## GALOIS THEORY 2015/2016 EXERCISE SHEET 13

Questions marked with an asterisk are optional. Think of them as a christmas present.

- (1) Give an example, with justification, of a polynomial in  $\mathbb{Q}[x]$  whose Galois group is  $D_6$ .
- (2) Give an example, with justification, of a polynomial in  $\mathbb{Q}[x]$  whose Galois group is  $S_7$ .

1

(3) \* Show that any subgroup of  $S_n$  which acts doubly transitively on  $\{1, \ldots, n\}$  and contains a transposition is equal to  $S_n$ . Deduce that for any *n* there is an extension of  $\mathbb{Q}$  which is Galois with group  $S_n$ .

Comments, corrections, questions etc to netandogra@gmail.com.