Algebraic Topology, Exercise sheet No 7

1. Let G be a topological group. (I.e., G is a group whose underlying set has a topology, such that the maps $g \mapsto g^{-1}$ and $g \times h \mapsto gh$ are continuous.)
   (a) Let f, g be loops in G at e. Define a loop $f \circ g$ by pointwise composition: $(f \circ g)(s) = f(s)g(s)$. Show that this defines a multiplication on $\pi_1(G, e)$ whose unit is the constant path $[c_e]$.
   (b) Use the algebraic lemma from the lecture to deduce that $\pi_1(G, e)$ is abelian.
   (c) Conclude that $\pi_1(G, g)$ is abelian for any $g \in G$.
2. Let X be the union of the unit sphere in 3-space with the straight line segment from the north pole to the south pole. Find $\pi_1(X)$.
3. Let X be the union of the unit sphere in 3-space with the unit disk in the $x - y$ plane. Find $\pi_1(X)$.
4. Describe all covering spaces of the 2-torus, up to equivalence.
5. $\star$ Find all covering spaces with three sheets of the figure eight.