Corrections to D. J. H. Garling's
'A course in Mathematical Analysis', Vol. 1

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I thank Fons van der Plas for contributing about 25 of the following corrections!

- p.5, l.-14: I strongly dislike the choice of having $\subset$ denote **proper** inclusion, however "logical" that may seem.

  Very few authors do this. (Right now I can only think of Gaal's 'Point set topology'.) Most authors let $\subset$ denote not-necessarily-proper inclusion, and deviating from this majority practice can only cause confusion.

  Anyway, a natural workaround is to avoid $\subset$ altogether and only use $\subseteq$ and $\subsetneq$ (as also T. Tao does in his two-volume analysis course).

- p.7, l.-5: I don't like the notation $C(B)$ for the complement $\mathbb{R}\backslash B$ and think one should stick to B^c.

- p.15, Exercise 1.4.1: In (c) and (d): Replace C, D by E, F, respectively.

- p.65, l.13: Garbled formula $\psi(-m)l) = \psi(-m)\psi(l)$. Probably there just is a bracket missing: $\psi((-m)l) = \psi(-m)\psi(l)$

- p.65, l.-10: Replace $\psi(jn')$ by $\psi(j'n)$.

- p.67, l.-3: 'upper bound for U' should be 'upper bound for L'.

- p.70, l.7: Here is the definition $D(x) = \{r \in \mathbb{Q} \mid r < x\}$. One may question that this makes sense. $\mathbb{R}$, as introduced in Theorem 2.9.5, is the set of Dedekind cuts. By definition, a cut just is a set of rationals satisfying some axioms. Thus for $x \in \mathbb{R}$, x and $D(x)$ are the same thing, and the notational distinction is hard to sell. (Of course one may say that $D(x)$ is a set of rationals, wheras writing $x \in \mathbb{R}$ one forgets about this fact and considers x as a structureless 'point' in $\mathbb{R}$. But still…)

- p.71, l.7: The middle term '$\{r \in \mathbb{Q} \mid j(r) < -x\}$' doesn't really make sense since $-x$ is not yet defined. (In fact, this line *is* the definition of $-x$.) At best, it serves as motivation for the third term.

- p.72, l.16: Replace '$D^+x.D^+y$' by '$D^+(x) \cdot D^+(y)$'.

- p.73, l.9: Replace '$x \in \mathbb{Q}$' by '$r \in \mathbb{Q}$'.

- p.74, l.-12: Replace $(y - s^n)/(2^n - 1)y$ by $\frac{y-s^n}{(2^n-1)s^n}$.

- p.74: The proof of Theorem 2.10.11 uses Lemma 2.10.12, where the hypothesis $0 < \varepsilon < 1$ is made. This must be taken into account when choosing θ, η:

$$0 < \eta < \min(\frac{y - s^n}{(2^n - 1)s^n}, 1), \qquad 0 < \theta < \min(\frac{s^n - y}{ns^n}, 1).$$

- p.84, l.10: Replace '$l > 0$' by '$\varepsilon > 0$'.

- p.84, l.11: Replace '$1/n < 1/n_0$' by '$1/n \leq 1/n_0$'.

- p.87, Item (v): It is simpler to just write $a_n b_n - ab = a_n(b_n - b) + b(a_n - a)$. The terms on the r.h.s. converge to zero by (ii) and (iv), respectively. Thus the sum goes to zero by (iii).

- p.87, Item (vi): This proof would be simpler if one noted earlier, preferably in the context of Prop. 3.2.3, that boundedness of $\{a_n\}_{n \geq n_0}$ for some n_0 implies boundedness of $\{a_n\}$.

- p.90, problem 3.2.4: add 'as $n \to \infty$'. (Omitting the variable is sloppy.)

- p.91 l.21: Replace $\{x\} = x - [x]$ with $\{x\} = x - \lfloor x \rfloor$.

- p.92, l.-3: Closing bracket missing in $k(j(x)$

- p.93, l.-12: $C/\ \sim$ is very ugly (too much white space). Inserting one negative space (\!) gives $C/\!\sim$, which is a bit better. I actually prefer $C/\!\!\sim$ (two negative spaces). There likely are other instances of this.

- p.96 l.18: Replace $a_M > r$ with $a_m > r$.

- p.108, l.7: Replace w_j by z_j.

- p.108, l.-8 nd l.-6: $\mathbb{N}^+$ has never been defined. The $\sum_{j=0}^{\infty}$ in l.-7 suggests that the author means $\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$, but then the statements $s_{2n} = 0$ and $s_{2n+1} = 1$ are false and should be replaced by $s_{2n} = 1$ and $s_{2n+1} = 0$.

- p.109, l.-13: Replace $\sum_{j=1}^{n}$ by $\sum_{j=0}^{n}$.

