that Alice responds with a given the prompt A, and that Bob responds with b given the prompt B, respectively. Both probabilities depend on λ. For local simulation, the probability of getting outputs a and b for inputs A and B, given local variable λ, is equal to the product of the individual probabilities for Alice's and Bob's response:

$$p_\lambda(a, b|A, B) = p_\lambda(a|A) \cdot p_\lambda(b|B). \tag{3.14}$$

On the contrary, for the PR-correlation between in- and outputs, this is not the case. Without the given local variable λ, this correlation satisfies:

$$p(a, b|A, B) \neq p(a|A) \cdot p(b|B) \tag{3.15}$$

This difference is explained by the fact that in the case of the shared random variable λ, Alice's output for a given input and given λ cannot depend on Bob's in- or output, and conversely. In that case, the responses are uncorrelated, or conditionally statistically independent, so they factorize as the product of the separate probabilities of Alice and Bob (Bub, 2016, p. 66).

Conditional statistical independence is equivalent to two conditions on the probabilities. The first is called 'outcome independence' and means that the probability for the two possible outcomes, given the hidden variable and a certain input, is independent of the outcome of the outcome of a measurement on an entangled particle. The second condition is called 'parameter independence' and means that the probability for the two possible outcomes, given the hidden variable and a certain input, is independent of the direction of measurement, or input, of the entangled particle (Bub, 2018b, pp. 75-76).

This distinction assures that Alice and Bob have no control over their outputs, but guarantees that their choice of measurement can be chosen freely, or is random (Bub, 2018b, p. 76).

The difference between the uncorrelated joint probabilities, which depend on the shared local variable, and the PR-correlated joint probabilities, is what limits the probability for success when simulating the PR-correlation with local resources (Bub, 2016, p. 66).

Calculating the expectation value given the shared local variable λ can be achieved by combining equations 3.7 and 3.14, which gives:

$$\begin{aligned}\langle AB\rangle_\lambda &= p_\lambda(-1,-1|A,B) - p_\lambda(-1,1|A,B) - p_\lambda(1,-1|A,B) + p_\lambda(1,1|A,B)\\ &= p_\lambda(-1|A)\cdot p_\lambda(-1|B) - p_\lambda(-1|A)\cdot p_\lambda(1|B)\\ &- p_\lambda(1|A)\cdot p_\lambda(-1|B) + p_\lambda(1|A)\cdot p_\lambda(1|B)\\ &= (p_\lambda(1|A) - p_\lambda(-1|A))\cdot(p_\lambda(1|B) - p_\lambda(-1|B))\\ &= \langle A\rangle_\lambda \langle B\rangle_\lambda.\end{aligned} \tag{3.16}$$

Here $\langle A\rangle_\lambda$ and $\langle B\rangle_\lambda$ denote the expectation values for A and B weighed with the probabilities corresponding to λ for 1 and -1 (Bub, 2016, pp. 66-67).

With this result, the value K for the local simulation can be determined for a certain shared local variable λ:

$$\begin{aligned} K_\lambda &= \langle A\rangle_\lambda \langle B\rangle_\lambda + \langle A\rangle_\lambda \langle B'\rangle_\lambda + \langle A'\rangle_\lambda \langle B\rangle_\lambda - \langle A'\rangle_\lambda \langle B'\rangle_\lambda \\ &= \langle A\rangle_\lambda \left[\langle B\rangle_\lambda + \langle B'\rangle_\lambda\right] + \langle A'\rangle_\lambda \left[\langle B\rangle_\lambda - \langle B'\rangle_\lambda\right]. \end{aligned} \tag{3.17}$$

The possible numerical values for K_λ are determined by considering that $\langle A\rangle_\lambda$, $\langle A'\rangle_\lambda$, $\langle B\rangle_\lambda$, and $\langle B'\rangle_\lambda$ all take their values between -1 and 1. This means that if the sum $[\langle B\rangle_\lambda + \langle B'\rangle_\lambda]$ takes its maximum value of 2, the difference $[\langle B\rangle_\lambda - \langle B'\rangle_\lambda]$ is equal to 0, and the other way around, if the difference $[\langle B\rangle_\lambda - \langle B'\rangle_\lambda]$ takes its maximum value of 2, the sum $[\langle B\rangle_\lambda + \langle B'\rangle_\lambda]$ is equal to 0. Analogously for the minimum values of the sum and difference of the expectation values $\langle B\rangle_\lambda$ and $\langle B'\rangle_\lambda$, when one of them is equal to -2, and the other one is 0, and conversely. Combined with the maximum and minimum values for $\langle A\rangle_\lambda$ and $\langle A'\rangle_\lambda$, this gives that K_λ is bounded by -2 and 2. Averaging K_λ over λ does not change the inequality, because $0 \leq p(\lambda) \leq 1$. So for simulating the PR-correlation with local resources, we have:

$$-2 \leq K_L \leq 2. \tag{3.18}$$

This is the CHSH-version of Bell's theorem as given by Bub (Bub, 2016, pp. 67-68). Inserting this upper boundary for K_L into the CHSH-inequality as given in equation 3.12 gives

$$p_L(\text{successful simulation}) \leq \frac{1}{2}\left(1 + \frac{2}{4}\right) = \frac{3}{4} \tag{3.19}$$

as the maximum probability of simulating the PR-correlation with local resources (Bub, 2016, p. 68).

