Scientific final report on the Pionier project of Klaas Landsman: '*Quantisation, noncommutative geometry, and symmetry*' (NWO-EW no. 616.062.384; duration 01/09/2002 – 15/08/2009)

Prof.dr. N.P. Landsman

Institute for Mathematics, Astrophysics, and Particle Physics (IMAPP)

Faculty of Science

Radboud University Nijmegen

Heyendaalseweg 135

6525 AJ NIJMEGEN

Email: landsman@math.ru.nl.

Telephone: 024-3652874

I. General information

1. Appointments

Prof.dr. N.P. Landsman, 01/09/2002-31/08/2007: principal investigator (PI)

Dr. C. Vilar Campos Carvalho, 01/09/2003-31/08/2005 (postdoc)
Drs. C.J.M. Heunen, 15/08/2005-14/08/2009 (PhD student)
Drs. P. Hochs, 01/04/2003-31/07/2007 (PhD student)
Drs. N. Kowalzig, 01/09/2003-31/08/2004 (PhD student)
Dr. M. Müger, 01/07/2003-30/06/2006 (postdoc)
Dr. H.B. Posthuma, 01/09/2005-31/08/2006 (postdoc)
Dr. W. van Suijlekom, 01/01/2007-31/12/2008 (postdoc)
Dr. E. Hawkins, 1/09/2007-30-04-2008 (postdoc)[1]

2. Guests

Drs. W. van Suijlekom, 01/09/2002-31/10/2002
Dr. R. Popescu, 12/05/2003-10/06/2003
Drs. E. van Erp, 17/05/2003-15/06/2003 and 11/11/2003-26/11/2003
Prof.dr. J. Trout, 01/10/2005-31/10/2005
Prof.dr. Varghese Mathai, 20/11/2006-01/12/2006 (jointly with GQT cluster)

3. Associated researchers
(not funded by the project but falling within its scope)

Drs. R.D. Bos, 01/09/2002-28/02/2007 (FOM/NWO-EW Mathematical Physics Program)
Drs. N. Kowalzig, 01/09/2004-30/06/2009 (UvA, CNRS, UU)
Dr. E. Hawkins, 01/09/2005-31/08/2007 (EC, Marie Curie incoming fellow)
Dr. P. Matias, 01/01/2006 - 31/12/2006 (FCT, Portugese Research Felowship)

4. Academic careers of researchers within the project

My Pionier project played a pivotal role in either launching or securing the academic careers of most of the scientists in it; I regard this as a major, perhaps the most important outcome of it:

• Vilar Campos Carvalho got tenure at the Instituto Superior Técnico in Lisabon in 2005;
• Heunen went to Oxford on a Rubicon grant from NWO in 2009;
• Hochs obtained research positions in mathematical physics, first at TNO and currently at Shell;
• Müger was appointed UD in mathematical physics at the Radboud University in 2005;[2]
• Posthuma obtained a Veni grant from NWO in 2006 (taken up at Utrecht) and was appointed UD in mathematics at the University of Amsterdam in 2008;
• Van Suijlekom obtained a Veni grant from NWO in 2009 and will be appointed UD in mathematical physics at the Radboud University in 2010;
• Hawkins was appointed Lecturer in Mathematics at York University in 2008.

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[1] Eli Hawkins initially came to Nijmegen as a Marie Curie Incoming Fellow in my group.

[2] This was the tenured position promised to NWO and the PI, initially by the UvA and subsequently by the RU.

At an earlier career stage, of the MSc students supervised by me on themes lying within the Pionier project *(thesis title in brackets)*, a number continued in academia:[3]

- F. Ypma (*Quasicrystals, C*-algebras and K-theory)* got a PhD in mathematics from Oxford;
- B.Norouzizadeh *(Coarse Geometry and K-Homology)* now has a PhD position at Göttingen;
- B. Mesland *(Algebraic K-theory, periodic cyclic homology, and the Connes-Moscovici index theorem)* obtained a PhD in mathematics from Bonn and is currently a postdoc in the Vici-project of G. Cornelissen at Utrecht;
- M. Vargas Rivera (*Noncommutative geometry and the integer quantum Hall effect)* obtained a PhD in applied mathematics from the VU;
- M. Caspers (*Gelfand Spectra of C*-algebras in topos theory*) is currently a PhD student in mathematics at the RU. His MSc Thesis won the Thesis Prize of the GQT-cluster in 2008.

Although the MSc Thesis of E. van Erp (*The Atiyah-Singer index theorem, K-theory, and quantisation*, July 2000) was finished just before the start of the Pionier project, he later reappeared a guest of the project, whilst doing a PhD at Penn State with Prof Nigel Higson. He is currently a postdoc at the University of Pennsylvania. See II.5 below for his contribution to the Pionier project.

5. Spin-off and follow-up

As will be detailed below, the six subprojects of my original research proposal have been successfully carried out. The following developments are worth spelling out separately:

1. A new research line, not anticipated at the time of submission (2001), has been included in the project since 2005, namely *quantum logic* (especially in connection with topos theory and intuitionistic mathematics, in which the Netherlands has a strong tradition, going back to L.E.J. Brouwer and A. Heyting and continued by A.S. Troelstra, I. Moerdijk, and others). Quantum logic was new to the Netherlands, as was the ensuing interaction between the mathematical physics community and the logic community. This interaction started locally in Nijmegen, taking place initially between the Mathematical Physics department and the Institute for Computer and Information Science (iCIS), notably with Prof. Bart Jacobs (with whom I jointly supervised a PhD student in this area, viz. Chris Heunen) and Dr Bas Spitters (a postdoc specialising in constructive mathematics). This was followed, after the arrival of Prof. Mai Gehrke, by informal joint activities also with the Algebra & Logic department (notably, a reading group on topos theory and categorical logic with Gehrke and her PhD student Dion Coumans). Meanwhile, this interaction is also notable at a national scale, as exemplified by the *Logic and Physics Sympoisum* at Utrecht (11/01/2008) and the *Workshop on Sheaves in Geometry and Quantum Theory* at Nijmegen (3-5/09/2008), the latter co-organised by myself. At Nijmegen, this research will be continued in collaboration with my new PhD student Sander Wolters, funded by the NWO-project *Topos theory, noncommutative geometry, and quantum logic* (NWO-EW no. 613.000.811). At Utrecht, the sabbatical taken up by Prof Hans Halvorson (Princeton) during 2008-2009, hosted by Prof Ieke Moerdijk, was related to this development as well. Specifically, in collaboration with Halvorson I now plan research on the possible role of topos theory and quantum logic in Rudolf Carnap's *Wissenschaftslogik* (a subject in the philosophy of science, which currently receives considerable attention).
2. Research in noncommutative geometry in the Netherlands has undoubtedly been strengthened by the Pionier project. This subject became embedded nationally as one of the three research themes of the GQT-cluster (see below) from 2006 onwards, and plays a central role in the current Vici-

[3] Tragically, H. Janssen *(Reconstructing Reality)* died in a traffic accident in 2008, two months after her M.Sc degree.

project *A tale of two geometries* of Prof. Gunther Cornelissen at Utrecht. At the University of Amsterdam, I acted as a co-promotor of Maarten Solleveld's thesis *Periodic Cyclic Cohomology of Affine Hecke Algebras* (6 March 2007), with Opdam as promotor.
The MRI Masterclass on *Noncommutative Geometry* during 2003-2004, which I organized jointly with Prof. Ieke Moerdijk (Utrecht), also deserves to be mentioned in this direction, as does its successor, the 2009-2010 MRI Masterclass on *Arithmetic Geometry and Noncommutative Geometry*, organized by G. Cornelissen. Both involve(d) substantial lecture courses from my side (i.e. *C*-algebras and K-theory* in 2003-2004 and *Noncommutative Geometry* in 2009-2010), helping to educate a new generation of young mathematicians in this direction.
At Nijmegen, the Veni-project *Noncommutative geometry of quantum gauge fields* of Walter van Suijlekom (NWO-EW no. 639.031.827) is a direct continuation of his research within my Pionier project. As already pointed out, the project prepared for his tenured appointment at the Radboud University Nijmegen in 2010. Van Suijlekom is an internationally leading researcher in the applications of noncommutative geometry to quantum field theory and elementary particle physics. His presence at Nijmegen is of central importance in the current collaboration between the departments of Mathematical Physics and Theoretical High-Energy Physics within the research institute IMAPP at the Faculty of Science of the Radboud University Nijmegen. In the near future, further collaboration also with the department of Experimental High-Energy Physics within IMAPP will make this institute probably unique in the world in hosting interconnected research on the Higgs sector of the Standard Model of elementary particle physics from three complementary points of view: *experimental*, *phenomenological*, and *mathematical*. The emergence of the latter perspective in Higgs research at Nijmegen is largely an outcome of the Pionier project and Van Suijlekom's role in it. This triangular approach to the Higgs sector will probably be further strengthened in the near future by the instalment of a new Chair *Beyond the Standard Model* at IMAPP, funded by the *Sectorplan Natuurkunde* (i.e. the national Master Plan for Physics). As my own future research plans emphatically include the application of noncommutative geometry to quantum field theory and elementary particle physics, in collaboration with van Suijlekom, this development is of great interest to me, and I indeed was among those who argued for this specific direction of the proposed new Chair.

