AANVRAAG VRIJE COMPETITIE NWO-EW OKTOBER 2008

1a Project Title. Topos theory, noncommutative geometry, and quantum logic

1b Project Acronym. Topos quantum logic

1c Principal Investigator. Prof.dr. N.P. Landsman
Radboud Universiteit Nijmegen, Faculty of Science,
Institute for Mathematics, Astrophysics, and Particle Physics (IMAPP),
Heyendaalseweg 135, 6525 AJ NIJMEGEN, THE NETHERLANDS

1d Renewed Application. No.

2a Summary. Topos theory and noncommutative geometry are areas of modern mathematics that may both be seen as vast extensions of topology, each providing its own generalized notion of space. In topos theory one regards the so-called locales of lattice theory as spaces, whereas the C*-algebras of functional analysis define spaces in the noncommutative sense. The aim of this proposal is to relate these different notions of space to each other and to quantum theory. That this can be done in principle has recently been shown by the applicant in collaboration with C. Heunen and B. Spitters: given some C*-algebra A, we defined a locale L(A) in a certain topos T(A) with various interesting properties, including computability in principle.

First, at least for certain classes of C^*-algebras this would provide interesting new examples of locales, whose structure can be analyzed using the different perspectives and toolkits of topos theory and noncommutative geometry. For example, the K-theoretic invariants of the C*-algebra A should define corresponding invariants of the locale L(A), and more generally anything that can be said about A will have repercussions for L(A).

Second, regarding the locale L(A) as a quantum phase space, the logical structure of the quantum system described by the C*-algebra A can be studied using the internal logic of topos theory. Since the latter happens to be intuitionistic, this proposal is also explicitly meant as a contribution to the emerging field of intuitionistic quantum logic (which is intended to clarify—if not replace—the bizarre nondistributive quantum logic proposed by Birkhoff and von Neumann in 1936).

We request funding for one PhD student (4 years) at a total budget of 192.495 euro's.

2b Abstract for laymen (in Dutch). Topostheorie en niet-commutatieve meetkunde zijn moderne gebieden van de wiskunde (ontwikkeld in de tweede helft van de 20e eeuw), die beide tot doel hebben een zeer algemeen ruimtebegrip te formaliseren. Een belangrijk voordeel van de topostheorie is daarbij dat dit doel is bereikt onder een duidelijk verband met de logica; daar staat tegenover dat het ruimtebegrip van de niet-commutatieve meetkunde weer uitstekend past bij de kwantummechanica. Het doel van dit voorstel is om deze twee wiskundige gebieden, die tot nu toe gescheiden zijn opgetrokken, met elkaar te verbinden door hun respectievelijke ruimtebegrippen te relateren. Dit is een doel op zich binnen de wiskunde, maar het is tevens een middel om de logische structuur van de

kwantummechanica op te helderen. Het is de bedoeling om op deze manier een opvolger te vinden van de in 1936 door Birkhoff and von Neumann voorgestelde 'kwantumlogica', die op een aantal punten onbevredigend is. De actualiteit van deze vraagstelling wordt onder meer gegeven door de mogelijke ontwikkeling van kwantumcomputers, waarvan de logische structuur uiteraard van groot belang is.

3 Classification. Wiskunde (Mathematics). MSC 2000: 03G30 (Categorical logic, topoi), 06D22 (Distributive lattices: Frames, locales), 81R60 (Groups and algebras in quantum theory: Noncommutative geometry), 81P10 (Logical foundations of quantum mechanics; quantum logic).

4 Composition of the Research Team.
PhD student (RU, 1.0 fte) *vacancy*
Prof.dr. N.P. Landsman (RU, 0.5 fte, promotor)
D. Coumans, M.Sc. (RU, 0.2 fte)
Prof.dr. M. Gehrke (RU, 0.1 fte)
Prof.dr. H. Halvorson (Princeton University and UU, 0.2 fte)
Prof.dr. I. Moerdijk (UU, 0.1 fte)
This application is intended to fund a 1.0 fte PhD student at the Radboud University. Dion Coumans is a PhD student of Mai Gehrke, both intending to collaborate on this project. Hans Halvorson is a Professor of Philosophy at Princeton University, currently spending a sabbatical at Utrecht with Ieke Moerdijk to work on topos theory. Moerdijk will act as a consultant, sounding board and source of ideas in this project.