3.6.5 Quantum simulation

After discussing the limits of local simulation of the PR-correlation, it is time to take a look at simulations involving entangled quantum states, as described in section 3.2.2; this is what we call quantum simulations.

Alice and Bob prepare pairs of photons in the maximally entangled state

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle. \tag{3.20}$$

They each get one photon of each entangled pair en make sure they keep track of the right order of the photons, so they will both use a photon of the same pair in each round of the game. Alice measures the polarization of her photons in directions 0 and $\frac{\pi}{4}$, denoted by observables A and A', when she gets the prompt S or T, respectively, and Bob measures the polarization of his photons in directions $\frac{\pi}{8}$ and $\frac{-\pi}{8}$, denoted by observables B and B', when he gets the prompt S or T, respectively. These angles of the polarization of the photons is represented in figure 3.3 (Bub, 2016, p. 70).

Between each of the pairs of observables A, B and A, B' and A', B, the angle is $\frac{\pi}{8}$. So when these are measured, the probability of getting the same result for both observables is $\cos^2(\frac{\pi}{8}) \approx 0,85$. Analogously, the probability that these measurements give different outputs is $\sin^2(\frac{\pi}{8}) \approx 0,15$. The angle between measurements in directions of

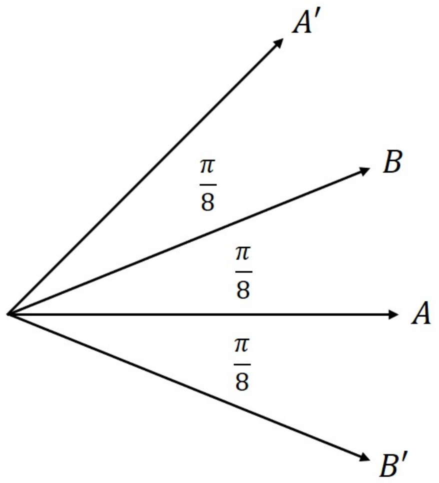

FIGURE 3.3: The polarization directions A, A', B, and B' for the optimal quantum simulation of the PR-correlation (Bub, 2016, p. 70).

the pair A', B' is $\frac{3\pi}{8}$. Therefore, measurements in these directions give the same output with probability $\cos^2(\frac{3\pi}{8}) = \sin^2(\frac{\pi}{8}) \approx 0,15$, and a different output with probability $\sin^2(\frac{3\pi}{8}) = \cos^2(\frac{\pi}{8}) \approx 0,85$ (Bub, 2016, pp. 70-71).

This success rate is significantly higher than the result where local resources were used. Furthermore, the probability for success is higher than the boundary of $\frac{3}{4}$ given by Bell's inequality. This violation implies that the use of entangled quantum states as a shared resource cannot be considered local. Therefore, it proves that, since the existence of quantum entanglement has been experimentally verified, we live in a non-local world, and there are correlations without a common cause explanation (Bub, 2016, p. 64).

3.6.6 Optimal quantum simulation

To determine the maximum probability of successfully simulating the PR-correlation with quantum resources, we need to determine the value of K_Q, i.e. the quantum mechanical value for K. Recall that the value for K is based on the expectation values of the products of observables as described in section 3.6.4. Again, use the values ± 1, instead of 0 and 1, as outputs for the different observables to calculate the expectation values of the products of observables. The calculation of the expectation values using entangled quantum states is given by:

$$\begin{aligned}\langle AB \rangle_{|\phi^+\rangle} &= p(\text{outcomes same}|A,B) - p(o\text{outcomes different}|A,B)\\ &= \cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\\ &= \cos\left(2\cdot\frac{\pi}{8}\right)\\ &= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.\end{aligned} \tag{3.21}$$

This gives the same values for $\langle AB\rangle_{|\phi^+\rangle}$, $\langle A'B\rangle_{|\phi^+\rangle}$, and $\langle AB'\rangle_{|\phi^+\rangle}$, but a different value for $\langle A'B'\rangle_{|\phi^+\rangle}$, since $\langle A'B'\rangle_{|\phi^+\rangle} = \sin^2\left(\frac{\pi}{8}\right) - \cos^2\left(\frac{\pi}{8}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (Bub, 2016, p. 71).