3. The research line on quantum symmetry in the Pionier project, mainly carried by Michael Müger, has found a follow-up in his NWO project *Categorical structures of infinite quantum systems with anyons* (NWO-EW no. 613.000.608). Pieter Naaijkens is employed on this project as a PhD student.
4. Much of 2004 was occupied with the preparation of the proposal *The Fellowship of Geometry and Quantum Theory* (GQT) for an NWO 'Wiskundecluster', of which I was the Principal Investigator. This proposal was approved in 2005, started on 1 September 2006, and was officially launched by Minister van der Hoeven on 10 October 2006. Of its three principal research themes, the first (*Poisson geometry, quantisation, and noncommutative geometry*) is closely related to my Pionier project.[4] More generally, this cluster has provided an enormous impetus to the areas of mathematical physics and mathematics I am interested in. To some extent, the GQT-cluster compensates for the discontinuation of the FOM-GBE Mathematical Physics Program (of which I was a Board member from 2001-2006). It has to be mentioned, though, that the unity of mathematical physics in the Netherlands is now largely lost at an organisational level, since 'classical' mathematical physics now resides with the so-called NDNS+ cluster.

[4] The other two themes are *Integrable systems, Frobenius manifolds, and the geometric Langlands program* and *Moduli spaces, mirror symmetry, and topological strings.*

6. Changing places

On 1 September 2004 the Pionier project was moved from the University of Amsterdam to the Radboud University Nijmegen, where I took up the Chair in Analysis (this was transformed into a Chair in Mathematical Physics in 2007). The reasons for this move were largely of a personal nature, but the project does not appear to have suffered much from this move and has arguably even been strengthened by it. For example, Prof. Gert Heckman of the Radboud University played an important role in the supervision of Peter Hochs, eventually ending up as his co-promotor. In addition, as already mentioned, I supervised Chris Heunen jointly with Prof. Bart Jacobs from computer science.
On the down side, at least as far my own research time is concerned, strong pressure was put on me to play a central role in the leadership of the mathematics department at Nijmegen (now part of the research institute IMAPP, of which I am currently deputy director). However, this has enabled me to create conditions at Nijmegen that seem to secure a healthy future for mathematical physics (and hence for the type of research in the Pionier project) at the Radboud University. This includes the tenured appointments of Müger (2006) and Van Suijlekom (2010) as UD's, as already mentioned, and also the appointment of Erik Koelink as a Professor of Analysis (2007), initially financed by the GQT cluster.[5]

7. Global investment of funds

The project moved from the University of Amsterdam to the Radboud University Nijmegen on 1-01-2004. Practically all available funds have been spent on salary costs of the above appointments (including computers and travel costs). A minor fraction has been spent on guests. The associated researchers listed here were externally funded members of my research group. They are mentioned here because their research has effectively been incorporated into the Pionier project (except financially). In my opinion, all project members have built up a satisfactory (and sometimes impressive) record of publications and scientific activities in general, as exemplified by their subsequent careers.

The project was extended beyond the usual 5 years in order to appoint Hawkins, Heunen and Van Suijlekom. The fact that this was (almost) possible within the total available budget, was a consequence of the early departure of Posthuma (who took up a Veni grant at Utrecht after one year, being on initial the budget for two years), and of the decision of Kowalzig not to move to Nijmegen with the project. This led to the withdrawal of project funding for his PhD position, which was subsequently financed by the UvA (01/09/2004-31/12/2007) and the UU (01/07/2008-30/06/2009), with intermittent research stays at the IHP in Paris, funded by the CNRS. He continued to be supervised by myself, from the Summer of 2008 jointly with Moerdijk at Utrecht. The PhD appointment of Heunen was financed jointly with the simultaneous Pionier project of Prof.dr. B. Jacobs (iCIS, FNWI, RU), also emerging from NWO-EW.

During the course of the Pionier project I organized four workshops on topics central it:[6]

- *Workshop on the Quantisation of Singular Symplectic Quotients* (Oberwolfach, 20-26/01/2003);
- *Mini-Workshop on Noncommutative Geometry* (Amsterdam, 21-23/05/2003);
- *Mini-Workshop on Index Theory on Singular Spaces* (Nijmegen, 28-29/11/2006);
- *Workshop on Sheaves in Geometry and Quantum Theory* (Nijmegen, 3-5/09/2008).

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[5] The (very) recent Research Assessment of mathematics at Dutch universities through the period 2002-2008 has led to the preliminary conclusion that indeed the Mathematical Physics department at the Radboud University is viable.

[6] Unfortunately, there was no money left for a final workshop at the end of the project.

The following national intercity seminars were co-organised by myself and backed the research in the Pionier project:

•*Seminar on Noncommutative Geometry* at Amsterdam (2002-2003, with E.M. Opdam); cf. http://remote.science.uva.nl/~npl/BC.html);

•*Seminar on Operads, Groupoids and Quantisation* at Utrecht and Nijmegen (2005-2006, with I. Moerdijk); see http://www.math.uu.nl./people/lukacs/seminar.html.

8. International collaboration

In the years 2003, 2004, 2005, and 2006, I was involved as a coapplicant in four European network applications that unfortunately and despite considerable effort failed (PI between brackets):

Operator Algebras and Noncommutative Geometry (Prof David Evans, Cardiff);
Noncommutative Geometry and Physics (Prof Florian Scheck, Mainz);
*Geometric Analysis (*Prof Elmar Schrohe, Hannover);
Deformation Quantisation (Prof Michel Cahen, Brussels).

Hochs collaborated with Paradan at Montpellier and with Varghese Mathai at Adelaide. Kowalzig collaborated with Neumaier (Freiburg), Pflaum (Boulder), and Kraehmer (Glasgow), within the Netherlands also with Posthuma (UvA). Müger collaborated with Tuset (Oslo), Roberts (Rome) and Halvorson (Princeton). Posthuma collaborated with Pflaum (Boulder) and Tang (Davis, CA). Van Suijlekom collaborated with Mahanta (Bonn), Kreimer (IHES, Bures sur Yvette), and with Landi, Pagani, and Reina (Trieste).

9. Outreach

Looking ahead from a more general perspective than the Pionier project alone, I felt an ever increasing responsibility to contribute to outreach to the general public and to schools in particular. One immediate goal is the rise of the number of mathematics students in The Netherlands and at Nijmegen in particular. This goal was, indeed, achieved, as from a meagre enrolment of about 10 first-year students in mathematics during the years 2000-2005, from 2006 the number steadily went up, being 54 in 2009-2010. More generally, public awareness of mathematics needs to be increased, the number of excellent mathematics teachers must increase dramatically, and enthusiasm of schoolkids for mathematics and physics needs a tremendous boost. Thus I have spent considerable effort on outreach activities, addressing audiences from secondary school students to teachers to the Minister of Science, Culture and Education. My popular book *Requiem for Newton* appeared in January 2005 published by Uitgeverij Contact, Amsterdam and is now in its 4th printing.[7] In addition, I wrote quite a fwe popular articles (in Dutch); see list following scientific publications. A list of other outreach activities includes:

- Organisation of a Mathematics Tournament at the Radboud Universiteit Nijmegen (for 500 scholchildren aged 15-17) in 2005, 2006 and 2007.
- Membership of the Resonansgroep Wiskunde of the Ministry of Science, Culture and Education.

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[7] Reviews have appeared in practically all relevant newspapers and periodicals: Radboud Magazine (december 2004, Bea Ros), Volkskrant (15-01-2005, Martijn van Calmthout), Trouw (22-01-2005, Joep Engels), Vrij Nederland (22-01-2005, Tomas Vanheste), Intermediair (17-02-2005, Kees Versluis), Nederlands Tijdschrift voor Natuurkunde (februari 2005, Claud Biemans), TUDelta (17-03-2005, Sjaak Priester), NRC-Handelsblad (02-04-2005, Fred Muller), De Physicus (oktober 2005, A.J. Vermeulen), Nieuw Archief voor Wiskunde (december 2005, Chris Zaal, Gunther Cornelissen), Catalogus Nederlandse Openbare Bibliotheken (D.G. van der Steen),

• Organization of a Master Class (vwo-6) “Bestaat Toeval?” in 2005 and 2006.

• Organization of a Regional Support Center for Wiskunde D in 2006.

• Courses on Probability Theory for mathematics teachers offering Wiskunde D in 2007.

• Chair of the PR-committe for mathematics at the Radboud University (2005-2007).

• Membership of the national PR-committe for mathematics (2006-2007).

• Organization of the IMAPP-Symposium *The Origin of the Universe* (13/10/2006).

• First “Universum-lezing” *No pain, no gain,* Utrecht, 21/11/2005.

• Science Café Debate with Robbert Dijkgraaf, Sijbrand de Jong, Piet Timmermans: *Is er een bèta-crisis?* (08/02/2006, Nijmegen).

• Plenary opening lecture, Nationale Wiskunde Dagen, Noordwijkerhout, 03/02/2006.

• NWO Leraar in Onderzoek project *Newtons afleiding van de wetten van Kepler* (2006-2008).[8]

• Membership of the Scientific Council of the national *Talentenkracht* program, researching the cognitive development of 3-8 year olds (2006-2008), and coapplicant of the project *Role of Language in Mathematics Talent Development in Kindergartners* (2008-12, with Prof L. Verhoeven).

II. Detailed results of research

1. Overview and highlights

The project has largely been carried as outlined in the proposal. A further theme (*Foundations of quantum theory and quantum logic*) has been added as from Summer 2005 onwards. A coherent team of scientists has been formed to investigate the subprojects listed in the proposal, and I would identify the following highlights to have emerged from the project:

• Correct formulation of the functoriality of quantisation [C6, C7] and its proof for free cocompact proper Lie group actions [11, 12, 37]. See § II.4.

