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Abstract: The paper[1] describes new achievements in application of parallel high performance computing for the finite element modelling of large scale 3D problems of geomechanics. We show an example of such modelling which was performed for the assessment of stability problems of the uranium ore mining at the deposit Rožná. We also briefly describe a finite element software GEM3/HPC which was developed for such types of modelling.

1 Introduction

This paper describes an application of large scale mathematical modelling for the solution of stability problems of the uranium ore mining at the deposit Rožná. The main aims of this application are the following:

- to assess the possibility of rockbursts during the mining on the new levels under 1000 meters below the surface,
- to compare stress fields which are induced by the use of two different mining methods – underhand and overhand stoping,
- to analyse stress changes during the forthcoming mining in the period 1999 - 2002.

The computations required for this modelling was performed with the aid of the GEM3/HPC software which was developed at the Department of Applied Mathematics of the Institute of Geonics AS CR. GEM3/HPC is a finite element software developed with a special emphasis on high performance computing and 3D applications in geomechanics. This software involves new research results in the numerical analysis concerning

- solution of very large linear systems,
- solution of singular systems arising due to the adopted boundary conditions,
- submodelling with composite finite element grids.

This paper describes the finite element analysis with GEM3/HPC from the point of view of both performance and interpretation of the computed data. The performance data are from running the GEM3/HPC software on IBM RS/6000 3CT computer and parallel IBM SP multicomputer. The conclusions obtained from the finite element analysis show the practical importance of such large scale mathematical modelling. More details can be found in papers [5], [6], [7], [10].

[1]The presented work is supported by the Grant no. 105/99/1229 of the Czech Grant Agency and the Grant COPERNICUS KIT 977006 of the European Commission .

2 Problem description and modelling principles

We shall describe mathematical modelling for the assessment of the stress field changes during the advance of mining at the Rožná deposit which is planned for the period 1999 - 2002. This mining will go under the level of 1000 meters below the surface in the deep uranium mine GEAM Dolní Rožínka and this fact imply several basic questions about the risk of rockburst, suitability of the so far used mining technology, mutual influence of mining in adjacent ore veins etc.. The preparation of a new classification of the deposit from the point of view of rock burst risk is also limiting condition for the state mining authorities to give permission for further mining.

Geological situation

The deposit Rožínka is situated 55 km northwest of Brno (Czech Republic). This deposit is situated in metamorphic sedimentary–effusive rocks of Precambrian age which belong to the eastern branch of Moldanubian (so called Strážek-Moldanubian). The uranium mineralization of hydrothermal origin is situated in zones of veins arising from longitudinal faults with inclination $45° - 70°$ to the west. These zones reach up to 10 km in the longitudinal direction and their thickness is mostly several meters, see Fig. 1. Two of the four ore zones in Rožná deposit are most important: 1st zone extending in bedrock of the deposit and 4th zone as roof boundary of the deposit. The 1st zone consists mostly of one 4 to 15 m thick fault zone with medium up to intense crushing of adjacent rocks. The 4th zone consists of several parallel dislocations reaching a 10 m thickness and filled with medium up to fine crushed adjacent rock. Basic mined minerals are uraninite and coffinite, uranium content in ore is $0.1 - 0.5$ %. The resultant product is the ammonium diuranate $(NH_4)_2U_2O_7$ with the uranium content exceeding 70% produced in an adjacent chemical processing plant.

