Fysica 2008:
The case for indeterminism.

Hans Maassen

April 18, 2008

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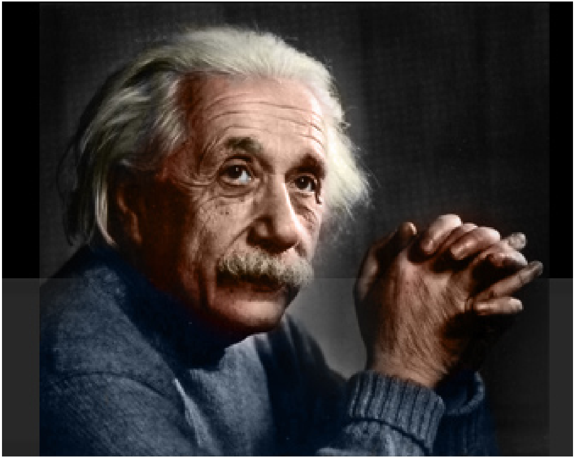
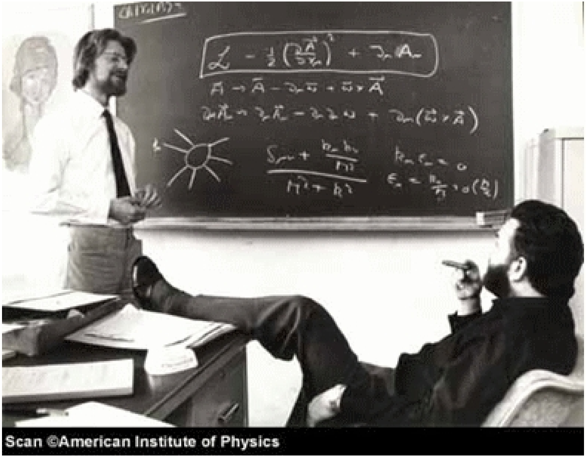
Such theories can not explain Aspect's 1982 experiment, in which Bell's famous inequality was broken.

Dramatis personae



Discoverer of entanglement (Einstein-Podolsky-Rosen-correlation)

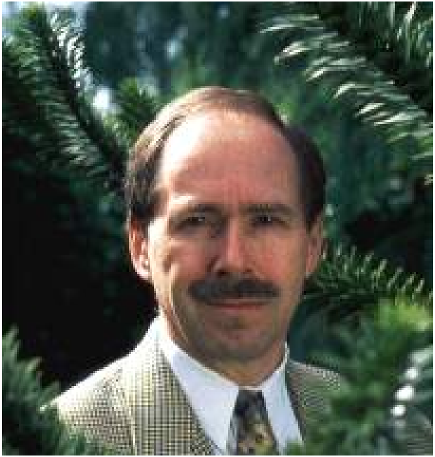


Discoverer of inequalities broken by EPR correlations

"If anyone ever uses this theory to send signals faster than light,

I hope he calls it the 'Bell Telegraph'."

Alain Aspect, who performed the experiment



A deterministic theory at the Planck scale?

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Hence

$$\text{Deterministic} \quad \Longrightarrow \quad \text{Realistic} \, .$$

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In local theories systems can be causally separated for a while.

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And what about statistical mechanics?

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Events $A : \Omega \to \{0, 1\}$ are now only predicted with some probability:

$$\mathbb{P}[A = 1] = \mathbb{E}(A) = \mathbb{P}\big(\{\lambda \in \Omega \,|\, A(\lambda) = 1\}\big) = \int_\Omega A(\lambda)\,\mathbb{P}(d\lambda) \ .$$

Such theories are also called realistic, an in the dynamic case they are still basically deterministic.

The Question

Does there exist

a local deterministic theory

underlying Quantum Mechanics?

The Answer

NO!

Not even a local realistic theory

(stochastic or otherwise).

Such theories will not be able to explain Aspect's experiment.