• Identification of the structure of renormalization Hopf algebras for gauge theories [33, 34] (§ II.2).

• Extension of equivariant geometric quantisation to Lie groupoid actions and the proof of the ensuing Guillemin-Sternberg conjecture [1, D1]. See §§ II.1 and II.4.

• Synthesis of (strict) deformation quantisation using C*-algebras and geometric quantisation using symplectic groupoids [6]; cf. § II.2.

• Proof of algebraic ‘higher’ index theorems on symplectic orbifolds [30].

• Development of intuitionistic quantum logic in the context of topos theory and the discovery of the localic Gelfand spectrum of a noncommutative C*-algebra [3, 9, 10, B1, C5, D4]; see § II.7.

• Construction of cyclic (co)homology of Hopf algebroids [D3, P4, P5]; see § II.6.

• New proof of the Doplicher - Roberts theorem of algebraic quantum field theory [B4]; cf. § II.6.

The details are as follows. Numbers [..] refer to the list of publications below. Due to the relative coherence of and mutual coherence between the subthemes of the project, most publications would actually fall under several headings. In order to avoid duplication, I have listed the principal one.

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[8] This was eventually taken up by my colleague Prof Gert Heckman, who supervised Maris van Haandel.

2. Quantisation and noncommutative geometry

(Bos, Hawkins, Landsman, Van Suijlekom): [D1, 1, 4, 5, 6, 14, 15, 16, 18, 21, 32, 33, 34, C1, C11, C12, C13, P1, P3, P7]

In 1989, Rieffel introduced an analytic version of deformation quantisation based on the use of continuous fields of C*-algebras. In [16] I explain how a wide variety of examples of such quantisations can be understood on the basis of a single lemma involving amenable groupoids. These include Weyl-Moyal quantisation on manifolds, C*-algebras of Lie groups and Lie groupoids, and the E-theoretic version of the Baum-Connes conjecture for smooth groupoids as described by Connes in his book *Noncommutative Geometry* (1994). Concerning the latter, I uses a different semidirect product construction from Connes. This enables me to formulate the Baum-Connes conjecture in terms of twisted Weyl-Moyal quantisation. The underlying mechanical system is a noncommutative desingularization of a stratified Poisson space, and the Baum-Connes conjecture actually suggests a strategy for quantizing such singular spaces.

The aim of my survey paper [18] is to explain the relevance of Lie groupoids and Lie algebroids to both physicists and noncommutative geometers, emphasizing my own contributions. Groupoids generalize groups, spaces, group actions, and equivalence relations. This last aspect dominates in noncommutative geometry, where groupoids provide the basic tool to desingularize pathological quotient spaces. In physics, however, the main role of groupoids is to provide a unified description of internal and external symmetries. What is shared by noncommutative geometry and physics is the importance of Connes's idea of associating a C*-algebra C*(G) to a Lie groupoid G: in noncommutative geometry C*(G) replaces a given singular quotient space by an appropriate noncommutative space, whereas in physics it gives the algebra of observables of a quantum system whose symmetries are encoded by G. Moreover, Connes's map G -> C*(G) has a classical analogue G -> A*(G) in symplectic geometry due to Weinstein, which defines the Poisson manifold of the corresponding classical system as the dual of the so-called Lie algebroid A(G) of the Lie groupoid G, an object generalizing both Lie algebras and tangent bundles. This will also lead into symplectic groupoids and the conjectural functoriality of quantisation. The first of these directions was taken up by Eli Hawkins, as I will now describe; for the second see § II.4.

In [6] Hawkins proposes that a Poisson manifold may be quantized by a twisted polarized convolution C*-algebra of a symplectic groupoid. Toward this end, he defines polarizations for Lie groupoids and sketch the construction of this algebra. A large number of examples show that this idea unifies previous geometric constructions, including geometric quantisation of symplectic manifolds and the C*-algebra of a Lie groupoid.
Previously, Hawkins had shown in [4] that noncommutatively deformed geometries, such as the noncommutative torus, do not exist generically. In [5], he analyses the phenomenon that in the standard example of strict deformation quantisation of the symplectic sphere *S^2*, the set of allowed values of the quantisation parameter is not connected; indeed, it is almost discrete. Hawkins identifies a natural additional axiom for strict deformation quantisation and proves that it implies that the parameter set for quantizing *S^2* is never connected.

In his PhD Thesis [D1] (see also [1]), Rogier Bos constructs Hermitian representations of Lie algebroids and associated unitary representations of Lie groupoids by a geometric quantisation procedure. For this purpose he introduced a new notion of Hamiltonian Lie algebroid actions. The first step of his procedure consists of the construction of a prequantisation line bundle. Next, he discusses a version of Kähler quantisation suitable for this setting. He proceeds by defining a Marsden-

Weinstein quotient for this setting and proves a “quantisation commutes with reduction” theorem. He explains how hia geometric quantisation procedure relates to a possible orbit method for Lie groupoids. His theory encompasses the geometric quantisation of symplectic manifolds, Hamiltonian Lie algebra actions, actions of families of Lie groups, foliations, as well as some general constructions from differential geometry.
The functional-analytical background to [1] is provided by [P1]. Here Bos introduces unitary representations of continuous groupoids on continuous fields of Hilbert spaces. He investigates some properties of these objects and discusses some of the standard constructions from representation theory in this particular context. An important role is played by the regular representation. He concludes by discussing some operator algebras associated to continuous representations of groupoids; in particular, he analyses the relationship of continuous representations of a groupoid and continuous representations of an associated Banach *-category.

Walter van Suijlekom is present through two sub-sub-themes within this subtheme, the first [15, 32, C13, P3] related to the quantisation of finite-dimensional systems from the point of view of noncommutative geometry, the second [33, 34, C11, C12] being concerned with quantum field theory from the same perspective.
In [32], his very first paper, van Suijlekom presents a new example of a finite-dimensional noncommutative manifold, namely the noncommutative cylinder. It is obtained by isospectral deformation of the canonical triple associated to the Euclidean cylinder. He discusses Connes' character formula for the cylinder. In the second part, he discusses noncommutative Lorentzian manifolds. Here, the definition of spectral triples involves Krein spaces and operators on Krein spaces. A central role is played by the admissible fundamental symmetries on the Krein space of square integrable sections of a spin bundle over a Lorentzian manifold. Finally, he discusses isospectral deformation of the Lorentzian cylinder and determine all admissible fundamental symmetries of the noncommutative cylinder.
In [15] (with Landi et al) he constructs theta-deformations of the classical groups SL(2,H) and Sp(2), and in [P3] (with Landi and Khalkhali), van Suijlekom shows that much of the structure of the 2-sphere as a complex curve survives q-deformation, and has natural generalizations to the quantum 2-sphere (which, with additional structures, he identifies with the quantum projective line). In parallel with the fact that positive Hochschild cocycles on the algebra of smooth functions on a compact oriented 2-dimensional manifold encode the information for complex structures on the surface, the authors formulate a notion of twisted positivity for twisted Hochschild and cyclic cocycles and exhibit an explicit twisted positivite Hochschild cocycle for the complex structure on the sphere. See also [C13].
In the quantum field theory/noncommutative geometry interface, van Suijlekom published a number of fundamental results, which propelled the approach to renormalisation theory due to Connes and Kreimer, so far developed for scalar field theories, in the physically desirable direction of gauge (Yang-Mills) theories (see also the summaries of these results in [C11, C12]). First, in [33] he studies the Connes-Kreimer Hopf algebra of renormalization in the case of gauge theories and shows that the Ward identities and the Slavnov-Taylor identities (in the abelian and non-abelian case respectively) are compatible with the Hopf algebra structure, in that they generate a Hopf ideal. Consequently, the quotient Hopf algebra is well-defined and has those identities built in. This provides a purely combinatorial and rigorous proof of compatibility of the Slavnov-Taylor identities with renormalization.
Second, in [34] van Suijlekom analyses the structure of renormalization Hopf algebras of gauge theories. He identifies certain Hopf subalgebras in them, whose character groups are semidirect

products of invertible formal power series with formal diffeomorphisms. This can be understood physically as wave function renormalization and renormalization of the coupling constants, respectively. After taking into account the Slavnov-Taylor identities for the couplings as generators of a Hopf ideal, we find Hopf subalgebras in the corresponding quotient as well. In the second part of the paper, he explains the origin of these Hopf ideals by considering a coaction of the renormalization Hopf algebras on the Batalin-Vilkovisky (BV) algebras generated by the fields and couplings constants. The so-called classical master equation satisfied by the action in the BV-algebra implies the existence of the above Hopf ideals in the renormalization Hopf algebra. Finally, he applies the entire construction to Yang-Mills gauge theory, which was the motivation in the first place.