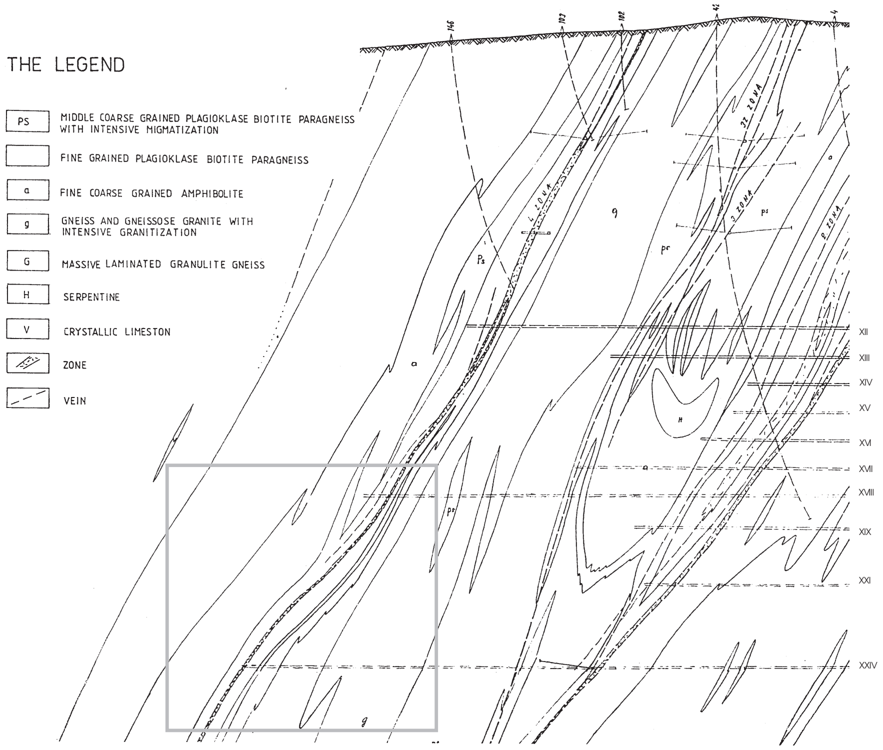

Figure 1: East-West cross-section of the deposit with 4th zone and the modelled area in window located in the left down corner (see [9]).

Mining technology

The mining at the Rožná deposit started in 1957 and in present the deposit is developed by several working shafts and main level crosscuts. The vertical distance of levels is 50 metres. In the levels there are longitudinal crosscuts in the footwall of the zones and short offsets running through the zones. The distance between the offsets is 50 metres. The stope dimensions are both 50 metres in longitudinal and vertical directions.

The total depth of the mine is 1 200 m below the surface. At present the mining will continue under the level 1000 metres. The methods used in underground mining depend on the geometry of the mineralized bodies and on the geomechanical characteristics of ore minerals and surrounding rocks.

The overhand stoping with filling was a prevailing method in the early stages of mining of the Rožná deposit. The underhand stoping method with caving and working under artificial roof is only used at the present time.

Applied mining methods have effect on extent and nature of particular zones of deformation. There is a substantial difference between the overhand stoping with backfill of worked-out area and the underhand slicing with caving. In the case of the underhand slicing immediate roof rocks are losing contact with footwall of original massif after working out of the ore bearing part and with a certain delay behind the worked level destruction of the immediate roof and its caving into gob area occurs. This qualitatively mighty disturbance of the roof has effect on bending and deformation of rocks. In the case of the overhand stoping with backfill the development of deformations depends on quality and type of backfill. Contrary to the underhand slicing a more pronounced disturbance of roof occurs only in process of compacting of backfill after a certain time delay. The coefficient of strengthening of backfill depends here on quality of the backfill work. A 50% efficiency of backfill can be expected.

Mining of mutually close structures happens very often in Rožná deposit. This can cause important impact on the stability of the influenced openings. The disturbance and deformation of rock strata depends on physical and mechanical properties of the interlaying rocks, dip and thickness of the mined underlying structure, sequence of works, mining speed, applied mining method, extent of mining, pillars left in worked-out area.

Domain of interest

The domain in which we will investigate development of stress fields has the dimensions 630x335x330 metres, is located in the depth of 900 metres under the surface and contains a part of the 4th zone with three uranium ore veins. The localization of this domain can be seen from Figure 1, detailed view the adopted coordinate system and its orientation can be seen from Figure 2.

