Radboud Universiteit

Masterthesis Physics and Astronomy

Institute for Mathematics, Astrophysics and Particle Physics (IMAPP)

State-levels and Boltzmann

The Macro-Micro-Distinction and its Relation to Human Projection in Physics

Kas Kornblum
Author

prof. dr. N.P. Landsman
Supervisor

prof. dr. N.P. Landsman
First Assessor

prof. dr. H.W. de Regt
Second Assessor

August 10, 2025

This thesis explores the mathematical and conceptual distinction between the micro- and macro-level in Statistical Mechanics, with a focus on how this distinction underlies the definition of entropy. Taking Boltzmann's groundwork of his 1877 paper, we investigate the role of coarse-graining - the partitioning of state-space into cells - as a necessary mathematical step in deriving macroscopic quantities from microscopic dynamics. This thesis aims to defend the argument that this process is not purely objective and reflects an inherent observer dependence, which is rooted in human empirical limitations of perception, scale and ability. Through mathematical and physical analyses, we examine how entropy, as derived by Boltzmann in 1877, depends on the choices regarding coarse-graining and therefore reflects a dependency on epistemic choices rather than being solely intrinsic features. Case studies such as the dilute gas and Baker's Map are taken as examples of how one derives entropy and how coarse-graining plays a role in this process. We conclude that entropy increase is not solely a physical phenomenon, but dependent on the observers choices that describe and simplify complex dynamical systems.

Contents

Chapter 1

Introduction

Physics is built upon a wide variety of perspectives on natural phenomena. These perspectives can depend on scale, both temporal and spatial, with variance in spatial scale being an intuitive guide to explore mechanisms that could underlie phenomena. Similar to how the discovery of DNA unraveled mysteries in biology, physicists have been varying the scales on which they explore nature, of which a substantial part in the microscopic and submicroscopic realm. Phenomena like temperature and pressure, which have long been studied as independent phenomena in Thermodynamics, have been connected to microscopic theories of Statistical Mechanics. Boltzmann's approach to connect these areas of study have laid the groundwork for physicists and mathematicians to describe and study these systems in nature and in thought.

The state level on which humans live, operate and observe is called the macro-state, with the etymology of 'macro' originating from the ancient Greek word $\mu\alpha\kappa\rho o\varsigma$, meaning 'long', 'on larger scale', 'far','large' [1] [2]. At first this seems counter-intuitive, with the macro-state referring to states that are directly observable, not in a relatively larger realm than the level we operate in. Our senses have developed to observe differences in things like temperature, pressure, but also radiation in the form of light. But where do those phenomena originate from? Could they be reduced to micro-relations, or are these phenomena mystically emerging and could they not be reduced to smaller, less complex parts?

This thesis aims to gain insight into these big philosophical questions using the scientific (physics) context of **entropy**. *What is the role of human projection in how entropy and the macro-micro-relation is defined in physics?*

In section 2 we aim to establish how entropy historically emerged by taking a closer look at Boltzmann, one of the founders of entropy. In section 3.1 we aim to shape a mathematical foundation to build upon throughout this paper and look at how Boltzmann got to entropy. In section 3.2 we zoom in on the specific act of coarse-graining that is needed to get to entropy and look at how the micro-macro-relation relates to this. In section 3.4 we take coarse-graining in a broader philosophical perspective and examine the role of human projection within this field. In section 3.3 we look at some examples of dynamical systems and look at how the above applies to each of these cases. In section 4 we aim to establish a conclusion and discuss how it relates to the research question. We end with an epilogue (chapter 5), which contains a collection of thoughts on and around the subject of this thesis.

Chapter 2

Historical background

Our scientific journey begins with a young Ludwig Boltzmann around the last quarter of the 19th century. The scientific community has expanded in the past centuries from the Scientific Revolution and the Age of Enlightenment. Philosophy and science are very much interwoven and it is in these flourishing times that Boltzmann is on a, later regarded as fruitful, thought track leading to his 1872 and 1877 papers on the relation between the Second Law of Thermodynamics and heat and probability theory (later known as Statistical Mechanics). Boltzmann was a fervent atomist and defended this position against many other major scientific figures at that time, like his friend Ernst Mach, but also Zermelo, Planck and Ostwald [3]. It was already in 1872 that Boltzmann started thinking beyond the equilibrium states of molecular systems. He developed the H-theorem, with which he connected the macroscopic irreversibility of molecular systems, like heat flowing from hot to cold, to their reversible microscopic dynamics. This H-theorem formed an explanation of the Second Law of Thermodynamics, namely that entropy of an isolated system increases over time. This explanation was builtwas bui upon Newtonian mechanics in the form of collision dynamics and under the condition of molecular chaos (Stosszahlansatz).

In Boltzmann's 1877 paper on the relation between the Second Law of Thermodynamics and heat and probability theory he continues this work, but in a more combinatorial and statistical way. He takes probability theory as a fundamental way to explain the relation between the Second Law and heat and probability theory. He does this by making a distinction between three different levels of state [4]. Boltzmann distinguishes the *complexion*, the *intermediate level* and the *macro-level*. In the complexion, the individual molecules, with properties like velocity, energy and momentum, are treated as individual entities. The *intermediate level* is the level where the distribution of one of the parameters, like energy or velocity, is known, without treating the molecules as individual entities. The *macro-level* is the level where physical quantities like pressure and temperature are linked to the intermediate distribution. Both the intermediate and macro level are covered in the contemporary macro-level concept. His proposal was that the macro-states that correspond to many micro-states are more likely to exist. This builds further upon his 1872 paper by defining the entropy as being dependent on the multiplicity of micro-states corresponding to the specific macro-state.

This in combination with probability theory, provided groundwork for a relation between Thermodynamics and Statistical Mechanics. Could the reason that heat dissipates and that systems in general seem to move from an ordered state to a chaotic one be explained by probability theory?

Chapter 3

Research

3.1 Theoretical framework

To study dynamical systems in the context of physics and philosophy, a proper description of such a system is necessary. Since Boltzmann's 1877 paper, these descriptions have become more advanced and extensive, as mathematics has developed new tools to describe and further classify them. In this chapter, a theoretical framework will be explained to build upon throughout this thesis. Later, this theoretical framework will be analyzed to better understand how entropy is defined.

In this chapter we aim to explain the mathematical structure of a *measure-preserving deterministic dynamical system* which exhibits ergodic behavior and obeys Poincare recurrence. These concepts will be explained in the next chapter alongside their relevance in this thesis.

3.1.1 Definition of dynamical systems

The mathematical framework for the definition of states starts with a proper definition of a dynamical system. To analyze a dynamical system, we need a well-defined mathematical structure. This structure should be defined and constrained in some ways. Such systems are described in the mathematical framework of a *measure-preserving deterministic dynamical system*

$$(X, \Sigma_X, \mu_X, T_t). \tag{3.1}$$

Here:

- X is a set representing all possible micro-states;
- Σ_X is a σ-algebra of subsets of X;
- μ_X is a probability measure on Σ_X;
- $T_t : X \to X$ is a measurable time evolution function where $t \in \mathbb{R}$ (continuous time) or $\mathbb{Z}$ (discrete time), that satisfies

$$T_{t_1+t_2}(x) = T_{t_2}(T_{t_1}(x)) \tag{3.2}$$

for all $x \in X$ and all $t_1; t_2 \in \mathbb{R}$ or $\mathbb{Z}$[5].

Also, the measure μ_X is invariant under transformation T_t. This means that for all $A \in \Sigma_X$ and all t, we have:

$$\mu_X(T_t^{-1}(A)) = \mu_X(A). \tag{3.3}$$

This ensures that the measurable subset Σ_X is *measure-preserving*, i.e. the time evolution of the system does not alter the 'volume' (probability measure) of any measurable subset of the state-space.[1] The reason why the measure-preserving time transformation is in terms of the inverse of T_t instead of the forward transformation $\mu(T_t(A)) = \mu(A)$, is for the measure to be invariant, the incoming measure $\mu(T_t^{-1}(A))$, should be the same as the current $\mu(A)$.
For each initial micro-state $x \in X$, there is a trajectory starting at x, $s_x : \mathbb{R} \to X$ or $s_x : \mathbb{Z} \to X, s_x(t) = T_t(x)$. A measure-preserved dynamical system is *deterministic*: the time evolution function $T_t(x)$ defines a unique trajectory for every initial micro-state $x \in X$, i.e. future states are fully determined by the present state. However, in the physical world these systems tend to be chaotic, which means that predicting system evolution in the far future is difficult[2].
A measure-preserving deterministic dynamical system obeys the *Poincaré recurrence theorem*[6]. This theorem states that in such a dynamical system, for any measurable subset $A \in \Sigma_X$, with $\mu(A) > 0$, the set of points in A that return arbitrarily close (in continuous systems) or exactly (in discrete systems) to A under iterations of T infinitely often, has full measure in A. This means that almost every point in A will return to A infinitely often and that the set of 'non-returning' points in A has measure zero.[3].