3. Quantisation of singular spaces

(Kowalzig, Posthuma): [13, 28, P7]

The basic examples of singular spaces to be studied were Marsden-Weinstein quotients and orbifolds (which may arise as such quotients, but may also be studied in general). Of all subthemes of the project as a whole, this is the one leaving most to be done in the future. Nonetheless:

In [13] Kowalzig et al. construct star products on Marsden-Weinstein reduced spaces in case both the original phase space and the reduced phase space are (symplectomorphic to) cotangent bundles. Under suitable assumptions on the original cotangent bundle T*Q, they show that the reduced phase space inherits a star product from T*Q. Moreover, they provide a concrete description of the resulting star product in terms of the initial star product on T*Q and prove that their reduction scheme is independent of the characteristic class of the initial star product. Unlike in other existing reduction schemes they are thus able to reduce not only strongly invariant star products. Furthermore, they establish a relation between the characteristic class of the original star product and the characteristic class of the reduced star product and provide a classification up to G-equivalence of those star products on T*Q which are invariant with respect to a lifted Lie group action. Finally, they investigate the question under which circumstances ‘quantisation commutes with reduction’ and show that in their examples non-trivial restrictions arise.

In the direction of orbifolds, in [28], Posthuma et al. study the cyclic homology theory of formal deformation quantisations of the convolution algebra associated to a proper etale Lie groupoid. They compute the Hochschild cohomology of the convolution algebra and express it in terms of alternating multi-vector fields on the associated inertia groupoid. They introduce a noncommutative Poisson homology whose computation enables them to determine the Hochschild homology of formal deformations of the convolution algebra. Then it is shown that the cyclic (co)homology of such formal deformations can be described by an appropriate sheaf cohomology theory. This enables the authors to determine the corresponding cyclic homology groups in terms of orbifold cohomology of the underlying orbifold. Using the thus obtained description of cyclic cohomology of the deformed convolution algebra, they give a complete classification of all traces on this formal deformation, and provide an explicit construction.
In [P8], Posthuma et al. give a complete computation of the Hochschild cohomology of both the “classical” and “quantum” algebra of observables on orbifolds, thereby completing their previous work. Furthermore - and this is the core of the paper - they give a description of the Yoneda product on these cohomologies. The authors relate these products to orbifold-like products as known from quantum cohomology, inspired by a similar conjecture in the “deformation quantisation” approach of Ginzburg and Kaledin to the McKay correspondence.

4. Functoriality of quantisation
(Bos, Hawkins, Hochs, Landsman, Posthuma, Varghese Mathai): [D1, D2, 11, 12, C1, C6, C7, 37]

This program was launched in [C6], where, notwithstanding known obstructions to this idea, I formulated an attempt to turn quantisation into a functorial procedure. I define a category PO of Poisson manifolds, whose objects are integrable Poisson manifolds and whose arrows are isomorphism classes of regular Weinstein dual pairs; it follows that identity arrows are symplectic groupoids, and that two objects are isomorphic in PO iff they are Morita equivalent in the sense of P. Xu. It has a subcategory LPO that has duals of integrable Lie algebroids as objects and cotangent bundles as arrows. I then argue that naive C*-algebraic quantisation should be functorial from LPO to the well-known category KK, whose objects are separable C*-algebras and whose arrows are Kasparov's KK-groups. This limited functoriality of quantisation would already imply the Atiyah-Singer index theorem, as well as its far-reaching generalizations developed by Connes and others. In the category KK, isomorphism of objects implies isomorphism of K-theory groups, so that the functoriality of quantisation on all of PO would imply that Morita equivalent Poisson algebras are quantized by C*-algebras with isomorphic K-theories. Finally, I argue that the correct codomain for the possible functoriality of quantisation is the category RKK(I), which takes the deformation aspect of quantisation into account.

In the follow-up paper [C7], I gave a concrete realisation of this rather abstract functoriality scenario, in specifying what the functor should be. Namely, I propose that it should be the index of a natural Dirac operator associated with the classical data, where the index is construed as a class in KK-theory. This proposal generalises earlier ideas of Bott in geometric quantisation. The 'quantisation commutes with reduction' conjecture of Guillemin and Sternberg then becomes a special case of the functoriality of quantisation. In fact, my formulation yields almost unlimited generalisations of the Guillemin--Sternberg conjecture, extending it, for example, to arbitrary Lie groups or even Lie groupoids. Technically, this involves symplectic reduction and Weinstein's dual pairs on the classical side, and Kasparov's bivariant K-theory for C*-algebras (KK-theory) on the quantum side.

Two such generalisations have meanwhile been studied. The PhD Thesis of Bos [D1], already discussed in § II.2, looks at the case of Weinstein dual pairs arising from groupoid actions, though under certain rather restrictive technical assumptions. My joint work with Peter Hochs [12], eventually finished by Varghese Mathai (who was a guest of the project in 2006) and W. Zhang [37], studied the technically challenging case of free cocompact proper Lie group actions without any compactness assumptions. The generalised Guillemin-Sternberg conjecture arising from the functoriality of quantisation then uses the equivariant K-homology of the underlying space and the K-theory of the group C*-algebra C*(G) in a crucial way. The equivariant index - which in the compact case takes values in the representation ring R(G) - is replaced by the analytic assembly map - which takes values in K_0(C*(G)) - familiar from the Baum-Connes conjecture in noncommutative geometry. Hochs and I prove the generalised Guillemin-Sternberg conjecture for all Lie groups G having a cocompact discrete normal subgroup, essentially by reduction to the compact case.
Subsequently, Hochs [12] proved the generalised Guillemin-Sternberg conjecture for semisimple groups G with maximal compact subgroups K acting cocompactly on symplectic manifolds, the case where the image of the momentum map in question lies in the set of strongly elliptic elements (i.e. the set of elements of g^* with compact stabilisers; the proof comes down to a reduction to the compact case). Finally, the conjecture was proved in the maximal possible setting, i.e. for all proper cocompact actions on orbifolds, in the magnificent paper [37] by Varghese Mathai and W. Zhang.

The work of Posthuma on conformal field theory also falls under the present heading, as his construction are closely related to the Guillemin-Sternberg conjecture for loop groups. In [P7] he constructs the conformal field theory associated to a compact torus T, i.e. the “nonlinear sigma-model”. Underlying the construction is a unitary modular functor, the construction of which follows from a “quantisation commutes with reduction” type of theorem for unitary quantisations of the moduli spaces of flat T-bundles and actions of loop groups. This theorem in turn is a consequence of general constructions on the category of affine symplectic manifolds and their associated generalised Heisenberg groups.

5. Quantisation and index theory
(Carvalho, Posthuma, van Erp): [2, 17, 29, 30, 31, 35, 36, C8, C10]

In [2] (partly based on her Oxford PhD thesis, further developed at Amsterdam under the present project) Catarina Carvalho gives a proof of the cobordism invariance of the index of elliptic pseudodifferential operators on sigma-compact manifolds, where, in the non-compact case, the operators are assumed to be multiplication outside a compact set. She shows that, if the principal symbol class of such an elliptic operator on the boundary of a manifold X has a suitable extension to $K^1(TX)$, then its index is zero. This condition is incorporated into the definition of a cobordism group for non-compact manifolds, called here “cobordism of symbols.” Her proof is topological, in that she uses properties of the push-forward map in K-theory defined by Atiyah and Singer, to reduce it to R^n. In particular, she generalizes the invariance of the index with respect to the push-forward map to the non-compact case, and obtains an extension of the K-theoretical index formula of Atiyah and Singer to operators that are multiplication outside a compact set. Her results hold also for G-equivariant operators, where G is a compact Lie group. Unfortunately, Carvalho left the project after two years without further results and has not returned to this research line at a later stage either.

My survey paper [17] on the relationship between quantum mechanics and representation theory introduced the Baum-Connes conjecture into The Netherlands. This has had a certain impact, not only on my Pionier project (notably the work of Bos and especially of Hochs, see the above account of their work) but also on the work of Prof Eric Opdam in Amsterdam and his group (cf. the PhD Thesis of Maarten Solleveld mentioned in § I.5). Quantum mechanics and representation theory (in the sense of unitary representations of groups on Hilbert spaces), were practically born together between 1925-1927, and have continued to enrich each other till the present day. Following a brief historical introduction, the paper focuses on a relatively new aspect of the interaction between quantum mechanics and representation theory, based on the use of K-theory of C*-algebras. In particular, the study of the K-theory of the reduced C*-algebra of a locally compact group (which for a compact group is just its representation ring) has culminated in two fundamental conjectures, which are closely related to quantum theory and index theory, namely the and Baum-Connes conjecture the Guillemin-Sternberg conjecture. Although these conjectures were both formulated in 1982, and turn out to be closely related, so far there has been no interplay between them whatsoever, either mathematically or sociologically. This is presumably because the Baum-Connes conjecture is nontrivial only for noncompact groups, with current emphasis entirely on discrete groups, whereas the Guillemin-Sternberg conjecture had so far only been stated for compact Lie groups. This has, indeed, subsequently been remedied by our work on the functoriality of quantisation described above.

The papers published by Hessel Posthuma and his coauthors (notably Pflaum and Tang) are a substantial step forward in our understanding of the relationship between deformation quantisation and index theory. In [29], on index theory on orbifolds, Posthuma et al use the concept of a twisted trace density on a cyclic groupoid, in order to construct a trace on a formal deformation quantisation of a symplectic orbifold. An algebraic index theorem for orbifolds follows as a consequence of a local Riemann--Roch theorem for such densities. In the case of a reduced orbifold, this proves a conjecture by Fedosov, Schulze, and Tarkhanov. Finally, they show how the Kawasaki index theorem for elliptic operators on orbifolds follows from this algebraic index theorem.