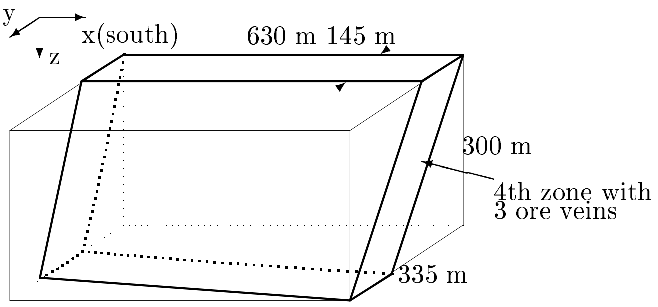

Figure 2: Domain of interest and the coordinate system.

Pre–mining state of stress

The pre–mining stress was measured by the hydrofracturing method, see [15]. This measurements reveal anisotropy and variability of the pre–mining stress caused by the deposit origin and existing faults. The measured stress substantially differs from the stress caused only by the weight of rocks.

For the aims of modelling, we consider two cases of the pre–mining stress. As a first choice, we consider the horizontal stresses

$$s_{xx} = k_{xx}\, s_{zz}, \quad s_{xy} = k_{xy}\, s_{zz}, \quad s_{yy} = k_{yy}\, s_{zz}$$

where $k_{xx} = k_{yy} = 1.25$, $k_{xy} = 0.75$ and s_{zz} is the vertical stress computed according to the formula $s_{zz} = -\gamma\, h$ where γ is the (averaged) unit weight of rock and h is the depth under surface.

The second choice is a hydrostatic stress defined by the same formulas with coefficients $k_{xx} = k_{yy} = 1$ and $k_{xy} = 0$.

Modelling of the mining process

The uranium ore is mined out by stoping with filling or caving which is a relatively complex process for detailed simulation. Therefore, we adopt a simplification which consists in restriction to four stages of mining. In each of these stages, we assume known dimensions of the mined out volumes as well as the volumes with broken material, see Fig. 3. Both volumes are assumed to be filled with fractured rocks – the goaf material. The sequence of mining can be seen from Fig. 5.

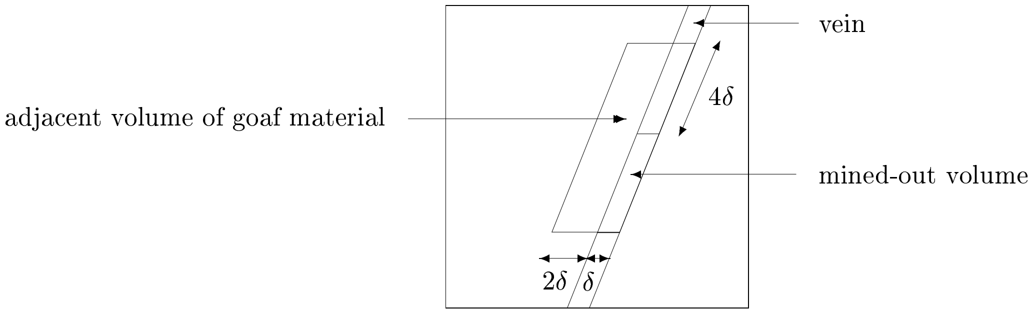

Figure 3: Dimensioning of the caved volume.

The modelling then consists in solving four boundary value problems with boundary conditions and volume loading chosen in such a way that they cause the pre–mining stress in the modelled domain if it is not influenced by mining. Assuming linear elastic behaviour of all considered materials, the modelling then consists in solving four boundary value problems of elasticity to obtain four stress states induced by the past and forthcoming mining.

3 Mathematical formulation and FE analysis

To compute the stress fields in the considered domain, we have to formulate the corresponding boundary value problems and apply the finite element method. To this end, we have to specify the problem domain, the material behaviour, the volume load and the boundary conditions. This specification will follow the considerations done in the previous Section.

Problem domain and FE grid

We will formulate the boundary value problem in a domain which will be a sufficiently large parallelepiped containing the whole investigated part of the deposit. Moreover, the problem domain must be sufficiently large to allow formulation of boundary conditions which correspond to the pre–mining stress state. In our case, the size of the problem domain will be 1430x550x600 metres with the top side situated horizontally in the depth of 800 metres under surface. This domain arises by enlarging the domain of interest, see Fig. 1 and 2, in the following way:

$$\begin{aligned} 1\,430 \text{ m} &= 200 + 630 + 600 \text{ m} \\ 550 \text{ m} &= 100 + 335 + 115 \text{ m} \\ 600 \text{ m} &= 100 + 300 + 200 \text{ m.} \end{aligned}$$

For the FE analysis, this domain is divided into hexagonal bricks which are further decomposed, each brick into six tetrahedra. For the modelling, we use FE grid of 124 x 137 x 76 nodes which is relatively fine in the area of interest. Here we can find the smallest bricks of 8 x 1 x 5 metres.