Now the value for K_Q is (Bub, 2016, p. 71):

$$K_Q = \langle AB\rangle_{|\phi^+\rangle} + \langle A'B\rangle_{|\phi^+\rangle} + \langle AB'\rangle_{|\phi^+\rangle} - \langle A'B'\rangle_{|\phi^+\rangle} = 4 \cdot \frac{1}{\sqrt{2}} = 2\sqrt{2}. \quad (3.22)$$

This value is also known as the 'Tsirelson bound', named after Boris Tsirelson (sometimes spelled Cirel'son). It is the optimal value for simulating the PR-correlation with quantum resources, but Bub does not prove in *Bananaworld* why this is the case (Bub, 2016, p. 68).

Together with the CHSH-inequality in equation 3.12, this value for K_Q gives the probability of successfully simulating the PR-correlation with quantum resources (Bub, 2016, p. 71):

$$p_Q(successful\ simulation) \leq \frac{1}{2}\left(1 + \frac{2\sqrt{2}}{4}\right) \approx 0,85. \quad (3.23)$$

3.6.7 Superquantum

Using quantum resources, the PR-correlation still cannot be simulated with a probability for success of 1. However, there might be another kind of no-signaling resources able to do so, Bub calls these 'superquantum' resources. Theoretically, achieving the maximum probability of success for the PR-correlation with nonlocal superquantum resources would require a value of $K = 4$. The first three terms need to take a value of 1, whereas the last term needs to be -1 to achieve this value for K (Bub, 2016, p. 69).

In the thought experiment of *Bananaworld*, bananas exhibiting such a superquantum correlation are possible. But the question is now why we seem to be committed by the Tsirelson bound in the real world. This question will be further discussed in section 3.9

3.7 Correlation arrays

Bub introduces a new notation for the PR-correlation. Given that Alice's and Bob's inputs for the correlation are A and B respectively, and their outputs are given by a and b respectively, each with possible values of 0 and 1, the correlation is given by:

$$a \oplus b = A \cdot B \quad (3.24)$$

where '$\oplus$' is addition modulo 2, and '$\cdot$' is multiplication in the usual sense (Bub, 2016, p. 89).

This correlation can also be represented in what is called a 'correlation array'. In the correlation array, all conditional probabilities for possible combinations of inputs are listed in a table. The correlation array for the PR-correlation is given in table 3.2. Often, the probability labels are dropped to make the array more readable (Bub, 2016, pp. 89-90).

It is important to note that the probabilities in each cell of the table must add up to 1. If you would have a deterministic correlation array, the only values for the probabilities

Alice / Bob		S 0	S 1	T 0	T 1
S	0	$p(00\|SS) = \frac{1}{2}$	$p(10\|SS) = 0$	$p(00\|TS) = \frac{1}{2}$	$p(10\|TS) = 0$
	1	$p(01\|SS) = 0$	$p(11\|SS) = \frac{1}{2}$	$p(01\|TS) = 0$	$p(11\|TS) = \frac{1}{2}$
T	0	$p(00\|ST) = \frac{1}{2}$	$p(10\|ST) = 0$	$p(00\|TT) = 0$	$p(10\|TT) = \frac{1}{2}$
	1	$p(01\|ST) = 0$	$p(11\|ST) = \frac{1}{2}$	$p(00\|SS) = \frac{1}{2}$	$p(11\|TT) = 0$

TABLE 3.2: The standard PR-correlation array (Bub, 2016, p. 89)

.

would be 0, and 1. The reason for this is that for a deterministic correlation, there is only one possible set of outputs for each different input (Bub, 2016, pp. 108-109).

3.8 Polytope of no-signaling correlations

We have seen three different kinds of correlations that satisfy the no-signaling condition, namely classical, quantum, and superquantum correlations. In this section, the relation between these correlations will be expressed geometrically (Bub, 2016, p. 106).

Figure 3.4 is what Bub calls "the key diagram for the narrative in the book" (Bub, 2016, p. 113). It is a schematic representation of the local polytope $\mathcal{L}$, inside a convex set $\mathcal{Q}$, which in turn is inside the polytope representing all no-signaling correlations $\mathcal{P}$ (Bub, 2016, p. 113).

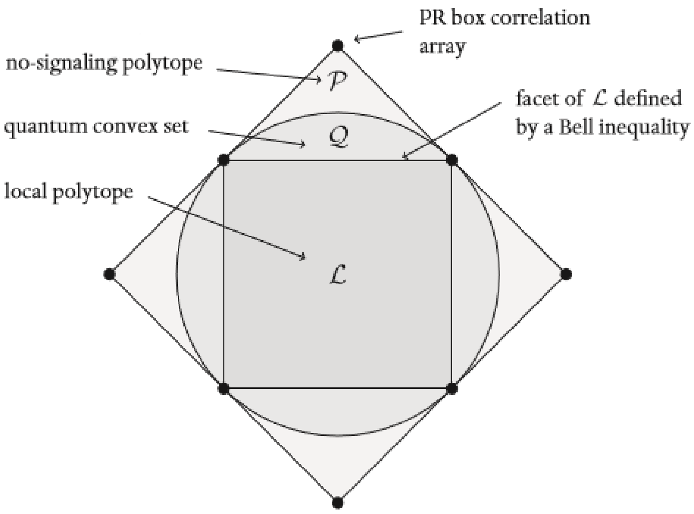

FIGURE 3.4: A schematic representation of the different sorts of correlations in Bananaworld (Bub, 2016, p. 107).

