# GALOIS THEORY 2015/2016 EXERCISE SHEET 9

Questions marked with an asterisk are optional.

(1) Let $L|K$ be a Galois extension, and $F$ a subextension. Show that if the Galois group of $L|K$ is abelian, then $F|K$ is Galois.

(2) Let $L$ denote the splitting field of $f(X) = X^4 - 3$ over $\mathbb{Q}$. What is the Galois group of $L|\mathbb{Q}$? Find all normal subgroups of the Galois group, and use this to find all subextensions $K$ with $K|\mathbb{Q}$ Galois.

(3) Let $K$ be a field, and let $f(X)$ be a separable polynomial of degree 3 in $K[X]$. Let $L|K$ be the splitting field of $f$. Suppose that every subextension $F$ of $L|K$ is Galois over $K$. What are the possible degrees of the irreducible factors of $f$? For each collection of possible degrees, give an example of such a field $K$ and polynomial $f$.

(4) * Let $K$ be a field, and let $f(X)$ be a separable polynomial of degree 7 in $K[X]$. Let $L|K$ be the splitting field of $f$. Suppose that every subextension $F$ of $L|K$ is Galois over $K$. What are the possible degrees of the irreducible factors of $f$?

Comments, corrections, questions etc to netandogra@gmail.com.