In the area outside the domain of interest the grid is gradually coarsened. The coarsening is not very high to avoid the appearance of elements with very high aspect ratio.

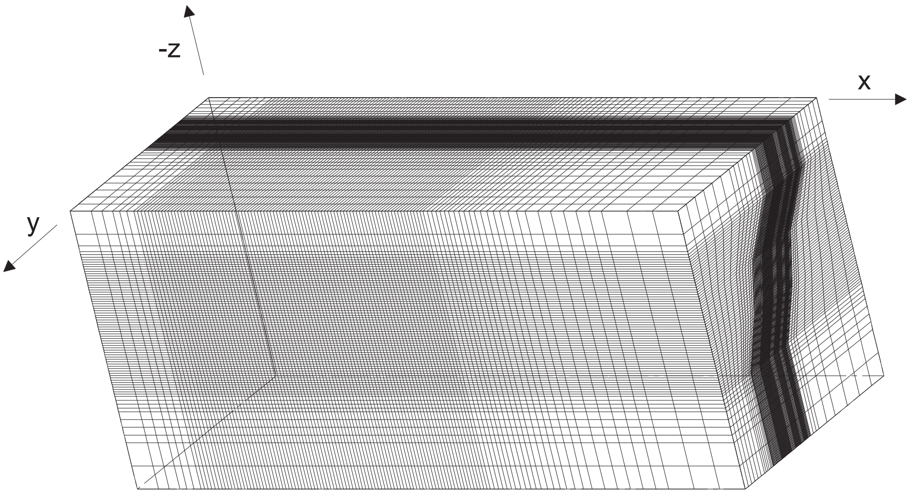

Figure 4: FE grid with 124x137x76 nodes (3 873 264 DOF)

Material behaviour

Two basic materials in the problem domain are the uranium ore and the surrounding rock. These materials are assumed to be elastic with the same material constants – elasticity modulus E=2000 MPa, Poisson ratio $\nu\ = 0.25$ and the unit mass $\varrho\ =2.7\ g\,cm^{-3}$.

The extracted volumes and their vicinity is filled by the goaf material. For the goaf material, we shall again assume linear elastic behaviour with the elastic constant E=2500 MPa, $\nu\ =0.4$ and $\varrho\ =2.0\ g\,cm^{-3}$. Note that a positive experience with this simplification was gained during the 2D modelling, cf. [14].

Boundary conditions and modelling procedures

Boundary conditions are specified to simulate the actions of the surrounding rock mass, existing in the pre-mining stress state, to the problem domain. Note that the problem domain must be chosen sufficiently large to guarantee that the surrounding rocks are in the pre-mining stress state.

We have several possibilities how to define the boundary conditions (BC). The first possibility is to use the pure Neumann boundary conditions which means that the surface forces depending on the pre-mining stress state are prescribed on the whole outer boundary. The values of such forces can be seen from Table 1.

side		outer normal	depth h	surface force
top	z = 0	(0,0,-1)	$h =\ 800$	$(0,0,1)\,\gamma h_0$
bottom	z = 600	(0,0,1)	$h =\ 1400$	$(0,0,-1)\,\gamma h$
east	y = 0	(0,-1,0)	$h =\ h_0 + z$	$(k_{xy}, k_{yy}, 0)\,\gamma h$
west	y = 600	(0,1,0)	$h =\ h_0 + z$	$-(k_{xy}, k_{yy}, 0)\,\gamma h$
north	x = 0	(-1,0,0)	$h =\ h_0 + z$	$(k_{xx}, k_{xy}, 0)\,\gamma h$
south	x = 1430	(1,0,0)	$h =\ h_0 + z$	$-(k_{xx}, k_{xy}, 0)\,\gamma h$

Table 1: Boundary conditions.

