Abstract. What is it that shapes arguments into mathematical proofs that are intelligible to us, and how is it that we can find proofs efficiently? – This is the informal question I intend to address by investigating, on the one hand, the abstract ways of the axiomatic method in modern mathematics and, on the other hand, the concrete ways of proof construction suggested by modern proof theory.

These theoretical investigations are complemented by experimentation with the proof search algorithm AProS: it searches, first of all, for natural deduction proofs in pure logic. This strategically guided search can then be extended in a direct way to cover elementary parts of mathematics. The subtle interaction between understanding and reasoning, i.e., between *introducing concepts* and *proving theorems*, is crucial and suggests principles for structuring proofs conceptually.

It is Hilbert's work that weaves these strands into a fascinating intellectual fabric and connects, in novel and surprising ways, classical themes with deep contemporary problems. These problems reach from proof theory through computer science to cognitive science and back.