- p.109, Coro. 4.2.2: It is true that in order to conclude convergence of $\sum_{j=0}^{\infty} c_j$ from convergence of $\sum_{j=0}^{\infty} a_j$ it suffices to have $0 \leq c_j \leq a_j$ for $j \geq j_0$. But in order to conclude that $\sum_{j=0}^{\infty} c_j \leq \sum_{j=0}^{\infty} a_j$, we need $0 \leq c_j \leq a_j$ to hold for **all** $j \geq 0$, i.e. $j_0 = 0$!!

- p.111, l.6: at the end of the line, there must be $(a_{j_0} r^{-j_0}) r^j$ (i.e. minus sign is missing).

- p.113, l.2: See Garling's erratum, which erroneously lists this under p.123.

- p.114, l.1: 'Exercise 3.1.9' does not exist. Presumably this should be Exercise 3.2.11.

- p.115, l.15: Replace $\sum_{j=0}^{\infty} |a_j|$ with $\sum_{j=0}^{\infty} |z_j|$.

- p.115: To prove of the first sentence in the proof of Prop. 4.3.1, we need not only $|x_j| \leq |z_j|$ and $|y_j| \leq |z_j|$, but also $|z_j| \leq |x_j| + |y_j|$.

- p.116, l.-13: Here a_{2n} must be a_{2n+2}, thus:

$$s_{2n+2} = s_{2n} - (a_{2n+1} - a_{2n+2}) \leq s_{2n}.$$

- p.116, l.-9: The a_{2n+1} must be a_{2n+2}.

- p.117, l.8-9: $s_n = \sum_{j=0}^{n} a_j z_j$ is used before it is defined.
- p.117, l.16: Abel's formula and its uses are better known as 'partial summation'.
- p.117, l.-10 and also l.-9: superfluous left bracket in $|(a_j - a_{j+1}|$
- p.117, l.-2: Since $\{a_j\}$ is decreasing, thus $a_j - a_{j-1} \leq 0$, the correct formula is:

$$\sum_{j=1}^{\infty} |a_j - a_{j-1}| = \sum_{j=1}^{\infty} (a_{j-1} - a_j) = a_0.$$

- p.120, l.–4: There must be $k = \sup\{\sigma(j) : \mathbf{0} \leq j \leq n\}$.
- p.128, item 5 of the list: uniform convergence has not yet been defined!
- p.128, item 6 of the list: Replace $e^z = e(z)$ with $e^z = \exp(z)$.
- p.131, l-10: *"It is an easy exercise to show that every interval is of one of these forms."* Add: 'or equal to $\mathbb{R}$'.
- p.133, l.11: Replace '$a \in \overline{A} = A$' by '$b \in \overline{A} = A$'.
- p.138, Exercise 5.3.1: The claimed statement is **not** equivalent to connectedness of A! If U_1, U_2 are as in the definition of connectedness (p.137) and one puts $F_i = U_i^c$ then connectedness of A is equivalent to the statement that F_1, F_2 closed, $F_1 \cup F_2 = \mathbb{R}$, $A \cap F_1 \cap F_2 = \emptyset \Rightarrow A \subseteq F_1 \vee A \subseteq F_2$. The condition $A \cap F_1 \cap F_2 = \emptyset$, without which there is no connection between A and F_1, F_2, is missing in Garling.
- p.138, Exercise 5.3.3: In the fourth line, replace '$b \in c_0, d_0$' by '$b \in [c_0, d_0]$'.
- p.139, l.-6: Replace 'First, $c > a$' by 'First, $s > a$'.
- p.140, l.11: After 'Suppose... bounded', there should be 'and let $\mathcal{U}$ be an open cover of B.'.
- p.140, l.19: The first closing bracket should be a brace.
- p.142, l.-12: Delete the assumption '$a \in \mathbb{R}$' that is not referred to in the rest of the statement of the Proposition.
- p.149, l.10: Delete 'and that $l \in \mathbb{R}$'. (No l appears in (ii) and (iii), whereas in (i) we have 'there exists l'.)
- p.149, l.15: Replace $N_\delta^*(b)$ by $N_\delta^*(b) \cap A$.
- p.157, l.2: Typo of *"... there exist ..."*.
- p.158, l.15: Replace 'Theorem 6.3.4' by 'Proposition 6.3.4'.
- p.159, l.12: '$= =$'.
- p.159, l.15: Replace 'e is not continuous' by 'f is not continuous'.
- p.161, l.3: Replace $a \in A$ with $x \in A$.
- p.163, in Corollary 6.4.6: Replace $k \in \mathbb{N}$ with $n \in \mathbb{N}$.