The second possibility consists in use of pure Dirichlet boundary conditions which arise from a pre-mining displacement field compatible with the pre-mining stress in the considered domain.

This two choices of the boundary conditions lead to two different modelling sequences, see Fig.5.

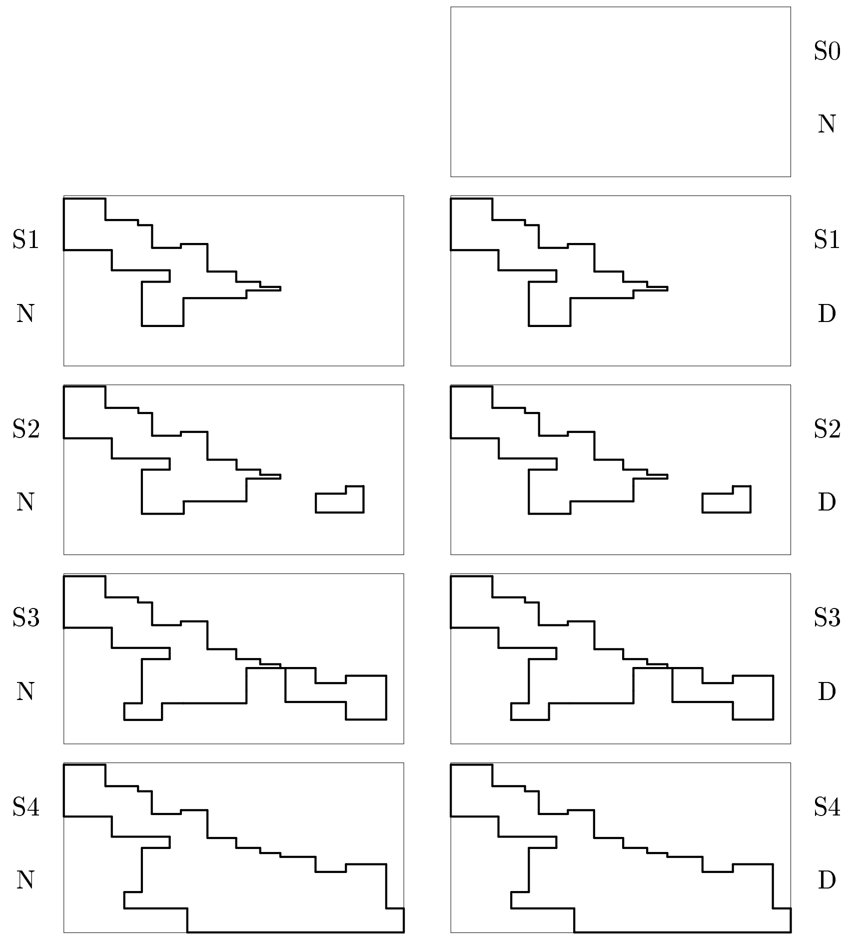

Figure 5: The Neumann's modelling sequence (left) and the Dirichlet's modelling sequence (right) with four stages of mining in one of the veins.

Neumann-type Modelling Sequence (NMS)

In four steps S1,...,S4 corresponding to the selected stages of mining, we solve the boundary value problems with pure Neumann boundary conditions. The solution from the previous step is used as an initial guess for the solution in the subsequent step for the steps S2, S3 and S4.

Dirichlet-type Modelling Sequence (DMS)

Here, we add the auxiliary step S0 which contains the solution of the pure Neumann boundary value problem for determining the pre–mining displacement. This displacement is then used for prescribing the boundary conditions in all the subsequent steps S1,...,S4 which corresponds again to the specified stages of mining. The solution from the previous step is used as an initial guess for the solution in the subsequent step for all the steps S1,...,S4.

Both sequences give comparable results (the difference in stresses is less than 7 %). The advantage of DMS is in saving the computational effort due to higher efficiency of iterative methods when solving the Dirichlet boundary value problems comparing with the solving of boundary value problems with the pure Neumann boundary conditions.