Ergodicity is a crucial concept when it comes to Statistical Mechanics. Intuitively, ergodicity can be seen when a systems trajectory explores the entire state space in a way where the time averages of observables equals their ensemble space averages. Ergodic theory applies to measure preserving maps for T to be called *ergodic*[8].
Mathematically, a measure preserving deterministic dynamical system $(X, \Sigma_X, \mu_X, T_t)$ is *ergodic* if every measurable subset $A \in X$ satisfies $\mu_X(A) = 0$ or $\mu_X(A) = 1$. This condition ensures that the system cannot be divided into smaller invarient subsystems, i.e. it behaves as a single statistical system. The probability measure of these subsets A has to be either negligible ($\mu(A) = 0$) or it covers everything ($\mu(A) = 1$). If there would exist any invariant subset A with $0 < \mu(A) < 1$, so not be 'all or nothing', this would mean that this invariant subset A would be decomposed in two disjoint regions, therefore violating ergodicity [9].
The crucial consequence of ergodicity is the fact that for an integrable observable $f : X \to \mathbb{R}$, we have

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(T_t(x))dt = \int_X f(x)d\mu_X(x) \tag{3.4}$$

[1] If you take to system to be, for example, a billiard table with billiard balls, and you define the state-space as a combination of velocity and position of all the balls. You can say that the state-space can evolve over time, meaning stretching and moving, but the volume of this state-space itself, and regions in this state space stays the same.

[2] Think about the weather for example. Particles seem to move in a predictable way, but it seems impossible to determine all possible interactions and variables and predict the weather precisely in one year

[3] Noteworthy is that in strict thermo-dynamical equilibrium, which is a theoretical phenomenon, a system that reached equilibrium will stay there forever, never returning back to its initial position. So Poincaré recurrence doesn't hold in this case. Although experimental data show that TD systems in equilibrium tend to slightly deviate from the equilibrium state[7]. Therefore ruling out strict TD equilibrium in nature, making it nonphysical[5].

for almost every $x \in X$. The crucial implication is that the long-term average over a trajectory equals the average over the whole space. Without ergodicity, Boltzmann's statistical interpretations would not hold [10].
An intuitive example is the diffuse gas, as explained in the case study in section 3.3.1, where the positions and velocities of the particles evolve in an ergodic manner. The statistical descriptions represent time-averaged behavior.

At the macro level the system is characterized by a set of macro-variables $\{v_1, ..., v_l\}(l \in \mathbb{N})$. These variables are measurable functions $v_i : X \rightarrow \mathbb{V}_i$, associating a value with each point in X. We use capital letters V_i to denote the values of v_i. A particular set of values $\{V_1, ..., V_l\}$ defines a macro-state $M_{V_1,...,V_l}$. We only write 'M' rather than '$M_{V_1,...,V_l}$' if the specific values V_i do not matter[5].

3.1.2 Definition of states in dynamical systems

In Statistical Mechanics, a *state* refers to a configuration of a physical system at a given time, which is characterized by parameters such as position and momentum of all particles involved. Since we are talking about systems in the physical world, the systems are dynamical, evolving according to physical laws. In his 1877 paper, Boltzmann considers the well known example of an ideal gas, composed of a large number of molecules that can exchange kinetic energy and momentum via elastic collisions. These molecules have properties like position x, mass m, velocity v and momentum p. When colliding, they can exchange their kinetic energy $E_{kin} = \frac{1}{2}mv^2$.

In this section we aim to summarize how Boltzmann constructs the mathematics and physics to end up with a definition for entropy. He does this by first developing a discretization of an ideal gas and expanding to the continuous case using mathematical methods to later arrive at a tight definition of entropy.

He starts with assigning discrete kinetic energies to the molecules from $0, \epsilon, 2\epsilon, ...p\epsilon$. This is similar to assigning discrete values to the velocity component of the molecules in the form of $0, \frac{1}{q}, \frac{2}{q}, ...\frac{p}{q}$. The act of assigning discrete values is, according to Boltzmann, for mathematical purposes, namely the use of combinatorics to count the number of ways the energy is distributed: he admits that this system does not correspond to anything physical and can be regarded as a 'fiction'[3]. The reason for the discretization is stated by Boltzmann as follows:

> This assumption does not correspond to any realistic mechanical model, but it is easier to handle mathematically, and the actual problem to be solved is re-established by letting p and q go to infinity.

After that, an energy distribution among the molecules is introduced by a sequence of integers $N_0, N_1, ...N_p$. Where each integer N_i represents the number of molecules occupying the energy state ϵ_i. The complexion is build up with the sequence $i_1, i_2, ...i_N$, which contains the number of discrete energy-'packages' per molecule. The insight of Boltzmann is that many micro-states (complexions) correspond to the same distribution. Boltzmann quantified this as the permutability Π of an energy distribution.

Discrete Boltzmanian statistics

This permutability arises from the discretization of the continuous phase space, containing position and velocity. This allowed Boltzmann to assign probabilities of the system to be in a certain cell. As this thesis aims to analyze the way entropy is established and how coarse-graining plays a role in this, a further elaboration on coarse-graining is given in section 3.2.1.

For now we focus on the permutability, which is defined as

$$\Pi = \frac{N!}{N_0!N_1!...N_p!}, \tag{3.5}$$

where N is the total number of molecules and N_i the number of molecules occupying a certain energy level. The permitted energy distributions are constrained by

$$\begin{aligned} \sum_{i=0}^{p} N_i &= N, \\ \sum_{i=1}^{p} iN_i &= P, \end{aligned} \tag{3.6}$$

where p is the maximum permitted energy and P is the total number of energy-packages. So the energy in the system is given by $E = P\epsilon$ The most probable distribution is the one where permutability Π is maximum. The probability of the system to be in a certain energy distribution W is therefore given by.

$$W = \Pi/J, \tag{3.7}$$

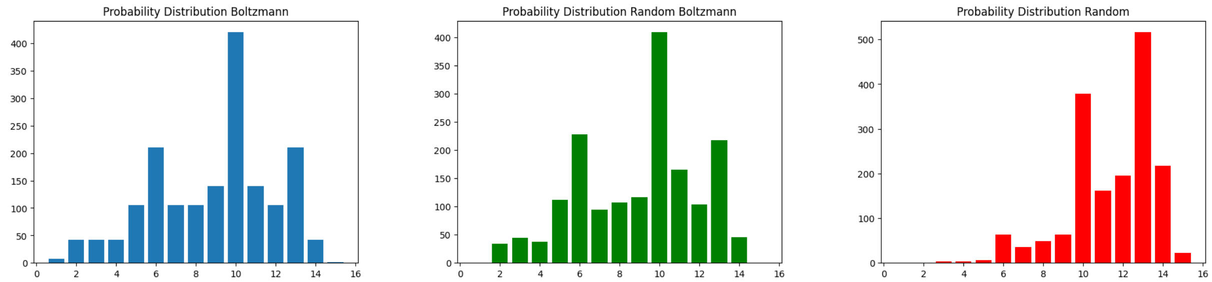
where J is the total number of complexions, i.e. the total number of micro-states.

Boltzmann demonstrates this method with a thought experiment involving drawing slips from an urn. His example is with $p = N = P = 7$. After running a brief simulation, we find the probability distribution is correct. Boltzmann explains it as follows:

> Suppose we have an urn containing an infinite number of paper slips. On each slip is one of the numbers 0, 1, 2, 3, 4, 5, 6, 7; each number is on the same amount of slips, and has the same probability of being picked. We now draw the first septet of slips, and note the numbers on them. This septet provides a sample state distribution with a kinetic energy of ϵ times the number written on the first slip for molecule 1, and so forth. We return the slips to the urn, and draw a second septet which gives us a second state distribution, etc. After we draw a very large number of septets, we reject all those for which the total does not equal 7. This still leaves a large number of septets. Since each number has the same probability of occurrence, and the same elements in a different order form different complexions, each possible complexion will occur equally often. By ordering

the numbers within each septet by size, we can classify each into one of the fifteen cases tabulated above. So the number of septets which fall into the class 0000007 relative to the 0000016 class will be 7:42. Similarly for all the other septets. The most likely state distribution is the one which produces the most septets, namely the 10th. [4]

By simulating the probability distribution of the distributions, using `random.randint()` as Boltzmann's urn, we find that the same as Boltzmann proposes in his paper. Boltzmann's method aligns



(a) The probability distribution in Boltzmann's 1877 paper.

(b) The distribution with Boltzmann's method assigning each molecule a value of 0–7 with $\frac{1}{7}$ probability, dismissing those where total energy does not add up to 7.

(c) The distribution using 7 energy packages randomly assigned to molecules with $\frac{1}{7}$ probability.

