A three dimensional UFD cancellation counterexample

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June 1, 2007

Central question: How to distinguish two rings? Central question: How to distinguish two rings? Or: how to distinguish two varieties?

Cancelation problem: $A \mathbb{C}$ -algebra. $A[T] \cong \mathbb{C}^{[n]} \xrightarrow{?} A \cong \mathbb{C}^{[n-1]}.$ Geometric formulation: $V \times \mathbb{C} \cong \mathbb{C}^n \xrightarrow{?} V \cong \mathbb{C}^{n-1}$

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Examples: Let $a_1, \ldots, a_n \in \mathbb{C}[X_1, \ldots, X_n]$. Then

$$a_1\frac{\partial}{\partial X_1}+a_2\frac{\partial}{\partial X_2}+\ldots+a_n\frac{\partial}{\partial X_n}$$

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Not locally nilpotent: $X \frac{\partial}{\partial X}$ on $\mathbb{C}[X]$.

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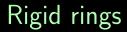
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Known fact: $\varphi \in Aut_{\mathbb{C}}(A)$, then $\varphi(ML(A)) = ML(A)$.



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Rigid rings

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So let us focus on getting such a rigid ring R!

polynomials"

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Then all f, g, h are constant. (Number Theory: Only proved for a = b = c!) We will choose $R = \mathbb{C}[X, Y, Z]/(X^a + Y^b + Z^c)...$ Let A be \mathbb{C} -algebra domain, and $D \in LND(A), D \neq 0$.

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Example: $\frac{\partial}{\partial X}$ on $\mathbb{Q}[X]$. One can even go more crazy: let $K = Q(\tilde{A}^D)$. Extend D to K[S]. Or even to $\bar{K}[S]$. Example: extend $\frac{\partial}{\partial X}$ on $\mathbb{Z}[X]$, to $\mathbb{Q}[X]$, and $\mathbb{C}[X]$.

Definition: $R := \mathbb{C}[X, Y, Z]/(X^a + Y^b + Z^c) = \mathbb{C}[x, y, z]$ where $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \le 1$.

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 $R \mathbb{C}$ -algebra $\longrightarrow \tilde{R} := R[q^{-1}] = \tilde{R}^D[s] \longrightarrow K[s]$ K[s] $\frac{\partial}{\partial s}$ $\frac{\partial}{\partial c}$ $\frac{\partial}{\partial s}$ D $D \neq 0$ p preslice, D(p) = q, D(q) = 0, $s := pq^{-1}$ K fraction field of \tilde{R}^D K algebraic closure of Kx = f(s), y = g(s), z = h(s) $0 = x^{a} + y^{b} + z^{c} = f^{a} + g^{b} + h^{c}$ Corollary implies: $f, g, h \in \overline{K} \cap R = R^D$.

Definition: $R := \mathbb{C}[X, Y, Z]/(X^a + Y^b + Z^c) = \mathbb{C}[x, y, z]$ where $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq 1$.

 $R \mathbb{C}$ -algebra $\longrightarrow \tilde{R} := R[q^{-1}] = \tilde{R}^D[s] \longrightarrow K[s]$ K[s] $\frac{\partial}{\partial s}$ $\frac{\partial}{\partial s}$ $\frac{\partial}{\partial s}$ D $D \neq 0$ p preslice, D(p) = q, D(q) = 0, $s := pq^{-1}$ K fraction field of \tilde{R}^D K algebraic closure of Kx = f(s), y = g(s), z = h(s) $0 = x^{a} + y^{b} + z^{c} = f^{a} + g^{b} + h^{c}$ Corollary implies: $f, g, h \in \overline{K} \cap R = R^D$. So $LND(R) = \{0\}$!!

Just one more thing to say:

THANK YOU