Submodelling

For the assessment of two different mining technologies (overhand and underhand stoping), we need sufficiently fine FE grid to be able to model stress changes in the vicinity of stopes. To this end, we shall use a composite grid FE analysis with the global (coarse) grid defined as before (Fig. 4) and the local (fine) grid of 41x113x49 nodes located in the investigated area where the situation must be described in more detail.

The use of composite grid substantially decreases the number of degrees of freedom comparing with the use of similarly refined regular grid with saving the advantages of regular grid. The solution of composite grid problem is enabled by using proper iterative procedure which alternatively combined solution of subproblems on global and local grids, see [8], [12].

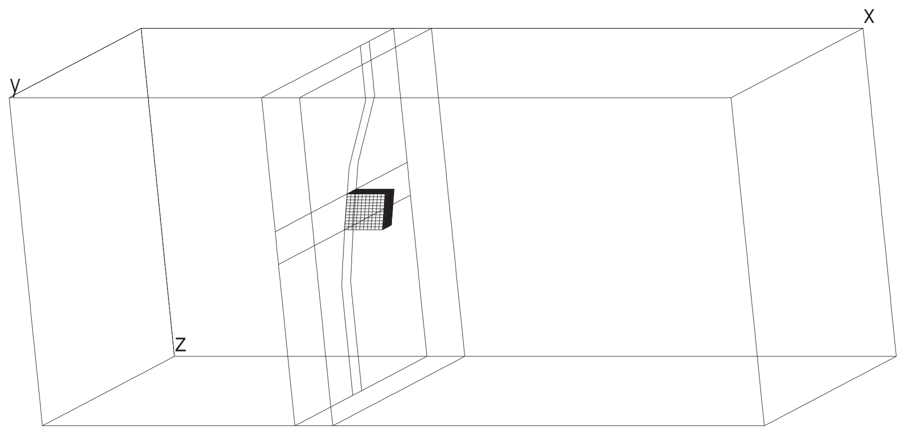

Figure 6: Submodelling - local grid with 4x4x4 times higher density in comparison with the global grid.

4 GEM3/HPC software

Numerical analyses required for the described modelling were performed using the GEM3/HPC software. This software is a powerful finite element program for three–dimensional engineering mechanics computations with a specialization in solving the geomechanical problems of mining or geotechnics. The special features of the program are the following:

- solution of 3D problem of *elasticity* (with linear or nonlinear material behaviour) and *plasticity*,
- domain discretization using *regular FE grids* which describe division into hexahedral bricks which are subsequently divided with the use of *linear tetrahedral finite element*,
- use of special *preprocessor* which is oriented to regular grids and the typical properties of geomechanical problems,
- existence of a library of *high performance iterative solvers* running on both sequential and parallel computer systems,
- *submodelling* supported by the use of composite finite element grids and *iterative balancing* the global and local subproblems,
- vizualization of the results in 2D cross–sections, query language and alphanumeric outputs for 3D subdomains.

GEM3/HPC is a command–driven computer program with a special emphasis on portability of programs and input/output data files and possibility to run the solvers or various types of supercomputers.

The performance of the solvers go far beyond the standard commercial software. This can be demonstrated also by the performance data from the use of the GEM3/HPC software for the described modelling which requires solution of large linear systems of nearly 4 million unknowns.

First results were obtained by using the conjugate gradient iterative method with preconditioning by the displacement decomposition and incomplete factorization, see [2]. This method implemented on IBM RS/6000 3CT computer has the following requirements concerning the computing (elapsed) time.

In the case of anisotropic pre–mining stress and the relative residual accuracy $\varepsilon = 0.001$ for solving the linear systems, the computing with the Neumann's modelling sequences needs 48.38 hours from which 93% was needed for solvers. The use of Dirichlet's modelling sequences reduce the computing times of solver by factor 1.5 which gives the total time 33.11 hours.

To further reduce the computer times of the solver we tested the parallelization by splitting the algorithm according to the displacement decomposition. Such decomposition is natural for elasticity and plasticity problems and allows to engage three processors with their local memories, see [3] and [4].