5 Research School. MRI

6a Description of the Proposed Research.
This proposal relates four different areas of mathematics and mathematical physics:
(1) Topos theory;
(2) Noncommutative geometry;
(3) Algebraic quantum theory;
(4) Quantum logic.

Before providing a brief introduction to each of these, as to the novelty of the research proposed here we would like to point out that the combination of (1) and (3) very recently originated with the applicant and collaborators [i]. This work was partly based on a series of papers by Banaschewski and Mulvey (culminating in 2006 with the appearance of [2]), in which the basic theory of C^*-algebras (forming part of the technical toolkit of noncommutative geometry) was generalized to topos theory. More broadly speaking, the idea of relating (1) and (2) has clearly been in the air for some time now (cf. [8]), but so far there have been few technical results in this direction [17]. Calling upon (3) in attempts to relate (1) and (2) appears to be completely novel and would bring physical intuition into a mathematical context, quite in the spirit of the Dutch GQT-cluster.

On the other hand, the interaction between (2) and (3) was already initiated by Rieffel in 1989 (cf. [22]), and has played a central role in the work of the applicant since the mid 1990s (see, e.g., [ii, iii]). More generally, quantum theory has formed a key source of ideas for noncommutative geometry ever since its inception [9, 10].

Partly because (3) and (4) both originated with the work of von Neumann [20], their relationship has been studied in detail [19]. However, the quantum logic we envisage in this proposal departs from von Neumann's version in being *intuitionistic*, and as such the link between (4) and (1) was initiated by Isham (with Butterfield and Döring) [14, 7, 11]. The strengthening and further exploration of this link by involving (2) and (3) is new with this proposal.

Survey of relevant fields

Topos theory originally came into existence in the circle of Grothendieck as a generalization of topology, offering a possible foundation of algebraic geometry [1]. It was subsequently related to (categorical) logic by Lawvere and Tierney; see, e.g., [15, 24]. Mac Lane and Moerdijk open their renowned textbook on the subject [v] with the words:

> A startling aspect of topos theory is that it unifies two seemingly wholly distinct mathematical subjects: on the one hand, topology and algebraic geometry and on the other hand, logic and set theory.

As a key example of this unification, we mention the notion of a *locale* or *frame*. This is a single lattice-theoretic object in a topos (originally conceived in conventional set theory) that can be seen in two different ways [16], [v], [24]: regarded as a locale it is a powerful generalization of a topological space, whereas seen as a frame it is a logical structure known as a *Heyting algebra*. As the name suggests (Heyting was a pupil of L.E.J. Brouwer, the Dutch mathematician who founded intuitionism), this is an algebraic description of a particular intuitionistic (propositional) logical theory in which the *Tertium Non Datur* or excluded middle third principle generally fails. More generally, a topos as a whole is home to a certain intuitionistic predicate logic with multivalued truth object.

Almost since its inception, there have been speculations about the possible relevance of topos theory to physics. In 1967 Lawvere proposed topos theory (more specifically, synthetic differential geometry, which is based on it) as a foundation of classical mechanics (cf. [3]). In this application, the use of intuitionistic logic is necessary in order to accommodate infinitesimals. Subsequently, in 1998 Butterfield and Isham established a deep mathematical connection between topos theory and quantum mechanics [14, 7] through their sheaf-theoretic reinterpretation of the Kochen–Specker Theorem (cf. [6]). Their work was recently extended by Döring and Isham, who went as far as proposing topos theory as a foundation for all of physics (including a possible future theory of quantum gravity) [11]. Our own work [i] was greatly indebted to these papers.