In [30], Posthuma et al construct a nontrivial cyclic cocycle on the Weyl algebra of a symplectic vector space. Using this cyclic cocycle, they construct an explicit, local, quasi-isomorphism from the complex of differential forms on a symplectic manifold to the complex of cyclic cochains of any formal deformation quantisation thereof. They then give a new proof of Nest-Tsygan's algebraic higher index theorem by computing the pairing between such cyclic cocycles and the K-theory of the formal deformation quantisation. Furthermore, they extend this approach to derive an algebraic higher index theorem on a symplectic orbifold. As an application, they obtain the analytic higher index theorem of Connes-Moscovici and its extension to orbifolds.
In [31] they subsequently construct an explicit quasi-isomorphism to study the cyclic cohomology of a deformation quantisation over a Riemannian étale groupoid. Such a quasi-isomorphism allows the authors to propose a general algebraic index problem for Riemannian étale groupoids, and to discuss solutions to that index problem when the groupoid is proper or defined by a constant Dirac structure on a 3-dim torus.
Finally, in [C10] Posthuma et al. use the concept of a twisted trace density on a cyclic groupoid to construct a trace on a formal deformation quantisation of a symplectic orbifold. An algebraic index theorem for orbifolds follows as a consequence of a local Riemann--Roch theorem for such densities. In the case of a reduced orbifold, this proves a conjecture by Fedosov, Schulze, and Tarkhanov. Finally, it is shown how the Kawasaki index theorem for elliptic operators on orbifolds follows from this algebraic index theorem.

Erik van Erp wrote his MSc Thesis with me in 2000 and subsequently did part of his PhD research at Penn State during a stay at the University of Amsterdam as a guest of the Pionier project. His thesis [35, 36] will be published in *Annals of Mathematics*, one of the most prestigious mathematics journals. To explain his achievement, recall that the Atiyah-Singer index theorem is a topological formula for the index of an *elliptic* differential operator. The topological index depends on a cohomology class that is constructed from the principal symbol of the operator. On contact manifolds, however, the important Fredholm operators are not elliptic, but merely *hypoelliptic*. Their symbolic calculus is noncommutative, and involves analysis on the Heisenberg group. For a hypoelliptic differential operator in the Heisenberg calculus on a contact manifold, he constructs a symbol class in the K-theory of a noncommutative C*-algebra of symbols. There is a canonical map from this analytic K-theory group to the deRham cohomology of the manifold, which gives a class to which the Atiyah-Singer formula can be applied. He then proves the index formula for these hypoelliptic operators. His methods derive from Connes' tangent groupoid proof of the index theorem, and also use s ideas from Boutet de Monvel for Toeplitz operators, and, more recently, from Melrose and Epstein for of Hermite operators. The construction of van Erp applies to general hypoelliptic pseudodifferential operators in the Heisenberg calculus. As in the Hermite Index Formula of Melrose and Epstein, his construction gives a vector bundle automorphism of the symmetric tensors of the contact hyperplane bundle. This automorphism is constructed directly from the invertible Heisenberg symbol of the operator, and is easily computed in the case of differential operators.

6. Quantum symmetry

(Kowalzig, Müger); [D3, 22-27, C9, B4, P4, P4, P5]

This theme had two largely independent tracks, one due to Michael Müger, closely related to operator algebras and category theory, and one developed by Niels Kowalzig, focusing on the cyclic (co)homology of Hopf algebroids. The symmetry-objects studied by Kowalzig may be seen as vast generalisations of those investigated by Müger, which makes the work of the former rather abstract and that of the latter more concrete and explicit. In the future, these research lines should converge.

Müger's research in the project revolved mostly about fusion categories, i.e., semisimple tensor categories that are linear over a field, in relation to subfactor theory, quantum field theory and representation theory, in particular modular categories and the study of algebras in and module categories over such categories. In the study of this area many subjects interact in a fascinating manner, and there is a host of interesting structural and classification problems. In [22] he proved a double commutant theorem for modular categories and deduced that every modular category is a direct product of prime ones. In [23], a sequel to his "Galois theory for braided tensor categories and the modular closure" (written as a postdoc supported by NWO on a different project of Moerdijk and myself), he proved that Galois extensions of braided tensor categories give rise to braided crossed G-categories, and studied the latter. In [26] he used these results to answer several long-standing questions in quantum field theory (CFT).
A group of four of Müger's papers is concerned with issues in the representation theory of groups and quantum groups: In [24], Müger proves that the center of a compact group can be recovered from the representation ring. In [25], written jointly with L. Tuset and J.E. Roberts, the authors give a streamlined presentation of the Tannaka reconstruction theory of discrete quantum including some new results. In another joint paper, [27] Müger and L. Tuset give an intrinsic characterization of those C*-tensor categories that are representation categories of discrete quantum groups. In [B4], which is a 60-page appendix to H. Halvorson's "Algebraic Quantum Field Theory" in the prestigous *Handbook of the Philosophy of Science*, Müger combines Tannaka's duality theorem with a result of P. Deligne and new ideas in order to give a considerably simplified proof of the abstract reconstruction theorem for compact groups, due originally to S. Doplicher and J.E.Roberts. The main application of this result is to algebraic quantum field theory, where it leads to the construction of fields and a (global) gauge group from the category of physical representations of an algebra of local observables. This theorem is one of the main achievements of post-1945 mathematical physics.

Kowalzig's PhD Thesis [D3] studies the cyclic theory (in the sense of homological algebra).[9] Although the definition of a so-called Hopf algebroid by itself is not due to Kowalzig, one finds a number of alternative definitions of this notion in the literature, which are similar but not (all) equivalent. This led him to the discovery of a weaker and more fundamental notion, namely that of a *left* Hopf algebroid (and its symmetric right counterpart). The structure of an antipode can then be seen as a certain map from the left to the right structure. He thereby shows that Hopf algebroids provide a promising noncommutative generalisation of both Lie groupoids and Lie algebroids (or, more generally, Lie-Rinehart algebras). This also sheds new light on the Connes-Moscovici algebras of transverse differential operators on etale groupoids. His central achievement, however, lies in the construction and computation of what he calls Hopf-cyclic cohomology for Hopf algebroids, which generalises the known cyclic homology and cohomology theories for étale groupoids and for

[9] While cyclic cohomology is the extension of classical DeRham cohomology to non-commutative geometry, Hopf algebroids form the non-commutative generalization of groupoids.

Lie-Rinehart algebras. He also considers the associated dual cyclic homology theory, which is a decidedly nontrivial matter because the underlying bimodule category fails to be symmetric. He also studies Hopf algebroids from the point of view of formal groupoids and formal integration (of algebroids to groupoids). Altogether, Kowalzig regards this theory as basic for the study of symmetry in noncommutative geometry, for it essentially provides a homological way to study noncommutative 'spaces' that play the role of symmetry objects.
Part of this work has been written up in the form of a joint paper with Posthuma [P4].
In Kowalzig's joint work with Kraehmer [P3], the origin and interplay of products and dualities in algebraic (co)homology theories is ascribed to a twisted Hopf algebra structure on the relevant universal enveloping algebra. This provides a unified treatment for example of results by Van den Bergh about Hochschild (co)homology and by Huebschmann about Lie-Rinehart (co)homology.

7. Foundations of quantum theory and quantum logic
(Heunen, Landsman): [D4, 3, 7-10, 19, 20, C2-C5, B1, B2, B3]

This theme has been added during the course of the project. About half of it [19, 20, B1, B2] is a continuation of my earlier work on quantisation and the classical limit of quantum theory. This part is mostly of a more conceptual nature and is unrelated to noncommutative geometry.
The other half [D4, 3, 7-10, C2-C5, B3] arose from the idea, due to Butterfield (Cambridge) and Isham (Imperial College) to apply topos theory to the foundations of quantum mechanics (in their work, the focus was on the Kochen-Specker Theorem). My contributions, jointly with Chris Heunen (a joint PhD student of myself and Prof Bart Jacobs) and bas Spitters, have two interacting features. One is that topos theory leads to a new approach to quantum logic, a subject originating in a remarkable paper of Birkhoff and von Neumann in 1936. However, while logic in general has made tremendous advances since then, even recent work on quantum logic appears to have been immune to such progress. Thus our goal was to incorporate the state of the art of categorical logic into quantum logic. The other feature is that we provide both a mathematical and a conceptual background to the interplay between quantum theory and noncommutative geometry that forms the basis of the Pionier project.

My paper [19] is a mixture of philosophical, historical, and mathematical ideas, appearing on invitation in the Einstein Centennial issue of the leading journal in the history and philosophy of physics. As I explain, Einstein's philosophy of physics (as clarified by Fine, Howard, and Held) was predicated on his *Trennungsprinzip*, a combination of separability and locality, without which Einstein believed objectification, and thereby "physical thought" and "physical laws", to be impossible. Bohr's philosophy (as elucidated by Hooker, Scheibe, Folse, Howard, Held, and others), on the other hand, was grounded in a seemingly different doctrine about the possibility of objective knowledge, namely the necessity of classical concepts. In fact, it follows from Raggio's Theorem in algebraic quantum theory that - within an appropriate class of physical theories - suitable mathematical translations of the doctrines of Bohr and Einstein are equivalent. Thus - upon my specific formalization - quantum mechanics accommodates Einstein's *Trennungsprinzip* if and only if it is interpreted à la Bohr through classical physics. Unfortunately, the protagonists themselves failed to discuss their differences in this constructive way, since their debate was dominated by Einstein's ingenious but ultimately flawed attempts to establish the incompleteness of quantum mechanics. This aspect of their debate may still be understood and appreciated, however, as reflecting a much deeper and insurmountable disagreement between Bohr and Einstein on the knowability of Nature.