Aspect's experiment

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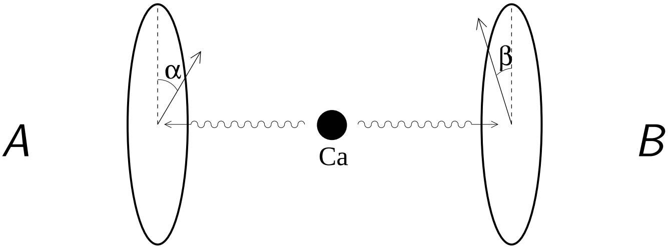

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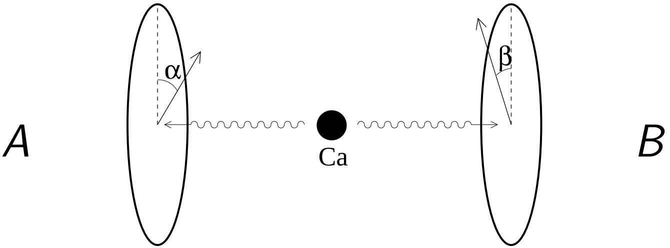

$$A(\alpha) = \begin{cases} 1 & \text{if } A\text{'s photon shows polarization } \alpha \in [0, \pi) \\ 0 & \text{if } A\text{'s photon shows polarization } \perp \alpha \end{cases}.$$

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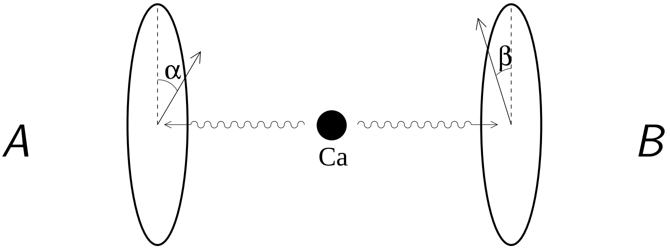

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$$B(\beta) = \begin{cases} 1 & \text{if } B\text{'s photon shows polarization } \beta \in [0,\pi) \\ 0 & \text{if } B\text{'s photon shows polarization } \perp \beta \end{cases} .$$

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and this is corroborated* by experiment.

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Bell's inequality is a quadrangle inequality in the space of events.

Bell's inequality

Theorem

For any four $\{0,1\}$*-valued functions* A_1, A_2, B_1, B_2 *on* $(\Omega, \mathbb{P})$*:*

$$\mathbb{P}[A_1 = B_1] \leq \mathbb{P}[A_1 = B_2] + \mathbb{P}[B_2 = A_2] + \mathbb{P}[A_2 = B_1] .$$

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Proof.

Pointwise! For any $\lambda \in \Omega$ a round-trip around the square

$$\begin{array}{ccc} A_1(\lambda) & - & B_1(\lambda) \\ | & & | \\ B_2(\lambda) & - & A_2(\lambda) \end{array}$$

meets an even number of equality signs. □

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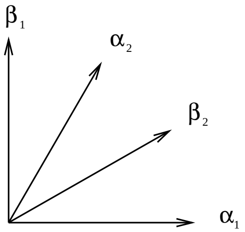

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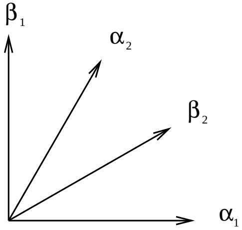

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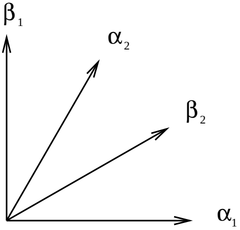

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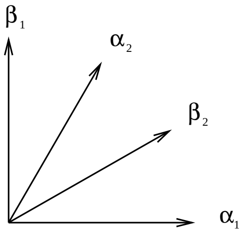

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But of course

$$1 > \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \; .$$

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Where does the $\sin^2$ term come from?