Figure 3.1 Comparison of different probability distributions in statistical mechanics.

correctly with the expected probability distribution of the distributions, i.e. the permutability of the distributions as seen when comparing 3.1a and 3.1b. However, alternative methods, like the stochastic sampling method of assigning the energy-packages with a $\frac{1}{7}$ chance on each molecule raise some questions about what it is that makes Boltzmann's method representative. Why is this the case and why does Boltzmann's method lead to the probability distribution that is expected when calculating Π and the alternative method not, as seen when comparing figure 3.1a and 3.1c? Boltzmann's method aligns correctly with the probability distribution. This seems intuitive when you consider the underlying argument; when you stop treating each atom as an individual, distinguished entity, you can re-order them, forming similar states. This essentially is the 'multiplicity' or 'permutability'.

From discrete to continuous

The way Boltzmann comes up with a discretization of the distribution of energy-packages on molecules is intuitive and induces mathematical simplicity. His explanation provides a way of seeing why the ideal gas, as an example of such systems, tends to occupy certain distributions, just because these distributions are more likely to exist. However, this case accounts for a fictitious case. Boltzmann makes his case for the more relevant continuous - non-fictitious - case, not by taking the size of the energy packages to be infinitely small, but by assigning to each molecule an energy interval $[i\epsilon, (i+1)\epsilon]$, withi being an integer [3]. The distribution is then given by $N_0, N_1, ...N_i...$, where N_i corresponds to the i-th interval. The permutability is then given by

$$\Pi = \frac{N!}{N_0!N_1!...N_i!...}. \tag{3.8}$$

Boltzmann relies on the Stirling approximation $\ln N! \approx N \ln N - N$ of the factorials for finding the most probable distribution, i.e. the distribution containing the most micro-states. He replaces this problem by finding the function f, for which

$$\sum_{i=0}^{\infty} \epsilon f(i\epsilon) \ln f(i\epsilon), \tag{3.9}$$

is a minimum under constraints

$$\sum_{i=0}^{\infty} \epsilon f(i\epsilon) = N, \tag{3.10}$$

$$\sum_{i=0}^{\infty} \epsilon(i\epsilon) f(i\epsilon) = E, \tag{3.11}$$

where N is the total number of particles and E is the total energy. The product $\epsilon f(i\epsilon)$ accounts for the number of particles in the small energy interval $[i\epsilon, (i+1)\epsilon]$.

He then replaces the sums with integrals, resulting in

$$I = \int_0^{\infty} f \ln f \mathrm{d}k, \tag{3.12}$$

with k the continuous kinetic energy, being minimum under constraints

$$\int_0^{\infty} f \mathrm{d}k = N, \tag{3.13}$$

$$\int_0^{\infty} k f \mathrm{d}k = E. \tag{3.14}$$

This is essentially the way to find the distribution f for which I is minimum. By taking the limit where the energy intervals become infinitely small and finding the function that minimizes I, Boltzmann finds the distribution which is now known as the Maxwell-Boltzmann distribution

$$f \propto e^{-\beta k}. \tag{3.15}$$

Generalization to three dimensions and poly-atomic molecules

Now, Boltzmann expands his case to three-dimensional poly-atomic gasses and liquids and including external forces. He generalizes to three spatial dimensions and includes additional degrees of freedom to account for molecular orientation. He does this to generalize his case. First, by expanding to three dimensions, the most probable distribution function changes to

$$I = \int f \ln f d^3v, \tag{3.16}$$

which is a minimum under conditions

$$\int f \mathrm{d}^3 v = N, \tag{3.17}$$

$$\int (mv^2/2) \mathrm{d}^3 v = E. \tag{3.18}$$

After this, Boltzmann introduces yet more dimensions of phase space, which account for the poly-atomic orientation around each center of mass. This includes transnational and internal energy. With this, the most probable distributions $f(q,p)$ and $F(Q,P)$ become

$$I = \int f \ln f \mathrm{d}^l q \mathrm{d}^l p + \int F \ln F \mathrm{d}^L Q \mathrm{d}^L P \tag{3.19}$$

which is minimum under constraints

$$\int f \mathrm{d}^l q \mathrm{d}^l p = n, \tag{3.20}$$

$$\int F \mathrm{d}^L Q \mathrm{d}^L P = N, \tag{3.21}$$

$$\int (k+\chi) f \mathrm{d}^l q \mathrm{d}^l p + \int (K+X) F \mathrm{d}^L Q \mathrm{d}^L P = E. \tag{3.22}$$

This is in confirmation with the Maxwell-Boltzmann distributions $f \propto e^{-\beta(k+\chi)}$ and $f \propto e^{-\beta(K+X)}$[3].

Permutability measure and entropy

From here, Boltzmann introduces the *permutability measure*, which is defined as

$$\Omega = \log \Pi_{max} \tag{3.23}$$

for a mono-atomic discrete ideal gas. The measure essentially defines the logarithm or the largest number of micro-states (complexions) of a system. This measure, dependent on Π_{max} sets the stage for the later defined entropy, which is defined as $S \propto \log \Pi$.

In continuous systems, Boltzmann defines the permutability measure as

$$\Omega = -\int f(x) \log f(x) dx \tag{3.24}$$

with the entropy defined as $S = k\Omega + C$, where k is a proportionality constant, later known as the Boltzmann constant.

With generalization for the continuous case with three dimensions, the permutability measure becomes

$$\Omega = -\int f(u,v,w)\log f(u,v,w)\mathrm{d}u\mathrm{d}v\mathrm{d}w. \tag{3.25}$$

For the continuous case with three dimensions and poly-atomic molecules, the permutability measure is defined as

$$\Omega = -\int\int\int\int\int\int f(x,y,z,u,v,w)\ln f(x,y,z,u,v,w)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}u\mathrm{d}v\mathrm{d}w. \tag{3.26}$$

The Boltzmann entropy is defined as the proportionality measure of multiplicity Ω that correspond to a given macro-state [11].

Boltzmann's paper laid the groundwork for the statistical definition of entropy. By using the discrete, combinatorial approach he showed that the equilibrium state follows statistically from the most probable state. He further expands this to the continuous case while expanding later to a phase space including three dimensions and poly-atomic forces. This makes for a statistical approach to the now famous Boltzmann Entropy equation

$$S = k\ln\Omega. \tag{3.27}$$

In the discrete case, Ω represents the number of micro-states. The entropy is defined using $\ln\Omega$, while in the continuous case, the number of micro-states is not countable. Therefore, the entropy is directly defined using distribution f, via

$$S = -k\Omega. \tag{3.28}$$

3.2 Coarse-Graining and Observer Dependence

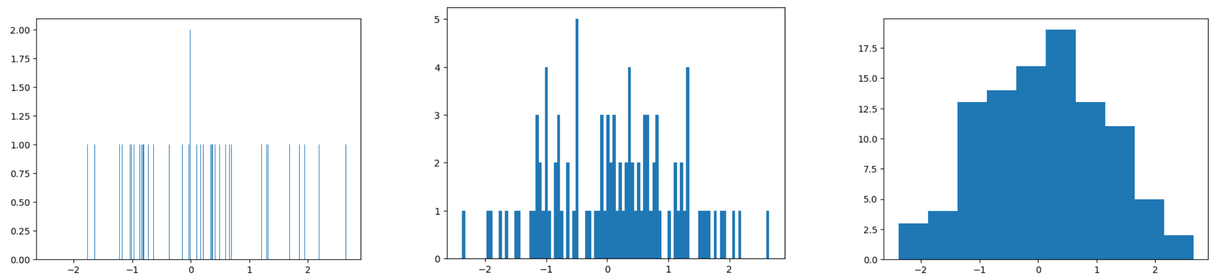
In this section we aim to look at how choices regarding coarse-graining influence the definition of entropy and how entropy behaves. This is linked to observer dependence, as an observer is actively choosing how to partition the state-space.

As seen before, the distinction between the micro-state and macro-state is important for a definition of entropy. In Thermodynamics, phenomena, like tendency to equilibrium, are explained using Statistical Mechanics through micro-relations. An important postulate of Boltzmann Statistical Mechanics (BSM), from which the modern approach originated, is that the macro-states *supervene* micro-states[5]. In other words, every micro-state corresponds to only one macro-state: the macro-state doesn't distinguish between micro-states. The largest macro-region is said to be the *equilibrium macro-region*, in which a dynamical system will inevitably spend the most of its time. The distinction between micro-states and macro-states is established through the act of coarse-graining.

Although Boltzmann does not explicitly mention the term *coarse-graining* in his 1877 paper, he was the one who first introduced the key concept that became later known as coarse-graining. Coarse-graining is essentially the partitioning of state-space into coarse cells (partitions) for the sake of being able to use combinatorial maths.

3.2.1 Coarse-Graining

Coarse-graining is the partitioning of a state-space into cells. This is similar to how a histogram distribution is built. If we take a normal distribution and we plot, say 100 points, on a histogram. When we make the cells, or bins, infinitely small, we end up with most of the bins containing zero points and 100 bins containing exactly one point. By doing this, all the information is contained about the exact value of the point, but this is not much of a distribution. By varying the bin size, or cell size, we get something that starts resembling a distribution. According to Atmanspacher (2016)



(a) 1000 bins

(b) 100 bins

(c) 10 bins

Figure 3.2 Histograms with varying bin sizes.