[B2] is a major survey of the the relationship between classical and quantum theory, including many new results and insights. This relationship is of central importance to the philosophy of physics, and any interpretation of quantum mechanics has to clarify it. My discussion of this relationship is partly historical and conceptual, but mostly technical and mathematically rigorous, including over 500 references. On the assumption that quantum mechanics is universal and complete, I discuss three ways in which classical physics has so far been believed to emerge from quantum physics, namely in the limit h -> 0 of small Planck's constant (in a finite system), in the limit of a large system, and through decoherence and consistent histories. The first limit is closely related to modern quantisation theory and microlocal analysis, whereas the second involves methods of C*-algebras and the concepts of superselection sectors and macroscopic observables. In these limits, the classical world does not emerge as a sharply defined objective reality, but rather as an approximate appearance relative to certain "classical" states and observables. Decoherence subsequently clarifies the role of such states, in that they are "*einselected*", i.e. robust against coupling to the environment. Furthermore, the nature of classical observables is elucidated by the fact that they typically define (approximately) consistent sets of histories. I make the point that classicality results from the elimination of certain states and observables from quantum theory. Thus I defend the point of view that the classical world is not created by observation (as Heisenberg once claimed), but rather by the lack of it.
Following up on the more technical parts of [B1], in [20] I clarify the role of the Born rule in the Copenhagen Interpretation of quantum mechanics by deriving it from Bohr's doctrine of classical concepts, translated into the following mathematical statement: a quantum system described by a noncommutative C*-algebra of observables is empirically accessible only through associated commutative C*-algebras. The Born probabilities emerge as the relative frequencies of outcomes in long runs of measurements on a quantum system; it is not necessary to adopt the frequency interpretation of single-case probabilities (which will be the subject of a sequel paper). My derivation of the Born rule uses ideas from a program begun by Finkelstein (1965) and Hartle (1968), intending to remove the Born rule as a separate postulate of quantum mechanics. Mathematically speaking, my approach refines previous elaborations of this program - notably the one due to Farhi, Goldstone, and Gutmann (1989) as completed by Van Wesep (2006) - in replacing infinite tensor products of Hilbert spaces by continuous fields of C*-algebras. Furthermore, instead of relying on the controversial eigenvector-eigenvalue link in quantum theory, my derivation just assumes that pure states in classical physics have the usual interpretation as truthmakers that assign sharp values to observables. In combination with our interpretational context, this technical improvement circumvents valid criticisms that earlier derivations of the Born rule have provoked, especially to the effect that such derivations were mathematically flawed as well as circular.

Turning to the work on topos theory, my principal achievement, jointly with Heunen and Spitters, is [9]. The aim of this paper is to relate algebraic quantum mechanics to topos theory, so as to construct new foundations for quantum logic and quantum spaces. Motivated by Bohr's idea that the empirical content of quantum physics is accessible only through classical physics, we show how a C*-algebra of observables A induces a topos T(A) in which the amalgamation of all of its commutative subalgebras comprises a single commutative C*-algebra. According to the constructive Gelfand duality theorem of Banaschewski and Mulvey, the latter has an internal spectrum S(A) in T(A), which in our approach plays the role of a quantum phase space of the system. Thus we associate a locale (which is the topos-theoretical notion of a space and which intrinsically carries the intuitionistic logical structure of a Heyting algebra) to a C*-algebra (which is the noncommutative notion of a space). In this setting, states on A become probability measures (more precisely, valuations) on

S(A), and self-adjoint elements of A define continuous functions (more precisely, locale maps) from S(A) to Scott's interval domain. Noting that open subsets of S(A) correspond to propositions about the system, the pairing map that assigns a (generalized) truth value to a state and a proposition assumes an extremely simple categorical form. Formulated in this way, the quantum theory defined by A is essentially turned into a classical theory, internal to the topos T(A).

This paper was extremely abstract. Partly due to a small but essential contribution by my MSc student Martijn Caspers (for which, as already mentioned, he won the GQT Thesis Prize), we discovered how to perform explicit computations. Thus the goal of our next paper [3] was to illustrate our abstract setup through the concrete example of the C*-algebra of complex n by n matrices. This leads to an explicit expression for the pointfree quantum phase space and the associated logical structure and Gelfand transform of an n-level system. We also determine the pertinent non-probabilisitic state-proposition pairing (or valuation) and give a very natural topos-theoretic reformulation of the Kochen--Specker Theorem.
In the next paper [10], we generalize these computations to a large class of C*-algebras (the so-called Rickart C*-algebras, which incorporates all von Neumann algebras), and thereby are able, at last, to compare our formalism with the quantum logic of Birkhoff and von Neumann. Since these pioneering authors, quantum logic had traditionally been based on the lattice of closed linear subspaces of some Hilbert space, or, more generally, on the lattice of projections in a von Neumann algebra A. Unfortunately, the logical interpretation of these lattices is impaired by their nondistributivity and by various other problems. We show that a possible resolution of these difficulties, suggested by the ideas of Bohr, emerges if instead of single projections one considers elementary propositions to be families of projections indexed by a partially ordered set C(A) of appropriate commutative subalgebras of A. In fact, to achieve both maximal generality and ease of use within topos theory, we assume that A is a so-called Rickart C*-algebra and that C(A) consists of all unital commutative Rickart C*-subalgebras of A. Such families of projections form a Heyting algebra in a natural way, so that the associated propositional logic is intuitionistic: distributivity is recovered at the expense of the law of the excluded middle. Subsequently, generalizing our earlier computation for n-by-n matrices in [3], we prove that the Heyting algebra thus associated to A arises as a basis for the internal Gelfand spectrum (in the sense of Banaschewski-Mulvey) of the 'Bohrification' of A, which is a commutative Rickart C*-algebra in the topos of functors from C(A) to the category of sets. We explain the relationship of this construction to partial Boolean algebras and Bruns-Lakser completions. Finally, we establish a connection between probability measure on the lattice of projections on a Hilbert space H and probability valuations on the internal Gelfand spectrum of A for A = B(H).

The development of the papers [3, 9, 10] is reviewed in [B1]. Heunen in addition worked on other aspects of quantum logic, partly in collaboration with Jacobs. In [7] Heunen turned his attention to compact categories, which have lately seen renewed interest via applications to quantum physics. Being essentially finite-dimensional, such categories cannot accommodate (co)limit-based constructions. For example, they cannot capture protocols such as quantum key distribution, that rely on the law of large numbers. To overcome this limitation, Heunen introduces the notion of a compactly accessible category, relying on the extra structure of a factorisation system. This notion allows for infinite dimension while retaining key properties of compact categories: his main technical result is that the choice-of-duals functor on the compact part extends canonically to the whole compactly accessible category. As an example, he models a quantum key distribution protocol and proves its correctness categorically.[10]

In [8] Heunen axiomatically defines (pre-)Hilbert categories, using axioms that resemble those for monoidal Abelian categories with the addition of an involutive functor. He then proves some embedding theorems: any locally small pre-Hilbert category whose monoidal unit is a simple generator embeds (weakly) monoidally into the category of pre-Hilbert spaces and adjointable maps, preserving adjoint morphisms and all finite (co)limits. An intermediate result that is important in its own right, is that the scalars in such a category necessarily form an involutive field. In case of a Hilbert category, the embedding extends to the category of Hilbert spaces and continuous linear maps. The axioms for (pre-)Hilbert categories are weaker than the axioms found in other approaches to axiomatizing 2-Hilbert spaces. Neither enrichment nor a complex base field is presupposed.

In [P2], Heunen considers categorical logic on the category of Hilbert spaces (or, more generally,on any pre-Hilbert category). He characterises closed subobjects, and proves that these form orthomodular lattices. This shows that quantum logic is just an incarnation of categorical logic, enabling him to establish an existential quantifier for quantum logic, and conclude that there cannot be a universal quantifier.

In [C4], finally, Heunen and Jacobs investigate quantum logic from the perspective of categorical logic. They start from minimal assumptions, namely the existence of involutions/daggers and kernels. The resulting structures then turn out to (1) encompass many examples of interest, such as categories of relations, partial injections, Hilbert spaces (also modulo phase), and Boolean algebras, and (2) have interesting categorical/logical properties, in terms of kernel fibrations, such as existence of pullbacks, factorisation, and orthomodularity. For instance, the Sasaki hook and and-then connectives are obtained, as adjoints, via the existential-pullback adjunction between fibres.

[10] One of the referees of Heunen's PhD Thesis, Prof Peter Johnstone from Cambridge, found a mistake in this paper. Hence the above summary of its results is subject to possible correction.

III. Publications

1. Refereed articles in scientific journals

1. R. Bos, Geometric quantisation of Hamiltonian actions of Lie algebroids and Lie groupoids, *Int. J. Geom. Methods in Physics* 4, 389-436 (2007).

2. C. Carvalho, A K-theory proof of the cobordism invariance of the index, *K-Theory* 36, 1-31 (2005).

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4. E. Hawkins, The structure of noncommutative deformations. *J. Differential Geom.* 77, 385-424 (2007).

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7. C. Heunen, Compactly accessible categories and quantum key distribution. *Logical Methods in Computer Science* 4, 1-26 (2008).

8. C. Heunen, An embedding theorem for Hilbert categories, *Theory and Applications of Categories* 22, 321-344 (2009).

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25.M. Müger, L. Tuset, J.E. Roberts, Representations of algebraic quantum groups and reconstruction theorems for tensor categories. *Alg. Repres. Theor.* 7, 517-573 (2004).