Noncommutative geometry was almost single-handedly created by Alain Connes [9]. It vastly extends topology and differential geometry (and, more recently, also arithmetic algebraic geometry [10]) by adapting their ideas and techniques (such as K-theory, De Rham cohomology and Atiyah-Singer index theory) to the setting of noncommutative algebras. Operator algebras, i.e. C^*-algebras and von Neumann algebras [23] (Connes' original field) have played a central role in noncommutative geometry from the beginning, including their link with quantum physics just mentioned [9, 10]. Indeed, inspired by the fundamental Gelfand theorem [12]—establishing a categorical duality between locally compact Hausdorff spaces and *commutative* C^*-algebras—in noncommutative geometry *arbitrary* C^*-algebras are treated as generalized spaces.

The remarkable fact that topos theory and noncommutative geometry each have their own—apparently independent—generalized concepts of space, begging to be related as it were, will play a crucial role here.

Algebraic quantum theory replaces the standard Hilbert space formalism of quantum mechanics [18] by the use of operator algebras. This turns out to have major advantages, especially when infinitely many degrees of freedom are involved, such as in quantum field theory and quantum statistical mechanics [13]. While these two areas have traditionally formed the main context of the algebraic formalism, the applicant has pioneered the use of algebraic quantum theory in quantization theory and the classical limit of quantum mechanics [ii, iii, iv]. Importantly, Bohr's philosophical 'doctrine of classical concepts' [5] has a transparent formulation in algebraic quantum theory, to the effect that the empirical content of a quantum theory described by a certain noncommutative C^*-algebra A is contained in suitable commutative C^*-algebras associated to A. In the simplest case, these are simply the (unital) commutative C^*-subalgebras of A. This reading of Bohr's philosophy formed the conceptual motivation for the basic construction in our paper [i].

Quantum logic is an attempt to capture the logical structure of quantum theory, generalizing or adapting the idea that the propositional logic of classical physics is described by the Boolean algebra of subsets of phase space. In his axiomatic formulation of quantum mechanics [18], von Neumann replaced classical phase space by Hilbert space, and (measurable) subsets of phase space by closed linear subspaces of Hilbert space (equivalently, orthogonal projections). As in the classical case, these form a lattice under inclusion as partial order. However, there is a major difference between the lattice of subspaces of phase space and the lattice of closed linear subspaces of Hilbert space, in that the latter fails to be distributive. Nonetheless, Birkhoff and von Neumann interpreted the lattice operations $\wedge$ and $\vee$ as 'and' and 'or', as in the classical case, and argued that the departure from distributivity (and hence from classical logic) simply meant that one faced a new kind of 'quantum' logic [4].

Unfortunately, this notion of quantum logic is problematic from both a logical and a physical point of view [i]; the advantage of its 'spatial' nature, in being based on sub*spaces* of Hilbert *space*, in our opinion forms insufficient compensation for this. The logical role of space as a carrier of propositions (as captured classically by the Boolean algebra of its subsets) is similarly obscured in noncommutative geometry, based as it is on C^*-algebras (and hence ultimately on Hilbert spaces). In contrast, because of its intimate relationship with categorical logic, topos theory provides concepts of space that do have a clear logical interpretation, such as a locale (at the level of propositional logic) or even a topos itself (at the level of predicate logic). As detailed below, our immediate goal is to relate these concepts to noncommutative spaces (i.e. C^*-algebras), so as to eventually develop a satisfactory spatial theory of quantum logic.

Research goals and methodology

The overall goal of the research program to which the PhD student applied for here should contribute, is to relate topos theory to noncommutative geometry by comparing their respective generalized concepts of space (i.e. locales and C^*-algebras). Doing so as proposed in [i] leads to very specific locales $L(A)$, defined by (unital) C^*-algebras A in the spirit of Bohr's doctrine of classical concepts. The physical interpretation of this construction is as follows: if A is the algebra of observables of a quantum system, then $L(A)$ is its 'quantum phase space', whose associated Heyting algebra $H(A)$ captures the (intuitionistic) logical structure of the system. In our opinion, this (rather than the quantum logic of Birkhoff and von Neumann) is the correct quantum analogue of the way the Boolean algebra of subsets of a classical phase space describes the (propositional) logic of classical physics.