Implementing this idea on three processors of the IBM SP multicomputer (the processors are the same as the processor of the previously used IBM RS/6000 3CT computer) with the ATM interconnection between the processors gave a further reduction of time required for solvers by factor 2.15 for the Dirichlet's modelling sequence.

To reduce the amount of communication, we tested third solver with variable preconditioner defined by the approximate solution of the separate displacement component submatrices of the stiffness matrix by the inner conjugate gradient method preconditioned by the same incomplete factorization of the submatrices as in before. The solution of the linear system is again naturally split into three independent tasks. The accuracy of inner iterations can be chosen relatively low without influencing much the convergence of the outer iterative process, e.g. we have used $\varepsilon_0 = 10^{-1}$. The communication now takes place only in outer iterations and the number of outer iterations is independent on the discretization density. The use of this solver give speed up of about 10 in comparison with the initial results for the Dirichlets's modelling sequence. In comparison with the first results for the Neumann's modelling sequence we obtain even higher speed up of about 14. It means that the sequence of five solutions of linear systems with nearly four million unknowns is now solved in 3.15 hours using three processors of the IBM SP multicomputer.

Note that the use of more sophisticated block incomplete factorization was tested within the COPERNICUS 940820 project with a further speed up between 2.7 and 4.4, see [1].

5 Visualization and interpretation of the results

The practical conclusions from the performed modelling are based on the computed stress fields. With respect to the aims of the modelling, we shall use the following scalar characteristic of the stress:

- horizontal stress component σ_{xx} which corresponds to the direction which is approximately normal to the veins,
- maximal shear stress $\tau_{\max}$ which is related to the rock strength via the Tresca strength criterion,
- safety factor f_{HB} according to the Hoek–Brown strength criterion ([11]) which can be used for more detailed study of the rock failure in the vicinity of mine openings. The safety factor f_{HB} is defined from the identity

$$-\sigma_1 = f_{HB} \{ -\sigma_3' - \sigma_c \sqrt{s + m \mid \frac{\sigma_3'}{\sigma_c} \mid } \}$$

  where $\sigma_1 \le \sigma_2 \le \sigma_3$ are the principal stresses, $\sigma_3' = \min\{\sigma_3, 0\}, \sigma_c \le 0$ is the uniaxial compressive strength and s, m are two material parameters. This criterion make sense in the case of compressive minimal principal stress $(\sigma_1 < 0)$ when $f_{HB} > 1$ indicates failure of the material. For the surrounding rocks, we use the parameters $\sigma_c = 100$MPa, $s = 0.1$ and $m = 8.5$.

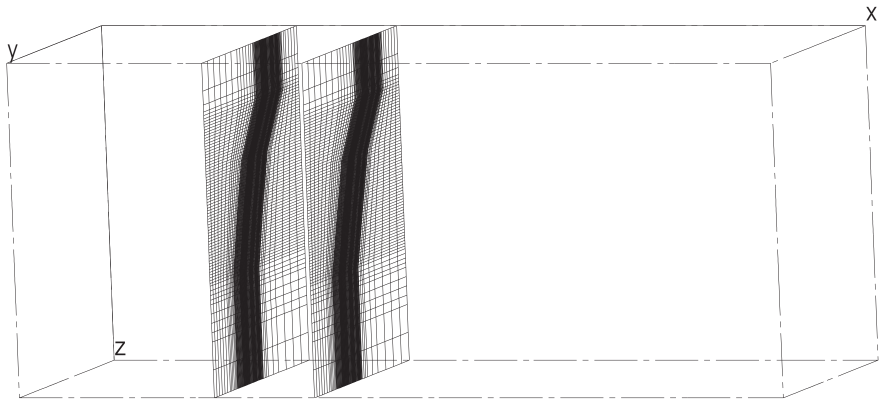

Figure 7: Localization of cross–sections $I = 35$ and $I = 60$.