To understand what this means, we first need to discuss the meaning of the word 'polytope'. Polytopes are geometric objects with 'flat' sides in $\mathbb{R}^n$, so they can exist in any number of n dimensions. For example, a two-dimensional polytope is called a 'polygon', which is a plane figure such as an equilateral triangle. An important characteristic of a polytope is that its boundary is given by different vertices joined by finite straight line segments. This discussion will be limited to considering 'convex' polytopes, which means that no straight line between any two points in the polytope goes outside the polytope. All polytopes are bounded by 'facets', which are multidimensional analogues of what is a face of a 'polyhedron', a three-dimensional polytope. A facet of a polytope always has one dimension less than the dimension of the polytope (Bub, 2016, pp. 106-107).

All correlations that can be simulated by using local resources are represented by the points inside the local polytope $\mathcal{L}$. In Bananaworld this local correlation polytope is a four-dimensional hyperoctahedron. A hyperoctahedron is a polytope defined by eight vertices in four dimensions. The vertices each represent a local deterministic correlation given by a particular correlation array, reduced to a correlation vector. This particular representation was proposed by Itamar Pitowsky (Bub, 2016, pp. 107-108).

Pitowsky defines the correlation vector as $p = (p_{11}, p_{12}, p_{21}, p_{22})$. The components p_{ij} are in this case the average values of the product of Alice's and Bob's output for a measurement on an entangled pair of particles, denoted by $s_{ij} = \pm 1$. $i = 1, 2$ represents the observable Alice chooses to measure, and $j = 1, 2$ represents the observable Bob chooses to measure (Pitowsky, 2008, p. 2). Just as in the derivation of the CSHS-inequality, possible outputs of the measurements are set to be ± 1 in this case.

If you consider all possible deterministic correlation arrays, so the ones with only possible values for the probability of 0 and 1, there are in total $4^4 = 256$ possibilities. Now 240 of those correlation arrays violate the no-signaling principle, and hence represent nonlocal deterministic correlations. For those correlations, the joint probabilities cannot be expressed as products of the individual local probabilities for Alice and Bob (Bub, 2016, pp. 108-109).

The remaining 16 correlation arrays are local deterministic correlations that do not violate the no-signaling principle. Each array can be represented by a point in an eight-dimensional space. The number of dimensions is directly related to the number of independent variables. Constraints used to reduce this number of dimensions are that the sum of the probabilities should be equal to 1, and no-signaling conditions (Bub, 2016, p. 110).

The smallest closed convex set containing all these 16 points, is the local correlation polytope $\mathcal{L}$. The edges of this polytope all have length 1, because the coordinates of the vertices are 0 or 1. Each point in the interior of the polytope represents a specific mixture of probability distributions of deterministic arrays. Only the vertices are not mixtures of this kind, because they are the extremal points of the polytope (Bub, 2016, p. 110).

The 16 correlation arrays can be reduced to eight different correlation vectors. So each correlation vector corresponds to a pair of correlation arrays. Instead of calculating the values for p_{ij} in the way Pitowsky does, taking the average values of the products s_{ij} Bub determines the correlation vectors directly from the correlation arrays. He gives each cell of the correlation array, which corresponds to certain values ij, a value -1 or 1, depending on whether the input value is different or the same as the output value respectively. By doing this, he finds the same eight correlation vectors as Pitowsky:

$$(1,1,1,1), (1,1,-1,-1), (1,-1,1,-1), (1,-1,-1,1),$$
$$(-1,-1,-1,-1), (-1,-1,1,1), (-1,1,1,-1), (-1,1,1,-1).$$

These eight vectors define the vertices of the four-dimensional hyperoctahedron $\mathcal{L}$ (Bub, 2016, pp. 111-112).

Considering the correlation arrays for the PR-correlation, gives eight more correlation vectors:

$$(1,1,1,-1), (1,1,-1,1), (1,-1,1,1), (-1,1,1,1),$$
$$(-1,-1,-1,1), (-1,-1,1,-1), (-1,1,-1,-1), (1,-1,-1,-1).$$

These vectors define eight points outside the local polytope. However, the eight local correlation vectors and the eight additional correlation vectors derived from the PR-correlation arrays together define the 16 vertices of a four-dimensional hypercube $\mathcal{P}$. This polytope contains all points representing no-signaling correlations, including the local correlations of $\mathcal{L}$ (Bub, 2016, pp. 112-113).

Between the local polytope $\mathcal{L}$ and the nonlocal polytope $\mathcal{P}$, there is a convex set of points $\mathcal{Q}$, of which the furthest points from the center are at the Tsirelson bound. $\mathcal{Q}$ is not a polytope, because its boundary does not consist of 'flat' facets. In figure 3.4 it is represented as a circle, but this is not completely correct either. In reality, the boundary of $\mathcal{Q}$ is a convex, but complicated three-dimensional region of points, with extrema at the Tsirelson bound (Bub, 2016, p. 113). Note that the extrema are on the Tsirelson bound, and explicitly, this means that the real bound of $\mathcal{Q}$ goes below this bound. Therefore, there are non-quantum correlations below the Tsirelson bound (Bub, 2016, p. 182).