The specific contribution expected from the present PhD student would in any case include the first part of the following program:

(1) Compute the locales $L(A)$ for familiar C^*-algebras A, such as:
   (a) $C(X)$ for arbitrary compact Hausdorff spaces X;
   (b) Matrix algebras;
   (c) AF-algebras;
   (d) Groupoid C^*-algebras;
   (e) Continuous fields of C^*-algebras with fibers in one of these classes.
(2) Exploit any possible information about the structure of A to draw appropriate conclusions about the associated locale $L(A)$.
(3) Analyse the logical structure of the quantum system described by A by studying the Heyting algebra $H(A)$ underlying $L(A)$.

The computations in (1) should apply the Joyal–Tierney description of locales in topoi of sheaves $\mathrm{Sh}(X)$ as fibre bundles over X (in the usual category of sets) [16] to the case at hand. Part (2) would rely on both traditional C^*-algebraic techniques (such as Bratelli diagrams in case (c), or Dixmier–Douady theory in case (e)) and tools of noncommutative geometry (notably K-theory). In addition, physical intuition might help, notably in case (d). This remark applies even more strongly to part (3), which otherwise relies on known techniques from intuitionistic logic as it has been developed in the context of topos theory (notably, the Mitchell–Bénabou language and its Kripke–Joyal semantics [v]).

Projects (2) and (3) are independent and may be done in arbitrary order; in fact, it would not be realistic to assume that our PhD candidate will be able to complete both. Depending on background and taste, (s)he will probably choose one or the other, the applicant and/or a second PhD student or postdoc to be funded by other means working on the complementary topic. In addition, also the evolving interaction between the envisaged research team (see §4) will undoubtedly guarantee that all three aspects of this research project will duly be taken care of.

Meanwhile, the applicant himself (probably in collaboration with Bas Spitters at Eindhoven and Steve Vickers at Birmingham) plans to investigate the possibility of refining the construction of $L(A)$ in [i] (in particular, by topologizing the poset $\mathcal{C}(A)$ of all unital commutative C^*-subalgebras of A, either as a set or as a category in the sense of Grothendieck). In case of either fast progress with (1), or else—the opposite case—stagnation, our PhD candidate might become involved in this.

Scientific interest and urgency

Topos theory and noncommutative geometry are two of the great mathematical theories of the second half of the 20th century. Their founders, Grothendieck and Connes, respectively, are both Fields Medallists with an enormous influence and a large following. Like Riemann, the goal of each has been to find an appropriate generalization of the concept of 'space'. Since they arrived at quite different solutions, it would seem to be of considerable importance to relate their efforts, particularly with a concrete application in mind like quantum theory. Modest as our contribution might be, the scientific communities that would be interested in the results of the program proposed here would presumably include category theory, noncommutative geometry and C^*-algebras, quantum logic, mathematical physics, and perhaps quantum gravity.

The urgency in time comes in part from the increasing interest in the interaction between quantum physics, category theory, and logic, as exemplified e.g., by the large number of papers and conferences per year in this context. The specific goal of this interaction is quantum computation and quantum information theory, areas of interest to physics, mathematics, computer science and technology. Furthermore, the recent book of Connes and Marcolli [10] has vastly expanded the scope of noncommutative geometry, especially in the direction of algebraic geometry and number theory (the areas for which Grothendieck originally devised topos theory).

Finally, on a different note, the fact that the applicant's Pioneer project is running out provides a certain urge for new funding.

Relationship to research elsewehere

As the goal of this proposal is to relate the four fields listed at the beginning of §6a above, it might be appropriate to explain its relationship to current research in those fields. In summary, at the early phase described here our project relates to important ongoing work in topos theory, whereas the other three fields are relevant at the stages at which they have consolidated themselves at least a decade ago. This may be reassuring for the PhD candidate applied for here, for whom it would be quite impossible to get involved in all of them at the pace of research at the frontier. In all cases, however, the applicant has excellent contacts with those active at each of these frontiers, so that leading researchers in each of the four areas mentioned could become involved as advisors, sounding boards, or even collaborators at any moment.