[12], the choice for partitioning is not arbitrary but contextually dependent.

An example of Coarse-Graining

To get a better understanding of this concept, consider the following thought experiment: imagine a round, empty table and a tube filled with sand grains. The sand-grains are dropped from a fixed position, for example, half a meter above the center of the table. Intuitively, we know that due to gravity the sand grains will fall and will be affected on their way down by air friction, bouncing into the table's surface, bouncing into each other. They will scatter seemingly random across the table's surface.

Suppose we now want to say something about the way the sand is distributed on the table top. We could get a very long list and write down exactly on which position each sand grain landed on the table. We assume the table top to be continuous, so we write this down with relatively precise measure. Now we have a list with roughly all the exact positions of the sand grains. However, such a list would offer little interpretative value. This is because none of the numbers on our list has any similarity to any other number on the list, the data is disjoint, and no meaningful pattern or structure would arise from this precise way of measuring each position.

Instead, we go for a different approach. We divide the table top into small partitions, also *cells*. We

count how many sand particles are in each cell and write this down on a second list. The second list contains less precise information about the position of the sand grains. However, on the second list we can see that the cells in the center of the table contain more sand grains than peripheral ones. When we plot this out, a distribution can arise, and we get a better notion of the system's overall macroscopic behavior.
Thus, by intentionally reducing the accuracy of the information about the positions of the sand grains, we end up with a more relevant notion of the system. That is the key insight of coarse-graining: sacrificing microscopic precision to gain macroscopic insights.

Now, how large should we make these cells in order to end up with relevant information about the system? If we make the cells too small, we end up with either 1 or 0 sand grains in each cell. If we make them too large, we lose a lot of information and fail to end up with a meaningful distribution.

Loss of information

The core concept of coarse-graining and the reason it is used in physics and mathematics is the simplification of a complex space. This simplification is the graining, or partitioning, of this space and could also be seen as the loss of information about the system and its contents. In the above example, the information about the positions of the particles is being simplified in some way, and it is only then that we can observe an interesting relationship within the data. So it seems that loss of information about the contents of the data tells us something about the data as a whole [4]

3.2.2 Observer dependence

In the act of defining parts of a system, there is a dependency on things that are inherently linked to the actor. In this section we look at what things could influence the way we look at systems (macro), their parts (micro) and the distinguishing between them. As we are talking about relativity between sizes, we have to look at scales and how scales could influence the observer. As we assume a space-time continuum, we take spatial and temporal scales and look at how they influence the observers perspective on systems. Furthermore we look at other things that influences an observer's perspective, namely meaning and usefulness of systems and their parts to the observer.

Spatial and temporal scale

As we humans live in the macro-world, we are empirically bounded by both spatial and temporal limits. Although we seem to be able to extend these limits with the help of technologies, we cannot ignore the fact that we can only observe the world directly in a relatively small spatial and temporal window.

[4] This makes me think about an images that only appears to us when we squint our eyes. We take away some information about detail to have the whole appear to us. This is also something that our brain is incredibly good at an inherently built for; information management. And with management meaning focusing on specific parts of information and more importantly; ignoring the rest of information.

The temporal window in which we can observe directly is at its largest a human lifespan, around 80 years, which is inherently distorted by the fact that we seem to have a subjective notion of time, that is constructed by how our brains processes time and how we ourselves construct memories. Even if we assume that we can have an objective notion of time during our lifespan, this lifespan is negligible in comparison with the presumed age of the universe; approximately $13.7 \cdot 10^9$ years. Although it is noteworthy that this temporal framework is constructed by ourselves, living in this relatively small timescale.

The spatial scales in which humans operate, also the macro in the macro-micro-relation, is also a narrow part of our notion of space. We roughly estimate the observable universe to have a diameter of 93 billion light years[13], approximately $8.8 \cdot 10^{26}m$. And the smallest known size to man is the Planck length at roughly $1.6 \cdot 10^{-35}$. As we humans tend to be able to directly observe on the scales of millimeters to a few kilometers, with exception of the stars, that we can directly observe.

These limitations however have not led us to be dejected and accept the short time-span that was given to us by nature. Humans have tried to extend this span by documenting information and finding more advanced methods of predicting the future documenting the past. However, I would like to note that, assuming the space-time framework as given, the further away something is from our point of perspective in this framework, the less certain we can empirically be about the thing we are establishing. But also in the scale frame of space-time; the larger or smaller something is, relative to our own operating scales, the more difficult it is to say something useful about it.
These limitations seem to be affecting to science in general, with the history of science teaching us that certain boundaries can be pushed. Still, it seems to me that the further we try to push these boundaries, the more asymptotic push-back we seem to receive. And even if we try to keep pushing and succeed in this, the less reliable and more distinct our notion of these realms are.

Meaning

In the study of physical systems, we often focus on defining states, probabilities and things like entropy, in formal and mathematical terms. It is important to acknowledge, that behind these frameworks, lies a fundamental and often neglected question: *What makes a state, variable or phenomenon meaningful to an observer?* This question becomes even more relevant when looking at coarse-graining, as this mathematical concept arises when the observer deliberately chooses which micro-details to ignore and which macro-variables to define. These choices are not arbitrary or objective, but are guided by empirical relevance, usefulness or even human empirical limitations.

Historically, scientific inquiry has been driven by such meanings: thermodynamic systems, like the heat engine, were studied, not only for their conceptual elegance, but because they promised to be practical. Similarly, macro-variables such as temperature or pressure are not chosen because they are ontological objective, but because they correlate with our conceptual epistemic restrictions. The act of defining macro-phenomena is then not only a technical necessity, but an act of interpretation.

In this section, we explore how meaning arises from mathematical concepts and the studying or physical systems. We investigate how certain configurations become *informative* or *relevant* based on human cognitive limitations, goals or context. Using examples from information theory and perceived randomness, we argue that meaning is not a property of the system itself, but from the relation between the system and its observer. This view challenges the existing assumption that entropy and order are purely objective and situates them in the philosophical contexts of anthropocentrism and ontology. Take the example below.

We take a state, say a sequence of the numbers $\{0, 1\}$.

$$\begin{gathered} 010010010100010100111001 \\ 011001010100101101100101 \end{gathered} \tag{3.29}$$

$$\begin{gathered} 010010000110010101101100 \\ 011011000110111100100001 \end{gathered} \tag{3.30}$$

These states seem random at first, although they are both created by a process that could be determined. But the second sequence is made by performing a translation of the string 'Hello!'. This example, from Zuchowski's book, is an example of what randomness *means* to us as conscious human interpreters. When we see a sequence that seems random, we have the tendency to attribute meaning to something. When that does not work, in the case of the sequence, we can say that it is random. But, when we get an insight of how the particular state is created, we can say something about the randomness of the sequence. In the case of sequence 3.30, the sequence that first seemed random now loses its randomness because of the information on how the sequence was created. Furthermore, the randomness disappears because of the meaning of the sequence. When we create the sequence,

$$\begin{gathered} 000000000000000000000000 \\ 000000000000000000000001, \end{gathered} \tag{3.31}$$

we could say that the chance of this sequence being created in a random matter is very small. Is this the case because this sequence has meaning to us? Not particularly, although it seems to have order. It contains not very much information, and takes not very much information to create; 'Forty-six zeros and one one', apposed to the random sequence 3.29, which would take the whole sequence to transfer.

3.3 Case Studies

In this section, we cover two examples of measure-preserving deterministic dynamical systems and we zoom in on the way the way the levels are chosen and the role of coarse-graining in the definition of entropy. We look at the often used example, already mentioned in this thesis, the dilute gas. We also look at the Baker's Map, because this example seems to be an intuitive case study of such a system.

3.3.1 Dilute Gas

We take a look at the example of a dilute gas. A dilute gas is a gas consisting of N interacting particles with $N \gg 1$. With the particles first being confined in a two or three-dimensional box, the box suddenly becoming larger. The volume of the, two or more dimensional, box goes from V to $2V$, with the total energy of the particles E and the number of particles N not changing [14]. The particles can be considered being hard discs or spheres, meaning that they interact with each other in a billiard ball way.
The Boltzmann equation for this system is

$$\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\nabla f(\mathbf{r},\mathbf{v},t) = \lambda^{-1}Q(f,f)\,; \tag{3.32}$$

where the $\lambda^{-1}Q(f,f)$ is the description of the interparticle collisions. We focus on the time evolution of the Boltzmann entropy $S_B(M_g)$ for different cell sizes $|\Delta_\alpha|$. These cells are the cells that emerge from coarse-graining. The paper of Garrido et al. suggests that for sufficiently large systems, the leading term of the entropy per particle does not depend on cell size $|\Delta_\alpha|$.
However, the intermediate non-equilibrium case does depend on cell size $|\Delta_\alpha|$ and is proportional to velocity coarse-graining Δv. The two mechanisms involved in this dependency are proposed to be;

1. dispersal of the particles in physical space due to free motion,

2. dispersal of the particles in velocity space due to collisions between particles.