- p.166, l.5: Replace $\sum_{j=m+1}^{n} |f(s)| < \epsilon$ with $\sum_{j=m+1}^{n} |f_j(s)| < \epsilon$.
- p.167, l.11: Replace $\sum_{j=0}^{\infty} a_j z_j$ with $\sum_{j=0}^{\infty} f_j z_j$.
- p.167: Theorem 6.6.1 is literally identical to Theorem 6.5.1. At least 'real-valued' should be changed to 'complex-valued', but probably also the functions are defined on subsets of $\mathbb{C}$?
- p.168, l.6: The '$r - s$' in the denominator should be '$s - r$'.
- p.172, l.7: Replace $\sum_{j=0}^{\infty} a_j z_j$ with $\sum_{j=0}^{\infty} f_j z_j$.
- p.173, l.-6: η is not defined. *"for some $\eta > 0$"* should be added after $(a - \eta, a + \eta) \subseteq I$.
- p.177, l.12: Superfluous bracket in $f(a + h) = b + (s(h)$.
- p.178, l.2: The word *"but"* seems misplaced, and a new sentence should start after $f'(a) \geq 0$.
- p.179, l.9-14: The enumeration here uses Arabic numbering (1., 2., 3., 4.), but in the remainder of the proof, these items are referred to using (i), (ii), (iii), (iv).
- p.180, Exercise 7.1.3: Replace 'f is differentiable at 0' by 'g is differentiable at 0'.
- p.186, Theorem 7.3.2 (Rolle): I dislike this proof, which obscures the matter by unnecessarily bringing in monotonicity. A cleaner argument is as follows:

  Let $A := f(a) = f(b)$. If f is constant, i.e. $f(x) = A$ for all $x \in (a, b)$, then $f'(x) = 0$ for all $x \in (a, b)$, and we are done. Assume f is non-constant. Then either $\sup_{x\in[a,b]} f(x) > A$ or $\inf_{x\in[a,b]} f(x) < A$ (or both). Assume the first holds. By Theorem 6.3.6, $S = \sup_{x\in[a,b]} f(x) < \infty$, and there is an $x \in [a, b]$ such that $f(x) = S$. Since $S > A$, we have $a \neq x \neq b$, thus $x \in (a, b)$. Thus the global maximum at x is a local maximum, thus $f'(x) = 0$ by Proposition 7.3.1. QED
- p.187, Theorem 7.3.5 (Mean value theorem): Garling omits the proof of $h_\lambda(a) = h_\lambda(b)$, which actually is quite tedious for his choice of h_λ. If instead we define $h_\lambda(x) = f(x) - \lambda(x - a)$ the computations are quite trivial: We trivially have $h_\lambda(a) = f(a)$. And

$$h_\lambda(b) = f(b) - \frac{f(b) - f(a)}{b - a}(b - a) = f(b) - (f(b) - f(a)) = f(a) = h_\lambda(a).$$

- p.192, last line: There must be $x \searrow 0$.
- p.193, l.-3: Replace $\sin' x = -\cos x$ with $\sin' x = \cos x$.
- p.194, l.6: A global factor w^2 has gone missing.
- p.194, lines 8 and 9: The factors $|w|^2$ are missing.
- p.194, l.-7: There must be $+\sin x$ instead of $-\sin x$ (as in the correct preceding equation).
- p.196, l.-1: *"..., respectively."* should be added.
- p.205, -9: Replace 'If $0 \leq x < 1$' by 'If $-\frac{1}{2} < x < 1$'. (If $-\frac{1}{2} < x < 0$ then $|x| < \frac{1}{2}$ and $|1 + \theta x| > \frac{1}{2}$, thus $\left|\frac{x}{1+\theta x}\right| < 1$ still holds.)
- p.212, l.-3: Replace $\sum_{j=1}^{k} M_j \chi_j$ with $\sum_{j=1}^{k} M(I_j)\chi_j$.
- p.213, l.6: Similarly, replace $\sum_{j=1}^{k} m_j \chi_j$ with $\sum_{j=1}^{k} m(I_j)\chi_j$.

- p.214, l.14: Replace $M_p l(J_p) l(J_p)$ with $M(J_p) l(J_p)$.
- p.216, l.-3: Add "$\in \mathbb{N}$" after *"Choose N"*.
- p.216, l.-1: Replace M_j and m_j with $M(I_j)$ and $m(I_j)$, respectively.
- p.216, l.-1: Superfluous closing brace.
- p.217, l.7: Missing closing bracket after $(f(b) - f(a)$.
- p.219, l.12: $G \cup B$ should be a partition of the interval *indices* (in $\mathbb{N}$), but G and B are incorrectly defined as the interval *boundaries* (in $\mathbb{R}$).
- p.232 in Theorem 8.7.3: Replace *"k times differentiable"* with *"n times differentiable"*.