**GALOIS THEORY 2015/2016 EXERCISE SHEET 11**

(1) Let $V_4$ denote the subgroup of $S_4$ generated by the elements of cycle type $(2,2)$. By considering the action of $S_4$ on the set of elements of cycle type $(2,2)$ by conjugation, show that $V_4$ is a normal subgroup of $S_4$, and the quotient $S_4/V_4$ is isomorphic to $S_3$.

(2) Let $\mathbb{C}(T_1, T_2, T_3, T_4)$ be the field of fractions of a polynomial algebra in four variables.

   (a) Let $K$ denote the fixed field $\mathbb{C}(T_1, T_2, T_3, T_4)^{S_4}$ with respect to the natural action of $S_4$ on the set $\{T_1, T_2, T_3, T_4\}$. Show that the polynomial
   $$F(X) = (X - T_1T_2 - T_3T_4)(X - T_1T_3 - T_2T_4)(X - T_1T_4 - T_2T_3)$$
   lies in $K[X]$.

   (b) Let $L$ be the splitting field of $F$, thought of as a subextension of $\mathbb{C}(T_1, T_2, T_3, T_4)|K$. Show that the Galois group of $L|K$ is $S_3$, and that the Galois group of $\mathbb{C}(T_1, T_2, T_3, T_4)|L$ is $V_4$.

   (c) Use this to write $\mathbb{C}(T_1, T_2, T_3, T_4)|K$ as a radical extension.

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