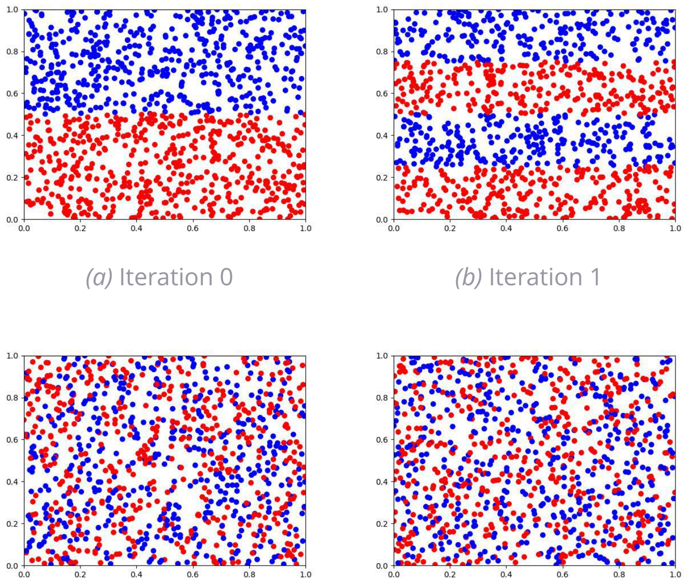
The free motion of particles in space seem to lead to a more uniform distribution of the particles in space, where the collisions seem to cause the velocities to be more uniformly distributed. Only when $(\Delta v)/l\Gamma$, where Γ is the particle scattering rate, the rate of entropy increase becomes independent of δv. It is noteworthy that Boltzmann looks at this limit in his H-theorem, and therefore ignores the fact that the entropy increase can depend on the size of coarse-graining cells.

3.3.2 Baker's Map

Another example is the The Baker's Map, which is dynamical system that is measure-preserving and may exhibit ergodic behavior in an intuitive way. It is based on a mathematical translation

$$(x_{n+1}, y_{n+1}) = \begin{cases} (2x_n, ay_n), & \text{for } 0 \leq x \leq \frac{1}{2} \\ (2x_n - 1, ay_n + \frac{1}{2}), & \text{for } \frac{1}{2} \leq x \leq 1 \end{cases} \tag{3.33}$$

, with $a = 0.5$ to create a half fold.
This translation is intuitively described as kneading because it looks like the way bakers knead their dough to mix all the different ingredients throughout the dough, hence the name Baker's map. As seen in figure 3.3, the particles that are first divided through $y = 0.5$, get mixed together.



(a) Iteration 0

(b) Iteration 1

(c) Iteration 15

(d) Iteration 25

Figure 3.3 Visualization of the Baker's map over multiple iterations.

In figure 3.3a, the first iteration of the translation can be seen, the layers are stretched out and then placed upon each other. In figures 3.3c and 3.3d the particles seem to be mixing together with each iteration and seeming to being evenly distributed among the map.

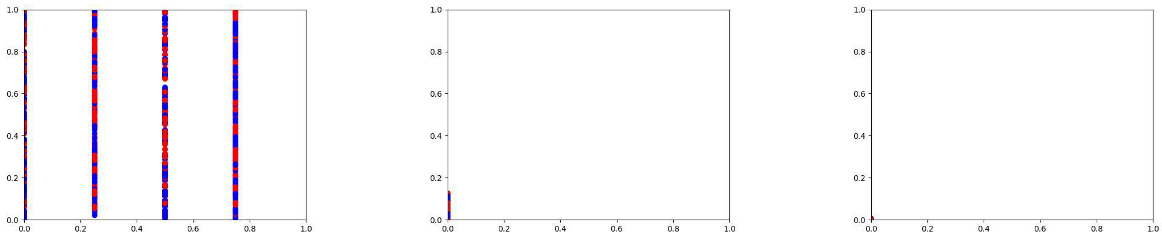
When examining properties of such a chaotic system, like entropy, one uses coarse-graining to calculate it. But what different coarse-graining methods are there, and do they need to be examined carefully before being used? As shown by Akritas et al. (2001), the different methods regarding coarse-graining yield drastically different results [15]. The main findings of this paper include the fact that differences in the coarse-graining unit cells yield different outcomes, even if the cell areas are equal. The findings are based upon triangular vs square unit cells. The paper finds that *autocorrelation functions*, which reflect how system properties evolve over time, can be well examined using coarse-graining methods. But when looking at *decay rates*, which reflect how fast correlations disappear, different coarse-graining schemes lead to very different results.

However, the paper of Schack et al. (1992), examines The Baker's Map, as a good example of chaotic Hamiltonian evolution, in an information-theoretic and thermodynamic framework. They do this by using two methods to follow a perturbation in the Baker's Map. They (i) track a perturbation in fine-grained detail, and (ii) use coarse-graining by averaging over a perturbation. Using the Landauer erasure cost, they find that coarse-graining is a quantitative justifiable method. They derive that:

> The information needed to specify a typical perturbed pattern far exceeds the entropy increase due to averaging over perturbations.[16]

They argue that coarse-graining is a justifiable method when looking at quantitative thermodynamic context: they claim it is an emergent necessity due to the Landauer cost of information and the inherent complexity of chaos. The additional information needed to track the system grows exponentially while the entropy from coarse-graining grows linearly. Therefore it is a justifiable method that seems to be a rational trade-off between information cost and available free energy.
Akritas et al. shows that entropy behavior is highly sensitive to *how* we coarse-grain, while Schack and Caves explain *why* we must coarse-grain.

When simulating the Baker's Map, one could experience a problem that occurs when simulating with a large number of iterations. While figures 3.3a, 3.3b,3.3c and 3.3d, the translation is clearly visible, after iterations $T_N > 48$, the points start moving into integer positions and eventually collapse towards the origin, as seen in figures 3.4. The immediate cause of this collapse was not



(a) Iteration 50 *(b)* Iteration 55 *(c)* Iteration 60

Figure 3.4 Baker's map evolution after many iterations.

directly clear. But upon further inspection and while trying to solve the problem, it became clear that the reason for this collapse had something to do with the fact that the Baker's Map is a chaotic system, in which the iteration represent some kind of zoom onto the small variations of the initial conditions. This tells us something about the way computers process these numbers, namely in bits, which are fundamentally a form of coarse graining [5]. This could be interpreted as a computational limitation and therefore a human limitation.

[5]This also made me think of the Mandelbrot set, which can be simulated and zoomed in. But the extensive zoom video's that are available are an illusion. The computer did not calculate all of the Mandelbrot set, but calculates again and again to create the illusion of a never ending fractal.

3.4 Philosophical Reflection on Human Projection in Physics

These case-studies have provided us with practical examples of how humans deal with experimental limitations in the context of entropy and chaotic systems. However, in this section we take some of these intuitions and place them in a broader philosophical context in which we explore thoughts on the role of anthropocentric projection in establishing a notion of the world around us. We start by expanding the concept of scales by taking it to an extreme using Timothy Morton's *hyperobjects*. We then briefly touch on the *reduction vs emergence debate* by exploring how this debate plays a role in the context of this thesis. After that we take the practical mathematical tool of *coarse-graining* and to establish a feeling of how it places itself in philosophical context. We then look at how emergent phenomena seem to be caused by *anthropocentric necessity*. We end this section with a broader notion on how the above intuitions tie together.

3.4.1 Scales and Hyperobjects

In *Hyperobjects: Philosophy and Ecology after the End of the World*, Timothy Morton proposes and defends the idea of *hyperobjects* - entities so massively distributed through space and time that they exceed human comprehension [17]. Some examples are nuclear waste, climate change and the lifespan of stars. Most of these things could be seen as irrelevant, because they do not directly influence the way we live, and we do not directly influence the way they happen, but some of these things, like climate change, do. This book came to me when I was discussing my thesis with a friend of mine who happens to study philosophy. He mentioned this book and we briefly discussed its main posits. I directly felt it could be placed in the philosophical context of this thesis, because it explores spatial and temporal scales in a different direction, namely greater. As mentioned in the introduction of this thesis, macro could mean 'on larger scale' or 'large', of which the concept of*hyperobjects* seems to be an extension. While the core posit of the book, climate change, is a very good example of an entity that is in the anthropocentric *blind spot*. The temporal and spatial limitations of a single human life, seems to be to small to get a good notion of really witnessing this entity, though its relation to the entity seems to be great enough that they have influence on each other. Another example, the life-span of the sun, makes us aware of the vastness of such hyperobjects, because the entity of the sun as a hyperobjects exceeds far beyond human history, making us humans subject to be engulfed in it. It seems impossible for us to be able to alter the spatial and temporal variables of the sun, however, it does have an influence on us.
The argument of Morton applies not only to objects of vast spatial and large temporal scales, but also to the topic of small spatial and short temporal scales. There is a similarity in the way humans seem to interact with and handle these objects. The human limitation of scale induces an epistemic boundary which is attempted to be overcome by gaining indirect knowledge of larger and smaller scales. However, the fact that a direct observation of these entities seems to be impossible, raises the question of the validity of this knowledge and the overall human limitations when it comes to non-human scales. In this argument, the scale is not just a physical dimension which could be

overcome, but it really is an epistemic boundary.