Topos theory has reached an almost final form with the appearance of Johnstone's compendium [15]. An important exception is recent work on 'geometricity' by Vickers [24], which is relevant to all steps (1)–(3) in the list of research goals above. As already indicated above, our mutual intention is to collaborate on this project. A first step in this direction has been our recent *Workshop on Sheaves in Geometry and Quantum Theory*, held at Nijmegen from 3–5 September 2008 (see `www.cs.ru.nl/~heunen/lgqt/`), which involved Vickers as a speaker. Another speaker at this conference was Pedro Resende (IST, Lisboa); his new ideas on the relationship between topoi, groupoids and quantales

[21] are of concern to at least our topics (1d) en (2), and, in the wake of recent mutual visits, collaboration will be sought as soon as the opportunity arises.

The frontier of *noncommutative geometry* is, roughly speaking, divided between the two halves of the recent book of Connes and Marcolli, viz. applications to physics (notably to quantum field theory and the Standard Model of elementary particle physics) and to number theory and (arithmetic) algebraic geometry. These developments are spectacular, but hardly relate to the present proposal: we only need the formalism of noncommutative geometry as written up in Connes' landmark book [9]. At an advanced stage of its development, this project might try to relate to both applications, but this is irrelevant to the PhD project explained here.

A similar comment applies to *algebraic quantum theory*: for the project described here it will be quite sufficient to take the field as it stood with the appearance of the books by Haag [13] on quantum field theory and quantum statistical mechanics and by the applicant on quantization and the classical limit [i].

Current research in *quantum logic* is mainly directed towards quantum computation, and takes place at the interface of theoretical physics, computer science and logic. Active centers include the Oxford University Computing Laboratory, the Center for Quantum Computation at Cambridge, and Waterloo (Canada), featuring both the Perimeter Institute and the nearby Institute for Quantum Computation at the University of Waterloo. None of these groups appear to be working in the direction envisaged here, but all would be interested, we presume, in our results.

Finally, the *quantum gravity* community is of perhaps unexpected importance here, as the research of Isham and collaborators [14, 7, 11] that originally inspired our program has its roots in it. Isham's group at Imperial College London remains a source of ideas, and in addition category theorists inspired by the problem of quantum gravity (e.g., John Baez at Riverside) provide both relevant ideas and a potential audience for us.

Continuity and local embedding

The present proposal continues work started by the applicant's current PhD student Chris Heunen and his M.Sc. student Martijn Caspers. The applicant has worked in all four areas mentioned [i]–[v]; the interaction between noncommutative geometry and algebraic quantum theory formed the core of his NWO-funded Pionier project (2002–2007). He has so far supervised six completed and two ongoing PhD Theses in these or related areas, as well as ten M.Sc Theses (see `www.math.ru.nl/∼landsman/` for a list). It has to be admitted that his active use of topos theory has been recent, but this lack of experience necessarily accompanies a change in research direction and in any case will be compensated for by the active role to be played by Ieke Moerdijk in this project. As to the general interaction between mathematics and theoretical physics, the applicant was the PI of the Dutch GQT-cluster.

The local environment for a project like this looks quite favourable at the moment. At Nijmegen, where our PhD student is to be appointed, Mai Gehrke arrived last year as the new professor of algebra, specializing in (algebraic) logic. She is actively interested in this project, as is her PhD student Dion Coumans. Bart Jacobs at the Computer Science department at Nijmegen is an expert in categorical logic; jointly with the applicant, he currently supervises Chris Heunen, a PhD student whose work in part paved the way for the present application. Also, Michael Müger is a specialist in the interface between category theory and mathematical physics. The mathematical physics group at Nijmegen has been greatly strengthened by the recent *veni* award to Walter van Suijlekom, an expert in noncommutative geometry and its relationship to quantum field theory. Adding to the expertise of Erik Koelink, Müger, and the applicant, his appointment has definitely established Nijmegen as a stronghold of noncommutative geometry in The Netherlands. In addition, this area is well represented at our GQT-partners Utrecht (Cornelissen, Crainic, and Moerdijk) and Amsterdam (Opdam, Posthuma).