The quantum calculation goes like this:

$$A(\alpha) = P(\alpha) \otimes I \ , \qquad B(\beta) = I \otimes P(\beta) \ ,$$

$$\begin{aligned}
\mathbb{P}[A(\alpha) = B(\beta)] &= 2\mathbb{P}_\psi[A(\alpha) = B(\beta) = 1] \\
&= \langle \psi, P(\alpha) \otimes P(\beta)\psi \rangle \\
&= \left| \left\langle \frac{1}{\sqrt{2}}(0, 1, -1, 0), \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \otimes \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \right\rangle \right|^2 \\
&= (\cos\alpha \sin\beta - \sin\alpha \cos\beta)^2 \\
&= \sin^2(\alpha - \beta) \ .
\end{aligned}$$

The Bell Game

A \ B	red	black
red	11010011000110100100011 00110010011100101100101 00110011011010010110101 0110101110010....... a_{11}	011101100111101001100001 110110001011101000111101 011010011001010111010010 110101010110011...... a_{12}
black	11010011010001101011110 01110010100101110101101 110001011..... a_{21}	110100011011001101001101 110000100101110000101001 100001000100101100101001 000101110000100.... a_{22}

Rules:

The following protocol is repeated many times:

- Alice and Bob both get a card (red or black). No spying! No talking!
- Dice are thrown
- Alice and Bob simultaneously say “”yes” or “no” (1 or 0).
- The cards are laid out. In the square of the board, determined by the cards, a 1 is written if Alice and Bob gave the same answer, a 0 otherwise.

Alice and Bob win the game if eventually they accumulate more ones in the (red,red)-square than in the other three together.

Theorem

Alice and Bob connot win the game "by classical means".

Proof.

The only thing they can do, is agree on some, possibly random strategy. A strategy is a specification what each of them will say if he/she gets a red/black card.

However, none of these strategies wins the game, by the same argument as above (even number of equaility signs).

Randomness does not help, since Bell's inequality is linear.

□

But ....

if Alice and Bob buy a set of polarizers,

and replace the dice by calcium atoms,

which they make emit a photon pair in each round of the game,

when they rotate their polarizers according to the color of their cards,

and answer the question: “does my photon get through?”,

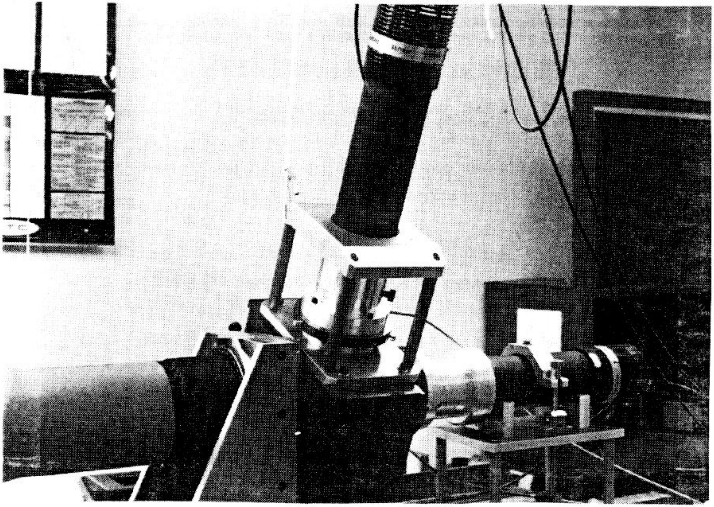
THEN THEY WIN!

Assumptions

The following assumptions suffice to derive Bell's inequality for the game.

- Locality: Alice and Bob don't look into each other's cards.
- Realism: For every $\lambda \in \Omega$ there is a full strategy A_1, A_2, B_1, B_2.
- Independence: There exists a deck of cards, statistically independent of each other and of λ.

The Orsay Experiment



The Orsay experiment

From a calcium source pairs of photons were produced.
Photons in the right and left wing of the setup were identified as belonging to the same pair by measuring their synchronicity.
In the 1982 experiment the polarization directions were randomly chosen *during the flight of the photons*, so that the measuring direction in one wing could not influence the outcome in the other.

In later years the experiment was done with protons, kaons, neutrons, cold atoms and atom-photon pairs. (Electrons are on their way.)
All were significant by many standard deviations.

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A fifth position is *logically possible* (Gill):

- Quantum Mechanics is right, but the game cannot be won.

Questions to 't Hooft:

Is your theory going to win the Bell game?

What position would you choose in the light of Bell's four possibilities?