3.4.2 Reduction vs Emergence

In the course of writing this thesis, the balance between the reduction vs emergence debate appeared often. Not so much in writing the thesis, as in slumbering background thoughts. In this section, we explore how this thesis lies in context of this debate and how it provides both camps with plausible arguments.

At first, this thesis and its subjects seem to be an exploration on how macro-level phenomena could be reduced to micro-level interactions, however, we find that in the relation between these levels there is more to explore. This is essentially what reduction and emergence are about, the relationship between different levels of state.

Reductionism and emergentism are both doctrines that are could be explained in their core by

1. Reductionism: the claim that higher-level phenomena or theories should ultimately be explainable in terms of lower-level physical constituents,
2. emergentism: the claim that some phenomena are ultimately not straightforwardly derivable from micro-level laws, by principle [18].

This distinction is somewhat polarized and it is already in the title of the book 'Reductionism, Emergence and Levels of Reality: The importance of being borderline', that the writers argue for a more united view regarding the positions in this debate.

Statistical mechanics and Boltzmann's contribution to the bridge law $S = k \log W$ could be seen as a direct argument for the reductionist camp[6] [10]. The dream of total reduction seems to be embedded in modern science, which could be seen in the dominant role that natural sciences have gained over the past centuries. An example of this dream is Laplace's Demon, which assumes the possibility of total deterministic reduction [20]. As Laplace mentions in his *philosophical essay on probabilities* of which the original work was already published in 1814:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it - an intelligence sufficiently vast to submit these data to analysis - it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

This notion, later named 'Laplace's Demon', could be seen as the ultimate dream of reduction. It could almost be regarded as the philosophical foundation of the physics and mathematics later done by Boltzmann. The 'respective situation' could be translated as 'positions and momenta'. In some way, the dream of Boltzmann, as defender of the atomist view, is somewhat aligned with

[6]The so called 'bridge law' was never directly proposed by Boltzmann, it was more or less the interpretation of Planck that led to the formulation of entropy in this way and forming the second law of thermodynamics [19].

the dream of ultimate deterministic reduction. However, there are practical challenges of 'knowing it all', found in this thesis with the example of the modeling of the baker's map (section 3.33), which seems to imply that computational modeling and keeping track of all the variables in such a system is limited. Also, the note on coarse-graining (section 3.2.1), implies that even if the practical limitations of 'knowing all' are overcome, there does not seem to be something useful to be said about a complex system, as information about the structure arises from the abandonment of detailed variables, by way of coarse-graining. This topic will be discussed in the next section.

But is this problem of determinism and prediction due to the *lack of skill* we have to really get all the information about a system? Would it be possible to reduce chaotic systems in such a way that we can predict this seemingly chaotic behavior? I had this thought experiment when I was a child, but I could then not find a conclusive answer to this question and I cannot find one now: If we were to know everything about everything, we could build or simulate a model-universe and predict everything about the future. Then the question arises, do we only need the technology and skills to do this, is this practically impossible, or is this inherently and logically impossible? I tend to the most radical and last option, as creating a model of the universe would mean that we get into some kind of meta-cycle, as this universe lives inside our universe, which would lead to an infinite cycle of universes in universes that seems a priori impossible to model. [21]

3.4.3 Coarse-graining and the information trade-off

This thesis aims to look at how coarse-graining is of influence on how one arrives at a definition of entropy. Coarse-graining, the partitioning of a space with the goal of focusing on larger-scale behavior. This partitioning and ignorance of detail to gain a more useful notion of the overall structure, could be placed in broader context.

It is only when one is an absolute reductionist that detail is everything and only everything. From that detail then, everything else arises.

For example in the way our brains operate to process stimuli. In evolution, data management has proven to be one of the most profound necessities when it comes to survival. Intelligent species that have developed multiple senses have to process the stimuli that are picked up by those senses and process them in their brain to form a proper response, as if their lives depend on this. This data management is complex and advanced, but it is largely based on selecting the right stimuli to respond to and therefore ignoring most of the stimuli [22]. It is in the ignorance of detail that puts the focus on useful stimuli.

Another example that came to my mind, is of something that we humans distinguish ourselves with from other animal species, namely the use of *vocabulary*. When we want to indicate something, for example a chair, we use the word 'chair' as a category of which the object we want to indicate has a sufficient similarities with the preconditions of the category 'chair'. You could see the word 'chair' as a macro-space, in which certain points can fall. Some are more near the edge, when you come across for example a office chair, this could fall under the category chair, but there is a finer detailed cell, which we label as 'office chair'. When we want to describe something to one another, we have to give up detail, otherwise it would take an eternity to describe a single object. Just as with coarse-graining, the way we use words come both from practical reasons as from the fact that a bigger structure does not appear if we only take into account pure detail. You couldn't tell apart a chair from a car because the details would be so vast that they would flood the notion of structure and correlation.

3.4.4 Anthropocentrism

The way humans are connected with the outer world is through sensory stimuli. Empiricism is the philosophical viewpoint that all knowledge is gained through these sensory data. From an individual human stancepoint, this seems to make sense, as every individual experiences the world through its own senses. When we combine this data with the fact that humans have found a way to communicate this data with each other, something bigger appears. We can talk about human knowledge and a human perspective on the outer world. The philosophical term for this perspective is *anthropocentrism*. As science tries to find a way to escape this anthropocentric perspective by introducing concepts and entities outside the self, compensating this bias seems to be a rather difficult task. In the context of this thesis, an inevitable anthropocentric task is the selection of macro-states. This selection seems to be largely based on meaning. The projection of meaning has in multiple ways implications for how humans choose macro-states. Macro-states are chosen in such a way that they have meaning to humans caused by the human macro-scale. A macro-state is a state that corresponds to multiple micro-states. These micro-states differ similarly to each other as to micro-states corresponding to other macro-states. However, on a human scale, we cannot distinguish the micro-states that correspond to the same macro-state and more importantly, *we not care to distinguish*. We only project meaning to the macro-variables, because these macro-variables can really mean something to us. We can feel temperature difference, we can feel pressure. We do not care about which atom is at what position x with what momentum p at time t.

If we take empiricism a step further and consider the Interface theory of perception by D. Hofmann, we could form an argumentation of how humans were led trough evolution to perceive the world the way they perceive it [23]. Within this thought of argumentation, one could interpret the choice of macro-states as being influenced by the way humans were shaped to perceive the outer world. Coarse-graining and the role of meaning could be seen as cognitive interventions of shaping the world around human perception.

3.4.5 Playing among the Gods

With the above somewhat loose philosophical chains of thought taken into consideration, there is a common thread to be seen: The tendency of the human species to break out of their anthropocentric and empirical limitations, to gain knowledge of the small, the big, the observable, the non-observable, the knowable and the unknowable.

Although the sciences have managed to change the world and the way we view the world by gaining knowledge about it, there seems to be a leap when it comes to the bigger philosophical questions. This thesis could be seen as a case study of this leap on the physics subject of *entropy*.

The phenomena of tendency to equilibrium, is that **due to** statistics, or is it **due to** a physical mechanism of interaction? Can we say that statistics is *withdrawn from* (a description of) phenomena, rather than the actual underlying and responsible mechanism?

The above question is an essential one. When you argue that probability theory is just a way to predict the future, the notion and fundamentals of it are way different than when you find it to be the underlying explanation of natural behavior. If you make it small: when throwing a dice, probability theory will give you nice predictions about large number behavior, and the dice will eventually, after a certain amount of throws, behave according to the prediction. However, the underlying mechanisms for the dice to fall on a certain face seem to have nothing to do with probability theory, as it seems to be a purely Newtonian mechanical process. When quantum theory was introduced, Einstein famously said: 'God does not play dice with the universe'[24]. He seems to view the universe as a deterministic system and rejects the randomness involved with quantum theory. It is in this quote, that there lies an assumption about the universe and its behavior in laws of nature, that this thesis aims to argue against. It is the attempt of the human species to break out of this *nature*, by uncovering its fundamentals, and it is the believe in this possibility that this thesis puts into question.

Chapter 4

Discussion & Conclusion

In this thesis we try answer the question how human projection and anthropocentrism could influence the way we study complex systems and how we think they could be reduced to their constituent parts. *What is the role of human projection in how entropy and the macro-micro-relation is defined in physics?*

4.1 Conclusion

We found that coarse-graining forms an important link to arrive at Boltzmann entropy. We began by laying out the mathematical foundation of a measure-preserving deterministic dynamical system and how it underpins Boltzmann's statistical framework. The multiplicity of the micro-states corresponding to a given macro-state led to the construction of Boltzmann entropy. However, this thesis argues that this construction is not entirely objective: the transition from micro- to macro-state requires a deliberate act of coarse-graining, which is essentially a simplification or averaging process that introduces observer-dependent elements into this physical description.