Furthermore, Ieke Moerdijk and Jaap van Oosten at Utrecht are experts on topos theory; the former has a history of joint projects with the applicant. Hans Halvorson, a professor of philosophy at Princeton, is spending the academic year 2008–2009 at Utrecht to work on topos theory with Moerdijk. Halvorson and the applicant have long-standing ties as members of the international philosophy of physics community, and there is a clear mutual desire for collaboration in the general area of this proposal.

6b Application Perspective. An improved understanding of quantum logic might be of help in technological applications of quantum theory, notably in the area of quantum computation (sometimes believed to the carrier of the next technological revolution).

It would, however, better suit the spirit of the present proposal to point out that the applicant has a track record in outreach and education, and will do his utmost to communicate the results of this project—in case it is funded and successful—to the general public. What is to be communicated here includes the fact that applications of (initially) 'pure' mathematics (such as topos theory) to physics (in this case, quantum theory) can be utterly unexpected. Even apart from the feeling that this should make one marvel at the architecture of the Universe and the very possibility of mankind penetrating its secrets, such instances ought to warm taxpayers and politicians alike to the funding of fundamental research.

7 Project planning. The better part of the first year of the PhD candidate will be spent studying topos theory and C^*-algebras, interrupted by small research problems preparing for the main project (1). These topics will be studied simultaneously and in close interaction with each other (which is unusual for both). The applicant has already supervised such a trajectory in the case of himself, Chris Heunen, and Martijn Caspers, and (together with Mai Gehrke) is currently introducing topos theory to Dion Coumans. This experience has given some insight on how to best approach this preparatory stage.

In addition, it would be appropriate for the PhD candidate to attend parts of the MRI Master Class 2009–2010 on *Arithmetic Geometry and Noncommutative Geometry*; see `www.math.uu.nl/people/cornelis/mc.shtml`. For example, courses on C^*-algebras and noncommutative geometry by Erik Koelink and the applicant will be part of the programme. One also hopes that by the time more general nationally organized courses and other opportunities for PhD students to meet have been set up, notably by MRI under its new director, and/or by GQT under its new board.

Subsequently, the second year will be spent on part (1) of the program listed under *Research goals and methodology*. The third year should be devoted to either part (2) or part (3), as explained above. The fourth year may be spent on the complementary project and/or on writing the thesis.

8 Expected Use of Instrumentation. None.

9 Literature.

Key publications of research team

[i] C. Heunen, N.P. Landsman & B. Spitters, A topos for algebraic quantum theory. `arXiv:0709.4364 v2` (2008). Submitted to *Commun. Math. Phys.*
[ii] N.P. Landsman, *Mathematical Topics Between Classical and Quantum Mechanics* (Springer, New York, 1998).
[iii] N.P. Landsman, Lie groups and Lie algebroids in physics and noncommutative geometry. *J. Geometry and Physics* **56**, 24–54 (2006).
[iv] N.P. Landsman, Between classical and quantum. *Handbook of the Philosophy of Science Vol. 2: Philosophy of Physics*, J. Butterfield & J. Earman (Eds.), pp. 417-554 (North-Holland, Amsterdam, 2007).
[v] S. Mac Lane & I. Moerdijk, *Sheaves in Geometry and Logic: A First Introduction to Topos Theory* (Springer, New York, 1994).