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35. E. van Erp, The Atiyah-Singer index theorem for subelliptic operators on contact manifolds, Part I. Accepted for publication by *Annals of Mathematics.*

36. E. van Erp, The Atiyah-Singer index theorem for subelliptic operators on contact manifolds, Part II. Accepted for publication by *Annals of Mathematics.*

37. Varghese Mathai and Weiping Zhang, *Geometric quantisation for proper actions*, arxiv/0806.3138.

2. Book chapters

B1. C. Heunen, N.P. Landsman, B. Spitters, Bohrification. *Deep Beauty*, Ed. H. Halvorson (Cambridge University Press, to appear, 2010). arXiv:0909.3468.

B2. N.P. Landsman, Between classical and quantum, *Handbook of the Philosophy of Science Vol. 2: Philosophy of Physics,* Eds. J. Butterfield and J. Earman, pp. 417-554 (North-Holland, Amsterdam, 2007).

B3. N.P. Landsman, Algebraic quantum mechanics. The Born rule and its interpretation. Quantisation (systematic). Quasi-classical limit. *Compendium of Quantum Physics*, Eds. D. Greenberger, K. Hentschel, and F. Weinert, pp. 6-9, 64-70, 510-513, 626-629 (Springer, Berlin, 2009).

B4. M. Müger, Abstract duality for symmetric tensor *-categories. (Appendix to: Hans Halvorson: 'Algebraic Quantum Field Theory'), *Handbook of the Philosophy of Science Vol. 2: Philosophy of Physics,* Eds. J. Butterfield and J. Earman, pp. 865-922 (North-Holland, Amsterdam, 2007).

3. Refereed conference proceedings

C1. R.D. Bos, Geometric quantisation of Hamiltonian actions of Lie algebroids. *XV International workshop on geometry and physics, Puerto de la Cruz, Tenerife, Canary Islands. Publication de la RSME*, vol. XX, pp. 2-9 (2007).

C2. C. Heunen, Semimodule enrichment. *Mathematical Foundations of Programming Semantics XXIV, Philadelphia, USA*. Bauer, A., Mislove, M. (Eds.), pp.193-209 (Elsevier, Amsterdam, 2008)

C3. C. Heunen and B. Jacobs, Arrows, like Monads, are Monoids, *Proceedings Mathematical Foundations of Programming Semantics* (MFPS 06) 22 (ENTCS vol 158), Elsevier Science Publishers, 2006.

C4. C. Heunen and B. Jacobs, Quantum logic in dagger kernel categories, *Proceedings of QPL'09, Electronic Notes in Theoretical Computer Science*, in press (2009)

C5. C. Heunen, N.P. Landsman, B. Spitters, The principle of general tovariance. *International Fall Workshop on Geometry and Physics XVI, Lisbon, Portugal. Eds. R. Picken et al, AIP Conference Proceedings*, Vol. 1023, pp. 93-102 (Springer, Berlin 2008).

C6. N.P. Landsman, Quantisation as a functor. In: *Quantisation, Poisson Brackets, and Beyond*, ed. T. Voronov, Contemporary Mathematics 315, pp. 9-24 (AMS, Providence, 2002).

C7. N.P. Landsman, Functorial quantisation and the Guillemin--Sternberg conjecture. In: *Twenty Years of Białowieza*: *A Mathematical Anthology*, *eds*. S.T Ali et al., pp. 23-45 (World Scientific, Singapore, 2005). arXiv:math-ph/0307059.

C8. N.P. Landsman, Quantisation and the tangent groupoid. In: *Operator Algebras and Mathematical Physics*, eds. J.-M. Combes, et al., pp. 251-265 (Theta Foundation, Bucharest, 2003). arXiv:math-ph/0208004.

C9. M. Müger, On Superselection theory of quantum fields in low dimensions. *Proc. XVIth International Congress on Mathematical Physics*. Prague, August 3-8, 2009. arXiv:0909.2537.

C10. N. Neumaier, M. Pflaum, H. Posthuma, and X. Tang, Index theory on orbifolds via deformation quantisation, *Oberwolfach Reports* 13/2006 p. 767-769 (2006).

C11. W.D. van Suijlekom, Renormalization of gauge fields using Hopf algebras. *Recent Developments in Quantum Field Theory, - Max Planck Institute for Mathematics in the Science*, Leipzig, B. Fauser, J.T., E. Zeidler (Eds.), pp. 137-154 (Birkhäuser Verlag, Basel, 2008).

C12. W.D. van Suijlekom, Multiplicative renormalization and Hopf algebras, arXiv:0707.0555. In *Arithmetic and geometry around quantisation*, Eds. O. Ceyhan, Yu.-I. Manin and M. Marcolli (Birkhauser Verlag, Basel, 2008).

C13. W.D. van Suijlekom, Landi, G., Noncommutative Bundles and Instantons in Tehran, *International Workshop on Noncommutative Geometry, IPM*, Tehran 2005, Iran, Marcolli, M. (Ed.) (World Scientific Publishing, Hackensack 2008).

4. PhD Theses

D1. Rogier Bos, *Groupoids in Geometric Quantisation* (RU, 28/09/2007). Promotor: prof.dr. N.P. Landsman.

D2. Peter Hochs, *Quantisation Commutes with Reduction for Cocompact Hamiltonian Group Actions* (RU, 08/02/2008). Promotores: prof.dr. G.J. Heckman and prof.dr. N.P. Landsman.

D3. Niels Kowalzig, *Hopf Algebroids and their Cyclic Theory* (UU, 30/06/2009). Promotores: prof.dr. N.P. Landsman and prof.dr. I. Moerdijk.

D4. Chris Heunen, C*ategorical Quantum Models and Logics* (RU, 07/01/2010). Promotores: prof.dr. B. Jacobs and prof.dr. N.P. Landsman.

5. Preprints and other (as yet) non-refereed articles

P1. R.Bos, Continuous representations of groupoids, arXiv:math.RT/0612639.

P2. C. Heunen, Quantifiers for quantum logic, arXiv:0811.1457.

P3. M. Khalkhali, G. Landi, and W.D. an Suijlekom, Holomorphic structures on the quantum projective line, arxiv:0907.0154.

P4. N. Kowalzig and U. Kraehmer, Duality and products in algebraic (co)homology theories, arXiv:0812.4312

P5. N. Kowalzig and H. Posthuma, The cyclic theory of Hopf algebroids, arXiv:0904.4736

P6. M. Müger, Tensor categories: A selective guided tour. Notes for a lecture series given during the workshop "Groupes quantiques dynamiques et catégories de fusion" at CIRM (Luminy, April 14-18, 2008). (arXiv:0804.3587).

P7. H. Posthuma, The Heisenberg group and conformal field theory. http://staff.science.uva.nl/~posthuma/heisenberg.pdf

P8. M. Pflaum, H. Posthuma, X. Tang and H.S. Tseng, Orbifold cup products and ring structures on Hochschild cohomologies.

6. Popular articles (in Dutch) by Klaas Landsman

Wiskunde aan de grenzen van de natuurkunde (oratie Universiteit van Amsterdam, *Nieuw Archief voor Wiskunde* 5/3 (2002), 24-32.

Wie was Thomas Young?, *Nederlands Tijdschrift voor Natuurkunde* 69 (2003), 40-44.

De dramatiek van de natuurkunde: toneelstukken van Frayn en Rijnders onder de loep, *Kunst en Wetenschap* 12 (2) (2003), 5-8.

Briljante gifmenger: een portret van Isaac Newton, *Academische Boekengids* 39 (juni 2003), 13-14.

De indexstelling van Atiyah en Singer, *Nieuw Archief voor Wiskunde* 5/5 (2004), 207-211.

Op het kruispunt (oratie Radboud Universiteit Nijmegen), *Nieuw Archief voor Wiskunde* 5/6 (2005), 206-214.

Waarom is er iets en niet niets? De visie van Hans Küng, *Nederlands Tijdschrift voor Natuurkunde* 72 (2006), 270-272.

Bestaat Toeval?, *Nieuwe Wiskrant* 26 (1) (2006), 21-26.

Op zoek naar de intellectueel, *Civis Mundi* 46 (1) (2007), 40-49.

Taal en werkelijkheid: Einstein tegen Bohr (BLIND!) http://www.ziedaar.nl/index.php?theme=10&id=263

Plato, wiskunde en het gymnasium, *Amphora* 25 (6) (2006), 8-10.

Blijf niet mokkend aan de kant staan, *Nieuw Archief voor Wiskunde* 5/8 (2007), 51-55.

Toeval is logisch, *Nieuwe Wiskran*t 26 (4) (2007), 42-47.

A random walk down Wall Street, *Nieuw Archief voor Wiskunde* 5/9 (2008), 18-23.

Where have all the students gone?, *Nieuw Archief voor Wiskunde* 5/9 (2008), 138-140.

Wanneer ga ik dood?, *Nederlands Tijdschrift voor Natuurkunde* 74 (2008), 98-101.

Wiskundetoernooi 2008: wat kun je nu eigenlijk met wiskunde? (met Dion Coumans en Mirte Dekkers), *Nieuwe Wiskrant* 28, 12-14 2008.

Christiaan Huygens: Traité de la Lumiere (met Fokko Jan Dijksterhuis), Jan Bos en Erik Geleijns (red.), *Boekenwijsheid*, pp. 177-185 (Walburg Pers, Zuthpen, 2009).

