Quantales and Spectra

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In these three independent but interrelated informal talks we consider ways in which the deductive context in which we are operating allows mathematical ideas to develop. The aim that we have is to examine the ways in which physical theories naturally involve expression within the non-commutative constructive context obtained by considering quantales. In particular, this provides a context in which the Gelfand representation may be extended to non-commutative C*-algebras and in which physical theories can consider quantum observables in a way generalising the classical case.

In the first talk, we shall focus on properties of the real and complex numbers, indicating the ways in which these lead to deductive contexts within which spectral theory may be developed, in particular to the concept of the spectrum of a non-commutative C*-algebra. In the second talk, we shall examine the constructs of non-commutative topology, in particular showing that the spectrum is a compact, completely regular quantal space. In the third talk, we examine the non-commutative theory of Penrose tilings and the insights into étale groupoids to which this leads.