3.9 Why the quantum?

The question "Why the quantum?" is the title of one of the chapters of *Bananaworld*. Bub explains that he got it from one of John Wheeler's "Really Big Questions". The question has different interpretations, for example why the world is quantum rather than classical, but also why the world is quantum rather than superquantum. This last question was also asked by Popescu and Rohrlich when they introduced their argument of the existence of superquantum no-signaling correlations. In this context, the question therefore is: "Why is there a Tsirelson bound?" (Bub, 2016, p. 181)

3.9.1 Information causality

In 2009, Marcin Pawłowki, Tomasz Paterek, Dagomir Kaszlikowski, Valerio Scarani, Andreas Winter, and Marek Żukowski showed that a principle called 'information causality' could explain the Tsirelson bound (Pawłowski et al., 2009). However, this principle does not answer the question "Why the quantum?" completely. Information causality only fixes a part of the boundary between quantum and superquantum correlations for two qubits (Bub, 2016, pp. 181-182).

There are many other principles that have been suggested to derive the Tsirelson bound from. Even though some are more successful than others, none of them is completely satisfying, as they only exclude some superquantum correlations, but not all (Bub, 2016, p. 183).

The no-signaling principle has come up many times. It limits the amount of information that a receiver can get about the data set of the sender. Information causality is a generalization of this principle. Consider the case in which Alice sends Bob a message of M classical bits. According to the information causality principle, Bob cannot know any more than M bits of Alice's set from that message. The no-signaling principle is the special case for $M = 0$, in which case there is no communication at all between Alice and Bob. Classical and quantum correlations satisfy information causality, but PR-correlations and a lot of superquantum correlations do not (Bub, 2016, p. 184).

The motivation for information causality as an information-theoretic principle is that it excludes those PR-correlations and a lot of superquantum correlations which seem 'to good to be true' (Bub, 2016, p. 187). The intuition that one cannot get more information from the sender than was actually send to you therefore underlyies this principle.

Chapter 4

Different views of quantum mechanics

4.1 Quantum information theory

The paper "A Mathematical Theory of Communication", written by Claude Shannon, can be seen as the beginning of information theory (Shannon, 1948). Shannon defined a more technical notion of 'information' in contrast with the everyday concept of 'information' that is associated with knowledge, language, and meaning (Timpson, 2013, p. 11). The technical notion of 'information' as described by Shannon is similar to the meaning of 'information' meant in *Bananaworld*, because it is concerned with the correlations of signals, which in this case come from experiments. This technical use of 'information' forms the basis of information theory.

In chapter 3 it was described how Bub introduced quantum information theory. The information theoretic approach can be recognized in the description of quantum mechanics in terms of inputs and outputs as (qu)bits with possible values 0 and 1. In the following sections, a number of things will be discussed that are relevant for further discussion and comparison with the minimal view of quantum mechanics.

4.1.1 Characteristics of Bub's view

The most important assumption in Bub's view of quantum mechanics is that the theory is fundamentally a theory of information. He proposes that quantum mechanics is about the structure of information, by which he means that quantum mechanics describes a structurally different kind of correlations from classical mechanics. These correlations can be described using information theory (Bub, 2016, p. 6).

After using the information-theoretic framework to describe quantum mechanics, there is another important characteristic of the view of quantum mechanics Bub presents in *Bananaworld*. Which is similar to the approach proposed by Popescu and Rohrlich (Popescu and Rohrlich, 1994). Instead of what many fundamental theories of quantum mechanics do, namely deriving the nonlocal aspects of quantum mechanics from the assumption of indeterminism, Popescu and Rohrlich, and also Bub, suggest to do it the other way around. They combine nonlocality and relativistic causality as axioms to derive intrinsic randomness as a theorem.

From this description of quantum mechanics 'from the outside', so without using a formalism derived from quantum mechanics itself, Bub (and Popescu and Rohrlich) showed that there are more nonlocal correlations that satisfy the no-signaling principle than only the quantum correlations. These theoretically possible superquantum correlations do not seem to exist in the real world.

4.1.2 Problems of Bub's view

This sections introduces a number of problems with the view of quantum mechanics that Bub described. It is not intended to describe all problems with his view, but is only meant to pinpoint some of the bigger problems. It is important to discuss these problems to make a fair comparison with the minimal view of quantum mechanics later.

Information about what?

This question was asked by Bell. Like 'measurement', he did not like to use the term 'information' either, because according to him it had not place in a description of physical reality (Bell, 1990, p. 34). According to Christopher Timpson, when considering that the quantum state represents information, there are two possible answers to this question. The first is that information is about what the outcome of an experiment will be, and the second is that information reveals something about a system prior to the measurement, so it reveals a hidden-variable (Timpson, 2013, p. 146).

