Operator Algebras 2014. Additional exercise

12.03.2014

0.1 Exercise Let V be a vector space over $\mathbb{C}$. A sesquilinear form is a map $\phi : V \times V \to \mathbb{C}$ such that for all $a, b, c \in V, \lambda \in \mathbb{C}$:

$$\phi(\lambda a + b, c) = \lambda\phi(a, c) + \phi(b, c), \qquad \phi(a, \lambda b + c) = \overline{\lambda}\phi(a, b) + \phi(a, c).$$

A sesquilinear form ϕ is called hermitian if $\phi(b, a) = \overline{\phi(a, b)}\ \forall a, b \in V$.

Prove: A sesquilinear form ϕ is hermitian if and only if $\phi(a, a) \in \mathbb{R}$ for all $a \in V$.

0.2 Exercise Give reasonably explicit proofs of these claims made in Murphy, pp.93-94 leading up to Theorem 3.4.1:

(i) N_τ is a closed left ideal.

(ii) The inner product in the last formula on p.93 is well-defined.

(iii) On p.94, line 4, $\varphi(a)$ is well defined.

(iv) The map $a \mapsto \varphi_\tau(a)$ is a $*$-homomorphism.