References

[1] M. Artin, A. Grothendieck & J.-L. Verdier, Théorie de topos et cohomologie étale des schémas (SGA4), *Lecture Notes in Mathematics* **269, 270**. Springer, Berlin, 1972.
[2] B. Banaschewski & C.J. Mulvey, A globalisation of the Gelfand duality theorem. *Ann. Pure Appl. Logic* **137**, 62–103 (2006).
[3] J.L. Bell, *A Primer of Infinitesimal Analysis* (Cambridge University Press, 1998).
[4] G. Birkhoff & J. von Neumann, The logic of quantum mechanics. *Ann. Math.* **37**, 823–843 (1936).
[5] N. Bohr, Discussion with Einstein on epistemological problems in atomic physics. *Albert Einstein: Philosopher-Scientist* (P.A. Schlipp, Ed.), pp. 201–241 (Open Court, La Salle, 1949).
[6] J. Bub, *Interpreting the Quantum World* (Cambridge University Press, 1999).
[7] J. Butterfield & C. J. Isham, A topos perspective on the Kochen-Specker theorem. II. Conceptual aspects and classical analogues. *Internat. J. Theoret. Phys.* **38**, 827–859 (1999).
[8] P. Cartier, A mad day's work: from Grothendieck to Connes and Kontsevich. The evolution of concepts of space and symmetry. *Bull. Amer. Math. Soc. (N.S.)* **38**, 389–408 (2001).
[9] A. Connes, *Noncommutative Geometry* (Academic Press, San Diego, 1994).
[10] A. Connes & M. Marcolli, *Noncommutative Geometry, Quantum Fields and Motives* (American Mathematical Society, Providence, 2008).
[11] A. Döring & C.J. Isham, A topos foundation for theories of physics: I-IV. *J. Math. Phys.* **49**, (2008). `arXiv:quant-ph/0703060` etc.
[12] I. Gelfand & M. Neumark, On the imbedding of normed rings into the ring of compact operators in Hilbert space. *Math. Sb.* **12** (54) 197–213 (1943).
[13] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras* (Springer, Heidelberg, 1992).
[14] C. J. Isham & J. Butterfield, Topos perspective on the Kochen-Specker theorem. I. Quantum states as generalized valuations. *Internat. J. Theoret. Phys.* **37**, 2669–2733 (1998).
[15] P.T. Johnstone, *Sketches of an Elephant: A Topos Theory Compendium. Vols. 1, 2* (Oxford University Press, Oxford, 2002).
[16] A. Joyal & M. Tierney, An extension of the Galois theory of Grothendieck. *Mem. Amer. Math. Soc.* **51**, 1–71 (1984).
[17] I. Moerdijk, Models for the leaf space of a foliation. *European Congress of Mathematics, Vol. I (Barcelona, 2000)*, Progr. Math. **201**, pp. 481–489 (Birkhäuser, Basel, 2001).
[18] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin 1932). English translation: *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Berlin 1955).
[19] M. Rédei, *Quantum Logic in Algebraic Approach* (Kluwer Academic, Dordrecht, 1998).
[20] M. Rédei & M. Stolzner (eds.), *John von Neumann and the Foundations of Quantum Mechanics* (Kluwer Academic Publishers, Dordrecht, 2001).

[21] P. Resende, Etale groupoids and their quantales. *Adv. Math.* **208**, 147–209 (2007).
[22] M.A. Rieffel, Quantization and C^*-algebras. *Contemporary Mathematics* **167**, 66–97 (1994).
[23] M. Takesaki, *Theory of Operator Algebras, Vols.* I-III (Springer, Berlin, 2003).
[24] S. Vickers, Locales and toposes as spaces. *Handbook of Spatial Logic*, eds. M. Aiello, I. Pratt-Hartmann, J. van Benthem, Chapter 8 (Springer, Heidelberg, 2007).

10 Requested Budget. We request funding for one PhD student, as follows:

(1)	Appointment:	177.495
(2)	Personal benchfee:	5000
(3)	Additional travel budget:	10000
(4)	**Total:**	**192.495**

The first two figures are standard for a PhD student. The additional travel budget should cover both regular conference attendances (1000 euro per year) and two longer periods abroad (3000 euro each). The applicant considers it to be of great importance that the candidate spends at least two longer periods abroad, for example at Birmingham (School of Computer Science, University of Birmingham), Cambridge (DPMMS and Center for Quantum Computation, University of Cambridge), Lisbon (Instituto Superior Técnico), London (Imperial College), Oxford (Oxford University Computing Laboratory), or Waterloo (Perimeter Institute and Institute for Quantum Computation, University of Waterloo). One such period could fall in the first half of the appointment, the other during the last two years.