ERRATUM and ADDENDUM

to the paper

Families of Motives and the Mumford–Tate Conjecture

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In Section 4.3 of the paper, there are some assertions that can be strengthened. As in that section, let S be a nonsingular geometrically connected variety over a base field K of characteristic 0, and let $\mathbf{M}$ be a family of motives over S, as in Definition 4.3.3. We fix an algebraic closure $\overline{K}$ of K and a prime number ℓ. Then we have an ℓ-adic (geometric) local system $\overline{\mathcal{H}}_\ell = \overline{\mathcal{H}}_\ell(\mathbf{M})$ over $S_{\overline{K}}$ and local systems of algebraic groups $G_\ell(\mathbf{M}/S) \subset \mathrm{GL}(\overline{\mathcal{H}}_\ell)$ (see just before Theorem 4.3.6) and $G^0_{\mathrm{mono},\ell}(\overline{\mathcal{H}}_\ell/S_{\overline{K}}) \subset \mathrm{GL}(\overline{\mathcal{H}}_\ell)$. The fibre of $\mathbf{M}/S$ at a point $s \in S(\overline{K})$ is a motive $\mathbf{M}_s$ over $\overline{K}$, and we denote by $G^0_\ell(\mathbf{M}_s) \subset G^0_\ell(\mathbf{M}/S)_s$ the ℓ-adic algebraic Galois group (see just before Theorem 4.3.8).

The last two sentences of Theorem 4.3.8 should be replaced by:

For every $s \in S(\overline{K})$ we have

$$G^0_{\mathrm{mono},\ell} \cdot G_{\mathrm{mot},\ell}(\mathbf{M}_s) = G_{\mathrm{mot},\ell}(\mathbf{M}/S)_s$$

and

$$G^0_{\mathrm{mono},\ell} \cdot G^0_\ell(\mathbf{M}_s) = G^0_\ell(\mathbf{M}/S)_s\,. \qquad (*)$$

Thus, in the last assertion the two groups G_ℓ that appear (in the printed text) should be replaced by their identity components G^0_ℓ; further, the condition that $G_\ell(\mathbf{M}_s)$ is reductive can be omitted.

The first sentence of Corollary 4.3.9(ii) should be replaced by:

For $s \in S(\overline{K})$ we have

$$s \text{ is Galois generic} \iff G^0_{\mathrm{mono},\ell}(\overline{\mathcal{H}}_\ell/S_{\overline{K}})_s \subset G^0_\ell(\mathbf{M}_s)\,. \qquad (\dagger)$$

So again the assumption that $G_\ell(\mathbf{M}_s)$ is reductive can be omitted.

To justify these improvements of the stated results, we first remark that we may assume that $s \in S(\overline{K})$ comes from a K-rational point, which we denote by s_K. (The group $G^0_\ell(\mathbf{M}_s)$

does not change if we replace K by a finitely generated extension, so we may just enlarge K such as to make s a K-rational point.) On fundamental groups we then get a diagram

$$\begin{tikzcd} 1 \arrow[r] & \pi_1(S_{\overline{K}}) \arrow[r, "i"] & \pi_1(S) \arrow[r] & \mathrm{Gal}(\overline{K}/K) \arrow[l, bend right, "s_{K,*}"'] \arrow[r] & 1 \end{tikzcd}$$

(For simplicity, let us say that we take s as our base point.) We have a representation $\rho\colon \pi(S) \to \mathrm{GL}(\overline{\mathcal{H}}_{\ell,s})$. The identity component of the Zariski closure of the image of ρ is the group $G^0_\ell(\mathbf{M}/S)_s$. The identity component of the Zariski closure of the image of $\rho \circ i$ is the group $G^0_{\mathrm{mono},\ell}$. The identity component of the Zariski closure of the image of $\rho \circ s_{K,*}$ is the group $G^0_\ell(\mathbf{M}_s)$. Because $\pi_1(S)$ is generated by its subgroup $\pi_1(S_{\overline{K}})$ together with the image of $s_{K,*}$, we obtain $(*)$. To see that "$\Leftarrow$" holds in $(\dagger)$, assume $G^0_{\mathrm{mono},\ell}(\overline{\mathcal{H}}_\ell/S_{\overline{K}})_s \subset G^0_\ell(\mathbf{M}_s)$. By $(*)$ it then follows that $G^0_\ell(\mathbf{M}_s) = G^0_\ell(\mathbf{M}/S)_s$, which just means that s is Galois generic.