If the information represented by the quantum state only says something about the outcome of the experiment, it is hard not to end up with an instrumentalist use of quantum mechanics. Instead of telling you something about the quantum state itself, this meaning of information only considers the quantum state as a device to calculate the probabilities of all possible outcomes of experiments. This answer is not very compelling as it does not result in a new interpretation of quantum mechanics (Timpson, 2013, p. 147).

The other answer to the question, about the information referring to a hidden variable, is not very appealing either. Problems of the quantum state, such as the collapse and nonlocality, are not solved when information is considered to refer to a hidden variable. So this would not solve the problems posed by more traditional ways of describing quantum mechanics either. Informationist would be aiming for an interpretation of 'information' that would solve these problems, whereas this interpretation only moves the problem from physical level to the level of knowledge about the system (Timpson, 2013, p. 147).

Even though there are ways to avoid the problems posed by these two answers to what the information refers to, they tend not to get any simpler. The question will therefore be an obstacle for quantum information theory for as long as it lacks a clear answer.

What the structure of information does not explain

Probably the hardest thing about the information theoretic approach to quantum mechanics is that it is hard to see how, and even if, it can explain important quantum mechanical properties and phenomena. Examples of these are superconductivity, and symmetry breaking, which are explained by quantum mechanics, but cannot be directly explained by information theory yet. The problem therefore is that one would expect that if these examples can be described in terms of quantum mechanics, and quantum mechanics can be described in terms of the structure of information, then one would expect that the examples can be described in terms of the structure of information, too. However, this is way less obvious that it seems, and this causes a problem for Bub's theory.

Another kind of problem in this category can be explained by the example of entanglement. Clifton et al. used C*-algebras, from which entanglement follows automatically given the mathematical machinery (Clifton, Bub, and Halvorson, 2003). However, if you dismiss the C*-algebraic framework and give a fully information-theoretic characterization of quantum mechanics, as Bub himself did in his work, there is no explanation for the existence of entanglement (Timpson, 2013, p. 171). So the approach of quantum mechanics as explained by Bub can be used to describe systems in which entanglement occurs, but it cannot explain why entanglement itself exists. Due to this point of critique it is hard to see how the structure of information can be a fundamental property of quantum mechanics.

Interpretational questions

One last problem with the theory described by Bub discussed here concerns the interpretation of quantum mechanics. Bub claims that quantum mechanics is fundamentally about the structure of information, but this does not necessarily lead to progress on the part of the interpretation of quantum mechanics. The idea of quantum mechanics as a theory of the structure of information leads to a new way of deriving the mathematical structure of quantum mechanics, but this structure can still be interpreted in many different ways (Timpson, 2013, p. 175).

Bub has an answer to this problem, for he claims that some interpretations are forbidden by the axioms used in information theory (Timpson, 2013, p. 179). The three axioms used in information theory are: (1) the one-world assumption, stating that a measurement has a single outcome, (2) the assumption that quantum mechanics applies to systems of any complexity, including observers, and (3) self-consistency, in particular agreement between an observer and a super-observer (Bub, 2018b, p. 226).

The Everettian interpretation rejects assumption (1), the one-world assumption, and believes in many worlds. This interpretation is not very plausible, according to Bub, when you consider, for example, the decision-theoretic account of the probabilistic correlations that are already known to characterize Hilbert space uniquely (Bub, 2018b, p. 226).

Another interpretation called quantum Bayesianism, or QBism in short, rejects assumption (3), the self-consistency assumption. This interpretation is also information-theoretic, because it understands all probabilities, including quantum probabilities, as personal judgments of an agent, based on how the external world responds to actions by the agent. This means that the perspectives of an observer and a super-observer do not need to be consistent (Bub, 2018b, pp. 226-227).

The last possibility of rejecting one of the axioms is rejecting (2), the assumption that quantum mechanics applies to systems of any complexity, including observers. Bub thinks rejecting this assumption comes closest to what Bohr and some other early proponents of the Copenhagen interpretations had in mind, and calls this the information-theoretic interpretation. The main idea of this interpretation is that quantum probabilities are probabilities of what will be found when a measurement is done with one ultimate observer as a reference. This means that just one observer is legitimate in the application of quantum mechanics, namely the perspective of the observer for whom an actual measurement outcome occurs at the macrolevel (Bub, 2018b, pp. 227-229).

4.2 Comparison

In this section of this thesis, two approaches or views of quantum mechanics will be compared. The goal is not to choose which one is better, but to see what the differences and/or similarities are and where they could complement each other.

4.2.1 Minimal view

To make a good comparison, it is important to define what is meant by the minimal view of quantum mechanics. A description as detailed as the one of quantum information theory will not be given here. Instead, some key properties of the minimal description will be given and taken as a basis to compare the two different views on. This explanation of the minimal view will be based mostly on what is taught in the mandatory quantum mechanics classes of the bachelor Physics at the Radboud University.

