# Algebraic Topology, Exercise sheet No 5

1. Let $p : (\tilde{X}, \tilde{x}_0) \to (X, x_0)$ be a covering map where $X, \tilde{X}$ are path-connected. Prove that $X \to \mathbb{N} \cup \{0, \infty\}$, $x \mapsto |p^{-1}(x)|$ is constant and equal to $[\pi_1(X, x_0) : p_*(\pi_1(\tilde{X}, \tilde{x}_0))]$.

2. Consider the subspace $X \subset \mathbb{R}^2$ given as the union over $n = 1, 2, \dots$ of circles of radius $1/n$ and origin $(1/n, 0)$. Show that $X$ is not semilocally simply connected.

3. Let $G$ be a group acting on the space $X$ in a properly discontinuous way. (I.e., every $x \in X$ has a neighborhood $U$ such that the sets $gU$, $g \in G$ are pairwise disjoint.) Prove:

   (a) The quotient map $X \to X/G$ is a normal covering map.

   (b) If $X$ is path-connected then $G$ is the group of covering maps of $p : X \to X/G$.

   (c) If $X$ is path-connected and locally path-connected then $G$ is isomorphic to $\pi_1(X/G)/p_*(\pi_1(X))$.