GALOIS THEORY 2015/2016 EXERCISE SHEET 3

(1) Let $L|K$ be a finite extension, and α a nonzero element of L. Let $f(X)$ be the minimal polynomial of α.
   (a) For β a nonzero element of K, what is the minimal polynomial of $\beta\alpha$?
   (b) For β as above, what is the minimal polynomial of $\alpha - \beta$?
   (c) What is the minimal polynomial of $1/\alpha$?
(2) Let $K = \mathbb{C}(A, B, C)$ (where A, B, C are formal variables). Let M be the splitting field of the polynomial
$$f(X) = X^3 + AX^2 + BX + C.$$
   (a) Show that f is irreducible.
   (b) What is the degree of $M|K$?
   (c) Let α, β and γ be the roots of $f(X)$ in M. Show that $M = \mathbb{C}(\alpha, \beta, \gamma)$.
(3) Find the degree of a splitting field of the polynomial $f(X) = X^{11} - T$ over $\mathbb{F}_3(T)$.
(4) Let $K = \mathbb{F}_p(T)$, and let a be an element of K. Let $f(X) = X^p - X + a$.
   (a) Show that, for all b in $\mathbb{F}_p$,
$$f(X) = f(X + b)$$
   (b) Suppose that f does not have a root in K. Let L be a splitting field of f. What is the degree of $L|K$? What is the size of $\mathrm{Hom}_K(L, L)$?

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