As said in the introduction, the minimal view is focused on 'doing' rather than 'understanding'. The important difference between these two is that 'doing' is about using the formalism and does not really care about where the formalism comes from as long as the results correctly describe their experiments. For 'understanding' on the other hand, the main goal is rather to find out why a certain explanation can be used and why its limits are where they are.

For 'doing', or 'using', quantum mechanics, important concepts are for example the Schrödinger equation, wave functions, eigenstates and eigenfunctions, boundary conditions, and operators. These are all more or less concepts which can be applied without understanding their meaning in reality. You have to familiarize yourself with the notation and relations between them, but asking questions about the interpretation of quantum mechanics is not the purpose of the minimal view (Sudbury, 1986).

4.2.2 Relating the two

Some of the relations between the two views of quantum mechanics are discussed by Bub in "Supplement: Some Mathematical Machinery" of *Bananaworld* (Bub, 2018b, pp. 233-267). That is also the part of the book in which most elements of the minimal view of quantum mechanics can be recognized. In this part, Bub explains some things about, for example, the Dirac notation, Pauli spin operators, and the Schrödinger equation. However, the link between this section and the rest of the book is limited to relating peelings of the bananas to measuring different observables on particles. After relating those, the discussion continues in a minimal direction, by deriving other properties, and stays silent about explaining those properties in an information-theoretic framework.

As discussed in the section about the problems of quantum information theory, this seems to imply that there is a possible relation between the phenomena explained by quantum mechanics and the information-theoretic description of quantum mechanics. However, this relation is still largely unknown, or at least is not discussed by Bub.

It seems here that there is still a large gap between describing quantum mechanics 'from the inside', by using a an appropriate mathematical framework based on Hilbert spaces, and 'from the outside', by describing quantum mechanics as the nonlocal correlations bounded by the Tsirelson bound. However, to complete our picture of quantum mechanics, this gap should eventually close, because in the end they are both descriptions of a single concept of quantum mechanics.

Chapter 5

Conclusion

What can finally be said about the fact that the questions addressed by Bub in *Bananaworld* are not discussed in the courses about quantum mechanics at Radboud University? One could argue that it is sufficient for the students to be taught how to use quantum mechanics. However, if this change of perspective on quantum mechanics is a new revolution in quantum mechanics, as Bub calls it, shouldn't it get more attention in the curriculum?

For me, reading *Bananaworld* has given me access to a whole new 'world of quantum mechanics', namely the possibility of describing it in terms of probabilistic correlations. Apart from the fact that this new kind of formalism is very interesting in itself, I think the questions it raises about the foundations of quantum mechanics are even more important. The discussions about the implications of this view may not have been finished, but the new questions it raises might just stimulate students to do what they are supposed to do in my opinion: think for themselves.

Teaching students which calculation to use in which situation might be hard enough for quantum mechanics, but focusing only on this part in the mandatory courses, and not even giving students the option to explore other approaches in optional courses, is in my opinion a missed opportunity.

I would therefore surely recommend my fellow students to read some of the recent papers published on the subject of foundations of quantum mechanics. Indeed, when you start looking into this, suddenly you see it popping up everywhere and realize that this is a very hot topic in contemporary physics, especially quantum computation and quantum information theory.

Eventually, I hope that foundations of quantum mechanics will receive more attention, both in general and at Radboud University, and more answers will be found to the questions still open. It would be great if in a number of years, the two approaches can be seen as describing one and the same theory, instead of being separated by a gap as it seems to be right now.

Appendix A

Review of *Bananaworld*

In this appendix I will give my opinion of the book *Bananaworld: Quantum Mechanics for Primates*. The focus will be on the way Bub presents the subject, the examples he uses, the structure of the book, et cetera. To review the book thoroughly, I came up with different criteria: goal, content, structure, presentation, and overall conclusion. These criteria will be discussed separately.

A.1 Goal

Bub's goal was to write a book for a public not only consisting of experts in physics and mathematics, but also of people interested in foundations of quantum mechanics without having the physical and mathematical background. It is quite hard to write a good book for such a broad public. You have to simplify many things, but also keep it interesting for the people who do have a background in physics or mathematics.

In my opinion, Bub partly succeeded. By explaining many of the more technical concepts in the "More"-sections and in the mathematical supplements at the end of each chapter, and the end of the book, respectively, he managed to take most technicalities out of the main parts of the book. However, there were still some difficult concepts which could not be avoided in the main text, and I would not expect people without a mathematical background to be able to grasp those.

Another goal of *Bananaworld* is to convince people that quantum mechanics really is about information theory, rather than about something else like wave functions, and I think he did succeed in achieving this. I will not claim that everyone who reads this book will convert from, for example, thinking that quantum mechanics is about quantized energy to agreeing with Bub that it is really about the structure of information. But I do think that after reading the book, you cannot convincingly deny that information theory is an important view on quantum mechanics that cannot be ignored.