Spiritualiteit tussen kwal en kosmos, Manon Duintjer (red.), *Zien is Geloven*, pp. 115-127 (Ambo, Amsterdam, 2009).

Terug naar de werkelijkheid, *Nieuw Archief voor Wiskunde* 5/10 (2009), 48-50.

De overval: Fox-IT (met Matthijs Coster en Bart Jacobs), *Nieuw Archief voor Wiskunde* 5/10 (2009), 91-93.

De “Vrije-Wil Stelling” van Conway en Kochen, *Nieuw Archief voor Wiskunde* 5/10 (2009), 228-234.

De Overval: All Options, *Nieuw Archief voor Wiskunde* 5/10 (2009), 245-249.

7. Scientific talks by Klaas Landsman

2009:

3 June, Waterloo, Perimeter Institute (Categories, Quanta, Concepts)
The Conway-Kochen-Specker Theorems

15 May, Utrecht (Symposium Determinisme en vrije wil)
Kwantummechanica, zuiver toeval en vrije wil: de stelling van Conway en Kochen

1 May, Washington DC (New Directions in the Foundations of Physics)
The Bohrification of quantum logic

3 April, Cambridge (CQC seminar)
The Bohrification of quantum logic

30 January, Göttingen (23th Workshop on the Foundations and constructive aspects of Quantum Field Theory) *Truth in algebraic quantum theory*

2008:

21 November, Utrecht (Seminar on the Philosophy of Mathematics)
Newton and Hilbert on the foundations of geometry: a case study in the philosophy of mathematics

28 October, Utrecht (One-day conference on the foundations of quantum mechanics)
Intuitionistic quantum logic

9 October, Granada (IVth International Workshop on Classical concepts and metaphysical presuppositions in quantum theory) *Intuitionistic quantum logic*

25 September, Nijmegen (iRUN Uitwisseling Krakow-Nijmegen)
From classical logic to quantum logic

29 May, Utrecht (Algemeen Wiskundig Colloquium)
Quantum logic revisited

21 May, Leiden (Colloquium Ehrenfestii)
Rethinking the Bohr-Einstein debate

9 April, Tilburg (Reduction, emergence, and physics)
Macroscopic observables and the Born rule in quantum mechanics

March 20, Nijmegen (Algebra & Logic Seminar)
Algebraic and logical notions of space

13 February, Nijmegen (Studentvriendelijk Wiskundig Colloquium)
The physical relevance of the foundations of mathematics

31 January, Groningen (General Physics Colloquium)
Rethinking the Bohr-Einstein debate

11 Januari, Utrecht (Logic and Physics Symposium)
Topos theory and quantum physics: towards spatial quantum logic

2007:

7 December, Hamburg (60. Geburtstag Klaus Fredenhagen)

The principle of general tovariance

November 13, Nijmegen (ISIS Colloquium)
Werner Heisenberg: torn between objectivity and subjectivity

3 October, Princeton (Deep Beauty: John von Neumann memorial conference)
The principle of general tovariance

5-8 September, Lisbon (XVI International Fall Workshop on Geometry and Physics)
The principle of general tovariance

June 7, Perimeter Institute, Waterloo, Canada (Operational quantum physics and the quantum-classical contrast) *Macroscopic observables and quantisation*

29 May, Nijmegen (Algemeen Natuurkundig Colloquium)
Determinism in physics

27 February, Paris, IHP (Groupoids in operator algebras and noncommutative geometry)
Groupoids, quantales and C-algebras*

29 January, TU Eindhoven (In zee met wiskunde D)
Logisch denken over kansen

22 January, TU Twente (In zee met wiskunde D)
Logisch denken over kansen

2006:

22 July, Oxford (Cats, Kets and Cloisters)
Between classical and quantum (survey talk)

4 May, Oxford (History and Philosophy of Science Seminar)
The Einstein-Bohr debate

2 May,Oxford (Quantum Field Theory Seminar)
Functoriality of quantisation and a Guillemin-Sternberg conjecture for noncompact groups and spaces

26 April, Nijmegen (Studentvriendelijk wiskundig colloquium)
De indexstelling van Atiyah en Singer

25 April, 15:00-17:00, Enschede (TUT, Colloquium vakdidactiek wiskunde)
Bestaat Toeval?

27 March, Delft (Nederlands Mathematisch Congres)
The Atiyah-Singer index theorem

27 January, Nijmegen (Onderwijs Anders, Algemene Docentendag FNWI)
Bestaat Toeval? De ongelijkheden van Bell

2005:

15 November, Nijmegen (Soeterbeeck-lunch)
Het Einstein-Bohr debat

28 October, 15:00-16:00, Freiburg (Seminar Fundamentale Wechselwirkungen)
Rethinking the Bohr-Einstein debate

26 October, Oberwolfach (Noncommutative Geometry and Quantum Field Theory)
The Einstein-Bohr debate

12 July, Utrecht (Grondslagencolloquium)
The appearance of the classical world

13 June, Nijmegen (Einstein week)
De zwaartekracht van Newton tot Einstein

22 May, Gent (Joint BeNeLuxFra Conference in Mathematics)
Functoriality of quantisation and the Guillemin-Sternberg conjecture

11 May, Nijmegen (Fysica, ten minste houdbaar tot...)
(In)determinisme in de fysica: wat zal de toekomst ons brengen?

5 May, Stillwater, Minnesota (Seven Pines Symposium IX: The Classical-Quantum Borderland)
Between classical and quantum

29 April, Maryland (New Directions in the Foundations of Physics)
Heisenberg's Umdeutung Today

8 February, Groningen (Mathematics Colloquium)
Index theory and quantisation

4 February, Noordwijkerhout (Nationale Wiskunde Dagen)
De indexstelling van Atiyah en Singer

7 January, Nijmegen (oratie Radboud Universiteit Nijmegen)
Op het kruispunt

2004:

20 October, Univ. of Pittsburgh (Foundations of Physics Handbook Workshop)
Between classical and Quantum

9 July, Paris26 (K-theory and Noncommutative Geometry)
The 'Quantisation commutes with reduction' conjecture for noncompact groups

June 29, Leusden (Wiskunde en Werkelijkheid)
Wiskunde of Religie: Spinoza versus Newton

June 23, Utrecht (MRI Workshop on Lie Groups in Analysis, Geometry, and Mechanics)
The Guillemin--Sternberg conjecture for noncompact groups

June 16, Frankfurt (Workshop zu Indextheorie und Quantisierung)
Die Vermutung von Guillemin--Sternberg für nichtkompakte Gruppen

May 27, Utrecht (Mathematics Colloquium)
The Atiyah-Singer index theorem (with J.J. Duistermaat)

12 May, Amsterdam (Mathematics Colloquium)
The Atiyah-Singer index theorem

16 March, Marseille-Luminy (Geometric Analysis)
Quantisation, the analytic assembly map, and the Guillemin--Sternberg conjecture for noncompact groups

30 March, Amsterdam (Seminar on Quantisation, Noncommutative Geometry, and Symmetry)
Geometric quantisation III

9 March, Amsterdam (Seminar on Quantisation, Noncommutative Geometry, and Symmetry)
Geometric quantisation II

17 February, Marseille-Luminy (Noncommutative Geometry in Mathematics and Physics)
Functoriality of quantisation and the Guillemin-Sternberg conjecture

3 February, Amsterdam (Seminar on Quantisation, Noncommutative Geometry, and Symmetry)
Geometric quantisation I

2003:

October 14, Amsterdam (Seminar on Quantisation, Noncommutative Geometry, and Symmetry)
From index theory to (geometric) quantisation

October 7, Amsterdam (Seminar on Quantisation, Noncommutative Geometry, and Symmetry)
From (deformation) quantisation to index theory

August 20, Villigst (Welle trifft Teilchen)
Klassische und Quantenmechanik

August 21, Villigst (Welle trifft Teilchen)
Werner Heisenberg und die deutsche Atombombe

July 11, Aachen (Mathematics Colloquium)
Quantum mechanics and representation theory: the new synthesis

23 May, Amsterdam (Mini-Workshop on Noncommutative Geometry)
Functoriality of quantisation, KK-theory, and the Guillemin-Sternberg conjecture

9 May, Amsterdam (Leve de Wiskunde)
De wiskunde achter de kwantumtheorie: Van atoomspectra tot niet-commutatieve meetkunde

1 May, Nijmegen (Nederlands Mathematisch Congres)
Quantum mechanics and representation theory: the new synthesis

April 11, Amsterdam (Seminar on Noncommutative Geometry)
The general Baum-Connes conjecture

4 April, Utrecht
The royal path to quantisation

13 February, Utrecht (Mathematics Colloquium)
Quantisation as a functor

23 January, Oberwolfach (Workshop on the Quantisation of Singular Symplectic Quotients)
Quantisation as a functor

15 January, Hamburg (Mathematical Physics Colloquium)
Quantisation as a functor

2002:

6 December, Amsterdam (Seminar on Noncommutative Geometry)
KK-theory

1 November, Amsterdam (Seminar on Noncommutative Geometry)
Connes's proof of the Atiyah-Singer index theorem II

18 October, Amsterdam (Seminar on Noncommutative Geometry)
Connes's proof of the Atiyah-Singer index theorem I

20 September, Amsterdam (Seminar on Noncommutative Geometry)
The Atiyah-Singer index theorem

6 September, Amsterdam (Seminar on Noncommutative Geometry)
Overview: from Atiyah-Singer to V. Lafforgue