Algebraic Topology, Exercise sheet No 1

Prove the following facts:

1. A closed subspace of a compact space is compact.
2. Every compact subspace of a Hausdorff space is closed.
3. If X is compact and $f : X \to Y$ is continuous then $f(X)$ is compact.
4. If X is compact and $f : X \to \mathbb{R}$ is continuous then f is bounded.
5. If X is compact, Y is Hausdorff and $f : X \to Y$ is continuous and bijective then f is a homeomorphism.
6. If X is compact Hausdorff then X has no strictly weaker Hausdorff topology.
7. A Hausdorff space X is compact iff $\bigcap_i A_i \neq \emptyset$ whenever $\{A_i\}$ is a family of closed subsets such that all finite intersections are non-empty.
8. The interval $I = [0, 1]$ is connected.
9. A path connected space is connected.
10. A homotopy equivalence $f : X \to Y$ induces an isomorphism $\pi_0(f) : \pi_0(X) \to \pi_0(Y)$ of the respective sets of path components.
11. A contractible space is path connected.
12. If X is contractible then $\pi_1(X, x_0) = \{e\}$ for all $x_0 \in X$.