A.2 Content

The book covers a lot of ground. In a way the subjects are all quite well related to each other. However, I think Bub maybe tried to explain too much. Therefore I chose to only summarize parts of six of the ten chapters of the book for this thesis. In the other chapters he discusses, for example, some more correlations that can be simulated with PR-bananas, and more examples of different kinds of simulation games. Even though those can be helpful towards explaining more aspects and applications of the

PR-bananas, I think they might distract from the important message of the book as well.

A.3 Structure

The use of short summaries in each chapter was very useful. The main points of the sections were repeated in those text boxes, and were not only useful to look back after reading a few more chapters, but also really helped to check if you understood the most important message of each section.

I have some doubts about the choice of the order of discussing some subjects. For example, explaining the concept of entangled photons so early, far before using them for quantum simulations, was strange to me at first even as a physics student. However, I saw eventually the logic of that choice, because the information about bits and qubits was very useful at that point of the argumentation. So it is quite hard to decide when to introduce which concepts, and even though I still do not agree with all choices, I think Bub did quite well.

Related to what I said about the content of the book, I think some parts are explained very elaborately, even though they might not be that important. Many, in my view, very important arguments are all put all together in the chapter "Bananaworld", whereas Bub describes many different examples of using PR-correlations extensively in different chapters. I think the focus could be more on the argumentation, but I can imagine other people have different preferences.

A.4 Presentation

The most important and novel aspect of the presentation of quantum information theory in *Bananaworld* is the thought experiment using bananas of the imaginary Bananaworld island. Even though it adds a fun element to the book, also because of the beautiful drawings, I think it does not really add anything to the argumentation. Bub is in my view not really consistent in using the bananas and the better known 'boxes'. Furthermore, he mentions that things that are proven with the thought experiments of the bananas also say something about the real world. However, eventually he discusses numerous characteristics and implications of the non-physical PR-bananas, which do *not* have an analogue in our real world. Therefore, the example of the bananas is still not much easier to grasp than the conceptual boxes.

A.5 Overall conclusion

My overall conclusion of the book is that it is a good introduction to quantum information theory, but the difficulty should not be underestimated. It contains many of the most important results from articles in the field, not only historically like the EPR-article, but also more recent results. Some more difficult articles are explained really well by translating them to the easier examples in the book. However, the banana-analogue of the boxes does not contribute very much except for the fun-part.

For me, the book has been my first introduction to quantum information theory. It has really broadened my view of quantum mechanics and has triggered my interest in the foundations of quantum mechanics. I read most parts of the book multiple times, and that contributed to understanding it. After getting the whole picture of the theory, it was easier for me to understand the smaller concepts and see the relation to the bigger picture, but I guess that applies to all serious books and theories.

Bibliography

Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox". In: *Physics* 1 (3), pp. 195–200.

— (1990). "Against 'measurement'". In: *Physics World* 3.8, pp. 33–40.

Bub, J. (2016). *Bananaworld: Quantum Mechanics for Primates*. Oxford University Press.

— (2018a). Personal communication between Jeffrey Bub and Klaas Landsman.

— (2018b). *Bananaworld: Quantum Mechanics for Primates*. Oxford University Press.

Clauser, J. et al. (1969). "Proposed Experiment to Test Local Hidden-Variable Theories". In: *Physical Review Letters* 23, pp. 880–884.

Clifton, R., J. Bub, and H. Halvorson (2003). "Characterizing Quantum Theory in Terms of Information-Theoretic Constraints". In: *Foundations of Physics* 33.11, pp. 1561–1591.

Einstein, A. (1948). "Quanten-Mechanik und Wirklichkeit". In: *Dialectica* 2.3/4, pp. 320–324.

Einstein, A., B. Podolsky, and N. Rosen (1935). "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" In: *Physical Review* 47 (10), pp. 777–780.

Gisin, N. (2014). *Quantum Chance: Nonlocality, Teleportation and Other Quantum Marvels*. Trans. by Stephen Lyle. Springer International Publishing.

Pawłowski, M. et al. (2009). "Information causality as a physical principle". In: *Nature* 461, pp. 1101–1104.

Pitowsky, I. (2008). "Geometry of quantum correlations". In: *Physical Review A* 77.6, p. 062109.

Popescu, S. (2014). "Nonlocality beyond quantum mechanics". In: *Nature Physics* 10, pp. 264–270.

Popescu, S. and D. Rohrlich (1994). "Quantum Nonlocality as an Axiom". In: *Foundations of Physics* 24.3, 379–385.

Shannon, C. E. (1948). "A Mathematical Theory of Communication". In: *Bell System Technical Journal* 27.3, pp. 379–423.

Sudbury, A. (1986). "Interpretations of Quantum Mechanics". In: *Quantum mechanics and the particles of nature*. Cambridge University Press. Chap. 5.5, pp. 210–226.

Timpson, C.G. (2013). *Quantum Information Theory and the Foundations of Quantum Mechanics*. Oxford University Press.