Deel Toets 1 2005

Opgave 4

1. (a) $f(z) = z^3 + i$. $f(x+iy) = (x+iy)^3 + i = x^3 + 3x^2yi - 3xy^2 - y^3i + i = (x^3 - 3xy^2) + (1 + 3x^2y - y^3)i$. Neem dus $u(x,y) = x^3 - 3xy^2$ en $v(x,y) = 1 + 3x^2y - y^3$.

   (b) $f(z) = \sin(z^3)$. Merk op (blz 110) $\sin(x+iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)$. Verder geldt $(x+iy)^3 = (x^3 - 3xy^2) + (3x^2y - y^3)i$ (zie vorige onderdeel). Dus we nemen $u(x,y) = \sin(x^3 - 3xy^2)\cosh(3x^2y - y^3)$ en $v(x,y) = \cos(x^3 - 3xy^2)\sinh(3x^2y - y^3)$

   (c) $f(z) = \frac{z^2}{1-z}$. $\frac{(x+iy)^2}{1-(x-iy)} = \frac{x^2-y^2+2xyi}{1-x-iy}\frac{1-x+iy}{1-x+iy} = \frac{(1-x)(x^2-y^2)-2xy^2}{(1-x)^2+y^2} + \frac{(x^2y-y^4+2xy(1-x))i}{(1-x)^2+y^2}$. Dus $u(x,y) = \frac{(1-x)(x^2-y^2)-2xy^2}{(1-x)^2+y^2}$ en $v(x,y) = \frac{(x^2y-y^4+2xy(1-x))i}{(1-x)^2+y^2}$.

2. (a) $\frac{\partial u(x,y)}{\partial x} = 3x^2 - 3y^2$. $\frac{partial v(x,y)}{\partial y} = 3x^2 - 3y^2$. $\frac{\partial u(x,y)}{\partial y} = -6xy$. $\frac{partial v(x,y)}{\partial x} = 6xy$.

   (b) $\frac{\partial u(x,y)}{\partial x} = \cos(x)\sinh(y)$. $\frac{partial v(x,y)}{\partial y} = \cos(x)\sinh(y)$. $\frac{\partial u(x,y)}{\partial y} = \sin(x)\sinh(y)$. $\frac{partial v(x,y)}{\partial x} = -\sin(x)\sinh(y)$.