The process of coarse-graining, which is necessary to extract macroscopic observables from microscopic data, is fundamentally tied to human perception and its limitations. Demonstrated by both mathematical formulations and analogies, for example the sand-on-table thought experiment, different choices of coarse-graining introduce different notions of macro-states. Especially in non-equilibrium situations.

This is emphasized by both case studies. We look at the example of the dilute gas, examined by Garido et al. (2024), which varies cell size $|\Delta_\alpha|$ in the intermediate non-equilibrium case and finds that, in this context, the entropy does indeed depend on this cell size. Also, the case study of the Baker's Map provides us with a chaotic complex system which can be used to study the influence of varying coarse-graining methods. The case of the Baker's Map is a good example to find how coarse-graining is necessary to derive entropy. The case study also provides us with a context in which different coarse-graining methods can be applied, as done by Akritas et al. (2001). These resulted in different macro-variables, including different entropies. Furthermore, the Baker's Map is a concrete example of the difficulty we face when computing and modeling complex systems.

Even a relatively simple translation, 3.33, turns into a chaotic complex system that requires a substantial amount of computational power to iterate indefinitely. This could philosophically be interpreted as an anthropocentric limitation.

A recurring theme is the dependency on the perspective of the observer - not only in subjective bias, but also fundamentally in limits of perception, cognition and temporal/spatial scale. The macro-level variables are chosen in such a way that they have meaning to humans, because it is at this level that our senses operate. Phenomena as temperature could be argued to not be ontologically privileged, but rather epistemically useful. These anthropocentric limitations cause the use of coarse-graining and the choice of macro-level variables based upon usability and intelligibility, not objectivity.
As discussed in chapter 3.4, which shapes a philosophical context around the derivation of entropy, physical models may not only describe the outer world, but they reflect our constraints as embodied, time-bound observers. When looking at entropy through the lens of hyperobjects (chapter 3.4.1), reduction vs. emergence (chapter 3.4.2) and the information trade-off (chapter 3.4.3), it becomes not only a measure of disorder or multiplicity, but also a projection of our own way of handling complexity through chosen ignorance, induced by human limitations.

We conclude that entropy, as understood by Boltzmannian Statistical Mechanics, is not only a physical property of a system, but also a reflection of the epistemic strategies used by human observers. The distinction between the macro- and micro-levels are far from an ontological given and are deeply tied to human perceptual limitations and are based upon conceptual frameworks and pragmatic goals. Coarse-graining, as a bridge between the two levels and as a mathematical necessity, is inherently anthropocentric. This is contextually emphasized by the case studies, which make the influence of coarse-graining choices concrete and analyzable. Therefore, we conclude that the entropy is not entirely an objective feature of complex systems or the world, it arises from the way we model and is based upon anthropocentric choices.

4.2 Discussion

This thesis finds itself between science and philosophy. While it tries to analyze the philosophical arena of metaphysics in the context of entropy and micro-macro-relations, it becomes clear that there is a substantial distinction between physics and philosophy. While this thesis aims to form a bridge between these areas, based upon scientific and philosophical literature, the area between these regions is sometimes vague; it is in this region that both areas are engaged in a fundamental struggle. This struggle forms an interesting but also slippery scene. This thesis could therefore be seen as a glance in both regions and a connection between them. This first part is a glance in the scientifically grounded area of entropy, the micro-macro-relation and coarse-graining. The second part is a philosophically grounded glance into the arena of metaphysics, ontology, reductionism and emergentism. The philosophical arena is inherently build upon debates and frameworks. The philosophical part of this thesis is therefore part of these debates and is inherently based upon personal experiences and biases. It is also Boltzmann who worked on this metaphysical area, especially in the latter part of his life, where he wrote about construction of concepts like atoms.

A substantial part of this thesis deals with coarse-graining - and by extension, entropy - is not purely an objective physical quantity but an epistemic construction reflecting anthropocentric choices. While this position is defended by mathematical analyses and philosophical argumentation, it may risk overstating the role of human projection. It could be argued that entropy and the use of coarse-graining is statistically robust under the ensemble of coarse-graining choices, making it more objective than this thesis admits. Especially within the boundaries of physics and in practical application of entropy, one could argue that it makes sense to look at systems this way and to construct entropy in the way it is done right now. Moreover, critique on coarse-graining for being a subjective act may conflate modeling choices with ontological claims about reality. A more nuanced distinction could be made between model-relative choices and emergent properties, avoiding falling into relativism.

While it is argued that the Baker's Map forms an interesting study area when it comes to a deterministic chaotic system of which the entropy can be determined, it is also an over-idealized mathematical system that does not have direct physical application. In context of this physics thesis it has its limitations. It could be questioned how the findings of Akritas et al. (2001) and Schack and Caves (1992) would apply to more realistic physical systems. Therefore it could be argued that the utility of Baker's Map lies more in its conceptual clarity than in its physical relevance.

Concrete outlooks, area's that could be explored within the context of this thesis, are:

- Further research could focus on the expansion of analyzing coarse-graining and entropy in the quantum realm. Especially because Quantum Statistical Mechanics (QSM) could be seen as a more general description of Statistical Mechanics. It forms an interesting area to study because of its connection to human reflection on measurement, decoherence and entanglement where *mixidness* plays a role. The quantum realm forms an ideal study field for the intertwining area's of physics and philosophy because of its limits and seemingly counter-intuitive notions. It could provide context and different perspectives to metaphysics, ontology, reductionism and emergentism.
- While in this thesis, the information-perspective of entropy and chaos is only briefly discussed, it could be worked out in a further research. Treating systems as information carriers and looking at how information changes while the system changes, could reveal a different perspective when it comes to correlation, chaos, entropy and coarse-graining. Shannon entropy, Kolmogorov complexity and thermodynamic entropy could help bridge the gap between information theory and statistical mechanics. It also could form an fruitful context for anthropocentric reasoning, as information itself could be argued to be the result of interaction between human and systems.

Chapter 5

Epilogue

This epilogue consists of loose thoughts and imaginings that arose while being submerged in the content of this thesis. Although they did not directly fit in the thesis itself, I would like to share them in the form of an epilogue.

- If the universe is like a book, a materialist would read it word for word and analyze it letter by letter. But the emergent phenomena when reading a sentence is the feeling it sparks within the conscious reader.
- Words are like the visible, attention-grabbing mushrooms that pop up the surface. Beneath hiding a vast network connecting them, forming the space of existence and non-existence. The network itself cannot be grabbed or touched, it is just the mushrooms that pop up and represent the network itself.
- The macro-states that matter to you as human, are completely dependent on how much you know about these micro-states. If you ask an average person what they see, and show them a picture of a car. They will answer completely differently than if you ask someone that is a car retailer. And if you ask a mechanic, the answer would be much different. The car retailer and the mechanic will know more information about cars, and therefore inherently see and experience different things about the car. When you ask an average person to jump into the car and start it, and ask if it works properly. They will have the possible micro-states and micro-states very much aligned. Either the car starts; "it works", or the car doesn't start; "it doesn't work". A mechanic will have more different available micro-states that fall into the macro-state "it doesn't work". If the car hesitates to start, or he hears a hick-up in the engine, he will probably say that "it doesn't work".
- Coarse-graining seems to be an act that is very human-like. It is an act of partitioning of the outer world. This act could be compared with how humans tend to partition, or rather characterize, things with words and language in general. The outer world could be seen as a meaningless, continuous gathering of molecules into something continuous like spacetime. But we as humans tend to attribute words to these molecules, when they have meaning to us. We characterize a chair as a chair, because we can sit on it, we can move it, all the molecules, or most of them, seem to move with it. We can also sit on the earth, but we cannot pick it up and take it somewhere, so it is not a chair.

- I read about the end of Boltzmann's life, in the preliminaries of the book written by Darrigol[3], with goosebumps. Boltzmann's determination and endless, very human, necessity of curiosity touched me and his turnaround in philosophical thought have led me to feel connected to him. I feel thankful for I had the opportunity to think about the same things as Boltzmann has, although I believe my thinking to be on a much superficial level as he did, evident from the degree attachment he felt towards his thoughts, something that eventually became too much for him.

- The task of writing down the exact location of a sand grain in the explanation above, is by definition impossible. In fact, the act of really *taking* a real number from a continuous region between 0 and 1, is impossible. Within continuous space, we coarse-grain automatically, by rounding the exact position, and therefore simplifying. In more mystical terms; only god - with god meaning the universe - knows the exact position of the sand grain, we can just barely guess.

- My opinion on the debate reduction vs emergence: First of all, the argumentation of a reductionist; if we have enough information about a system, we could know how the system evolves and we therefore can say everything about the system. Complex systems can be reduced in their less complex parts. I have multiple arguments against this: to me it seems impossible to know enough, or everything, about a system. We can just barely know everyting in thought, but not in the real world. Why? Because we are not the objective observer of the universe. We are in the universe, we are part of it. Inherently, the only thing that knows everything is the universe itself. Also, philosophy is not even clear about the fact that there is no un-observable world, so to really know everything seems like an impossible task. We can barely grasp anything that is going on, let alone know everything that is going on, let alone know everything that is going on and not going on.

