PD Dr. Steffen Sagave
Summer term 2015

Seminar S4D4
"Homotopical Algebra"

Tuesday 12-14, Room 0.006

In this seminar we study *model categories*, an axiomatic framework for homotopy theory introduced by Quillen [Qui67]. By definition, a model category is a category together with three preferred classes of morphisms that satisfy a list of axioms. Topological spaces, simplicial sets, and chain complexes of modules over a ring are examples for categories which can be equipped with the structure of a model category.

Our main references will be the survey article [DS95] by Dwyer and Spalinski and Hovey's book [Hov99].

Schedule

Talk 1, 07.04.2015
Axioms, homotopy relation, and the model structure on categories ([DS95, §3 and §4], [Hov99, §1], and [Rez] for the example)

Talk 2, 14.04.2015
Chain complexes of modules ([DS95, §7] and [Hov99, §2.3])

Talk 3, 21.04.2015
Quillen functors and the homotopy category ([DS95, §5 and §9] and [Hov99, §1.2 and §1.3])

Talk 4, 28.04.2015
Topological spaces with the Quillen model structure ([DS95, §8] and [Hov99, §2.4])

Talk 5, 05.05.2015
Topological spaces with the Strøm model structure ([Str72, Col06a, Col06b])

Talk 6, 12.05.2015
Simplicial sets I ([Hov99, §3] and [GJ99])

Talk 7, 19.05.2015
Simplicial sets II ([Hov99, §3] and [GJ99])

Talk 8, 02.06.2015
Model categories of algebras ([SS00])

Talk 9, 16.06.2015
Sequential spectra I ([BF78] and [Bou01, §9])

Talk 10, 23.06.2015
Sequential spectra II ([BF78] and [Bou01, §9])

Talk 11, 07.07.2015
The smash product of orthogonal spectra ([MMSS01])

Talk 12, 14.07.2015
Model structures on orthogonal spectra ([MMSS01])

References

[BF78] A. K. Bousfield and E. M. Friedlander. Homotopy theory of Γ-spaces, spectra, and bisimplicial sets. In *Geometric applications of homotopy theory (Proc. Conf., Evanston, Ill., 1977), II*, volume 658 of *Lecture Notes in Math.*, pages 80–130. Springer, Berlin, 1978. DOI:10.1007/BFb0068711.

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[Col06a] Michael Cole. Many homotopy categories are homotopy categories. *Topology Appl.*, 153(7):1084–1099, 2006. DOI:10.1016/j.topol.2005.02.006.

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[DS95] W. G. Dwyer and J. Spaliński. Homotopy theories and model categories. In *Handbook of algebraic topology*, pages 73–126. North-Holland, Amsterdam, 1995. DOI:10.1016/B978-044481779-2/50003-1.

[GJ99] Paul G. Goerss and John F. Jardine. *Simplicial homotopy theory*, volume 174 of *Progress in Mathematics*. Birkhäuser Verlag, Basel, 1999.

[Hov99] Mark Hovey. *Model categories*, volume 63 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 1999.

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[Qui67] Daniel G. Quillen. *Homotopical algebra*. Lecture Notes in Mathematics, No. 43. Springer-Verlag, Berlin, 1967. DOI:10.1007/BFb0097438.

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[SS00] Stefan Schwede and Brooke E. Shipley. Algebras and modules in monoidal model categories. *Proc. London Math. Soc. (3)*, 80(2):491–511, 2000. DOI:10.1112/S002461150001220X.

[Str72] Arne Strøm. The homotopy category is a homotopy category. *Arch. Math. (Basel)*, 23:435–441, 1972. DOI:10.1007/BF01304912.