- When I read about Laplace's Demon, it reminded me of my childhood. My bed stood in a small loft within my room, which I remember now as being a whole universe of its own. Many things I learned later, about philosophy and physics, take me back to my childhood in that bed, when my brain was still childlike and unbothered by cognitive interventions. I had many, almost spiritual, experiences about the universe and the development of my ego in that universe. When I read about what some of the great minds have come up with and have been able to put in writing or thinking, I feel a shallow connection with the minds that developed these concepts and the cloudy whole where these concepts come from.

- It was, when I read about the unimaginable amount of gas molecules in a macro-volume, that I realized how subjective and dependent we are to gasses and molecules. It was when I breathed in through my nose and felt the resistance of my nose with sucking in the gas molecules that I realized that my body is not floating in an objective vacuum but that I am subject to gasses while I am not even aware of it for most of the time. Sometimes you can feel knowledge, rather than just knowing it. It is then when you become drenched in this knowledge rather than observing it. I feel like that is what feeling knowledge feels like. It does something with your emotions and your body. You suddenly become not only cognitively aware, but emotionally aware of certain dogma's.

- If I would like to connect psychology and physics in context of this thesis, I see a connection between emergent phenomena along with the believe in emergence, and the psychological concept of feelings. Feelings seem to be something that an observer can have, and cannot be always reduced to a certain cause or be broken into parts. Feelings are emergent phenomena that arise suddenly, and if dealt with in a rational (reductionist) way, are elusive and intangible.

Bibliography

[1] (). Macro-, wiki, [Online]. Available: https://en.wiktionary.org/wiki/%5C%CE%5C%BC%5C%CE%5C%B1%5C%CE%5C%BA%5C%CF%5C%81%5C%CF%5C%8C%5C%CF%5C%82 (visited on 03/31/2025).

[2] (). Macro-, etymology, [Online]. Available: https://www.etymonline.com/word/macro- (visited on 03/31/2025).

[3] O. Darrigol, *Atoms, Mechanics, and Probability: Ludwig Boltzmann's Statistico-Mechanical Writings - An Exegesis*. Mar. 2018, pp. 1–612, isbn: 0198816170. doi: 10.1093/oso/9780198816171.001.0001.

[4] K. Sharp and F. Matschinsky, "Translation of ludwig boltzmann's paper "on the relationship between the second fundamental theorem of the mechanical theory of heat and probability calculations regarding the conditions for thermal equilibrium" sitzungberichte der kaiserlichen akademie der wissenschaften. mathematisch-naturwissen classe. abt. ii, lxxvi 1877, pp 373-435 (wien. ber. 1877, 76:373-435). reprinted in wiss. abhandlungen, vol. ii, reprint 42, p. 164-223, barth, leipzig, 1909", *Entropy*, vol. 17, no. 4, pp. 1971–2009, 2015, issn: 1099-4300. doi: 10.3390/e17041971. [Online]. Available: https://www.mdpi.com/1099-4300/17/4/1971.

[5] C. Werndl and R. Frigg, "Reconceptualising equilibrium in boltzmannian statistical mechanics and characterising its existence", *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, vol. 49, pp. 19–31, 2015, issn: 1355-2198. doi: https://doi.org/10.1016/j.shpsb.2014.12.002. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1355219814001233.

[6] J. Uffink, "Compendium of the foundations of classical statistical physics", English (US), in *Philosophy of Physics*. Elsevier, 2007, pp. 923–1074, isbn: 9780444515605. doi: 10.1016/B978-044451560-5/50012-9.

[7] G. M. Wang, E. M. Sevick, E. Mittag, D. J. Searles, and D. J. Evans, "Experimental demonstration of violations of the second law of thermodynamics for small systems and short time scales", *Phys. Rev. Lett.*, vol. 89, p. 050 601, 5 Jul. 2002. doi: 10.1103/PhysRevLett.89.050601. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.89.050601.

[8] A. del Junco, "Ergodic theorems", in *Ergodic Theory*, C. E. Silva and A. I. Danilenko, Eds. New York, NY: Springer US, 2023, pp. 79–107, isbn: 978-1-0716-2388-6. doi: 10.1007/978-1-0716-2388-6_176. [Online]. Available: https://doi.org/10.1007/978-1-0716-2388-6_176.

[9] M. Nicol and K. Petersen, “Ergodic theory: Basic examples and constructions”, in *Ergodic Theory*, C. E. Silva and A. I. Danilenko, Eds. New York, NY: Springer US, 2023, pp. 3–34, isbn: 978-1-0716-2388-6. doi: 10.1007/978-1-0716-2388-6_177. [Online]. Available: https://doi.org/10.1007/978-1-0716-2388-6_177.

[10] S. Chibbaro, L. Rondoni, and A. Vulpiani, “A first attempt to tame complexity: Statistical mechanics”, in *Reductionism, Emergence and Levels of Reality: The Importance of Being Borderline*. Cham: Springer International Publishing, 2014, pp. 45–69, isbn: 978-3-319-06361-4. doi: 10.1007/978-3-319-06361-4_3. [Online]. Available: https://doi.org/10.1007/978-3-319-06361-4_3.

[11] L. Zuchowski, *From Randomness and Entropy to the Arrow of Time*, ser. Elements in the Philosophy of Physics. Cambridge University Press, 2024.

[12] H. Atmanspacher, “On macrostates in complex multi-scale systems”, *Entropy*, vol. 18, no. 12, 2016, issn: 1099-4300. doi: 10.3390/e18120426. [Online]. Available: https://www.mdpi.com/1099-4300/18/12/426.

[13] *Size of the observable universe*, https://https://www.britannica.com/topic/observable-universe, note = Accessed: 2025-04-14.

[14] P. L. Garrido, S. Goldstein, D. A. Huse, and J. L. Lebowitz, *Time evolution of the boltzmann entropy for a nonequilibrium dilute gas*, 2024. arXiv: 2403.07519 [cond-mat.stat-mech]. [Online]. Available: https://arxiv.org/abs/2403.07519.

[15] P. Akritas, I. Antoniou, and E. Yarevsky, *Coarse graining: Lessons from simple examples*, 2001. arXiv: physics/0101077 [physics.class-ph]. [Online]. Available: https://arxiv.org/abs/physics/0101077.

[16] R. Schack and C. M. Caves, “Information and entropy in the baker’s map”, *Phys. Rev. Lett.*, vol. 69, no. 23, pp. 3413–3416, Dec. 1992. doi: 10.1103/PhysRevLett.69.3413.

[17] T. Morton, *Hyperobjects: Philosophy and Ecology After the End of the World*. Minneapolis: Univ of Minnesota Press, 2013.

[18] S. Chibbaro, L. Rondoni, and A. Vulpiani, “A random journey from monism to the (dream of) unity of science”, in *Reductionism, Emergence and Levels of Reality: The Importance of Being Borderline*. Cham: Springer International Publishing, 2014, pp. 21–44, isbn: 978-3-319-06361-4. doi: 10.1007/978-3-319-06361-4_2. [Online]. Available: https://doi.org/10.1007/978-3-319-06361-4_2.

[19] M. Planck, “Über das gesetz der energieverteilung im normalspectrum”, German, *Annalen der Physik*, vol. 309, no. 3, pp. 553–563, 1901. doi: 10.1002/andp.19013090310.

[20] P.-S. Laplace, *A Philosophical Essay on Probabilities*, trans. by F. W. Truscott and F. L. Emory. New York: Dover Publications, 1951, Originally published in 1814 as *Essai philosophique sur les probabilités*.

[21] S. Chibbaro, L. Rondoni, and A. Vulpiani, “From microscopic reversibility to macroscopic irreversibility”, in *Reductionism, Emergence and Levels of Reality: The Importance of Being Borderline*. Cham: Springer International Publishing, 2014, pp. 71–97, isbn: 978-3-319-06361-4. doi: 10.1007/978-3-319-06361-4_4. [Online]. Available: https://doi.org/10.1007/978-3-319-06361-4_4.

[22] L. A. Jordan and M. J. Ryan, “The sensory ecology of adaptive landscapes”, *Biology Letters*, vol. 11, no. 1, p. 20 141 054, 2015. doi: 10.1098/rsbl.2014.1054. [Online]. Available: https://doi.org/10.1098/rsbl.2014.1054.

[23] D. D. Hoffman, M. Singh, and C. Prakash, “The interface theory of perception”, *Psychonomic Bulletin & Review*, vol. 22, no. 6, pp. 1480–1506, 2015. doi: 10.3758/s13423-015-0890-8.

[24] A. Einstein and M. Born, *The Born-Einstein Letters: Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955*, I. Born, Ed. London: Macmillan, 1971, "God does not play dice with the universe" appears in a 1926 letter to Max Born.