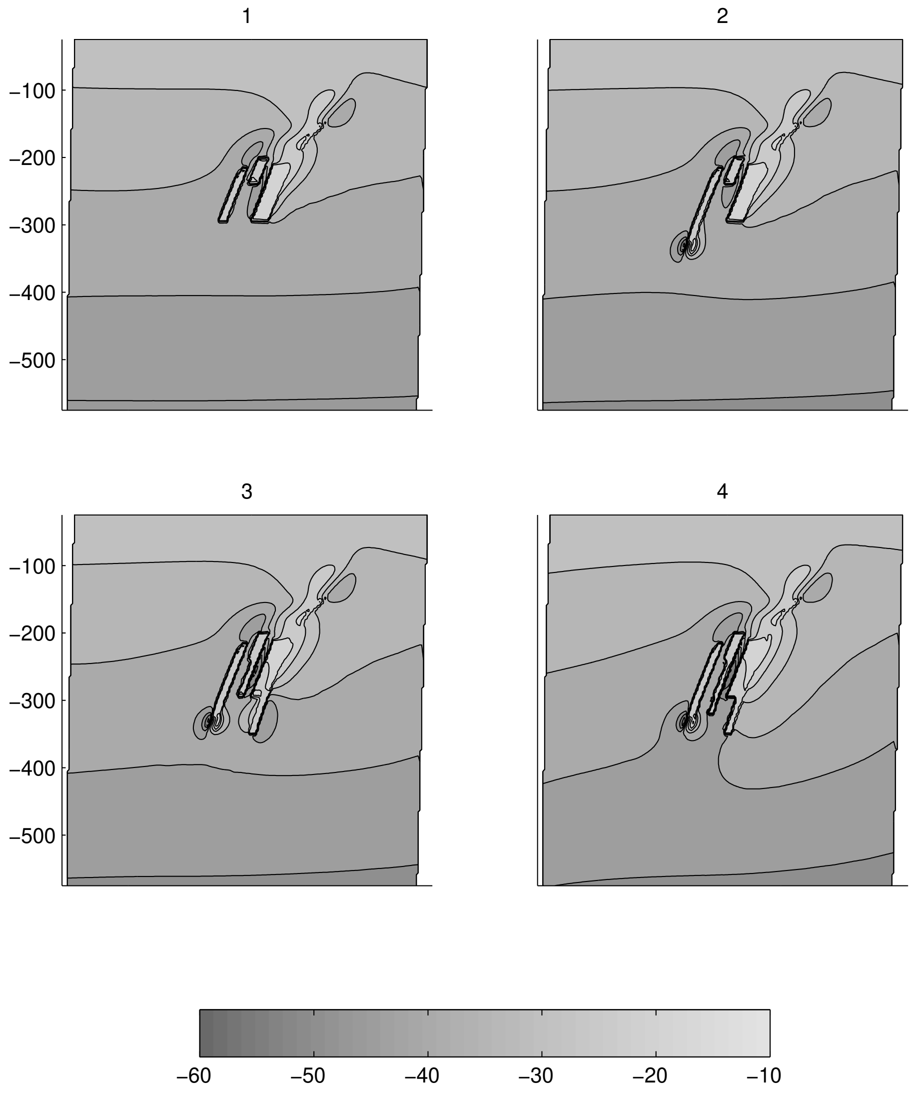

Figure 8: Development of the horizontal stress σ_{xx} in the cross section $I = 35$.

The visualization of the stress fields was performed in the cross–sections perpendicular to the axis x. The Figure 7 shows localization of two interesting cross–sections $I = 35$ and $I = 60$. The results in the selected cross–section can be visualized by drawing isolines, colored value levels, graphs etc. The verification of hypotheses which concerns the global 3D domain or its part can be then done with the aid of a query language GEMQ integrated with various types of alphanumeric outputs.

An example of visualization can be seen from Figure 8 which shows the development of the horizontal stress σ_{xx} in the cross section $I = 35$ in the case of the anisotropic pre– mining stress. The figure is presented in grey scale because the coloured figures could not be used in this paper.

From Figure 8, we can see that the modelled mining steps induces remarkable stress changes which are located to the neighbourhood of the working area, the most substantial changes are in the vicinity up to 15 metres of the mined out volumes.

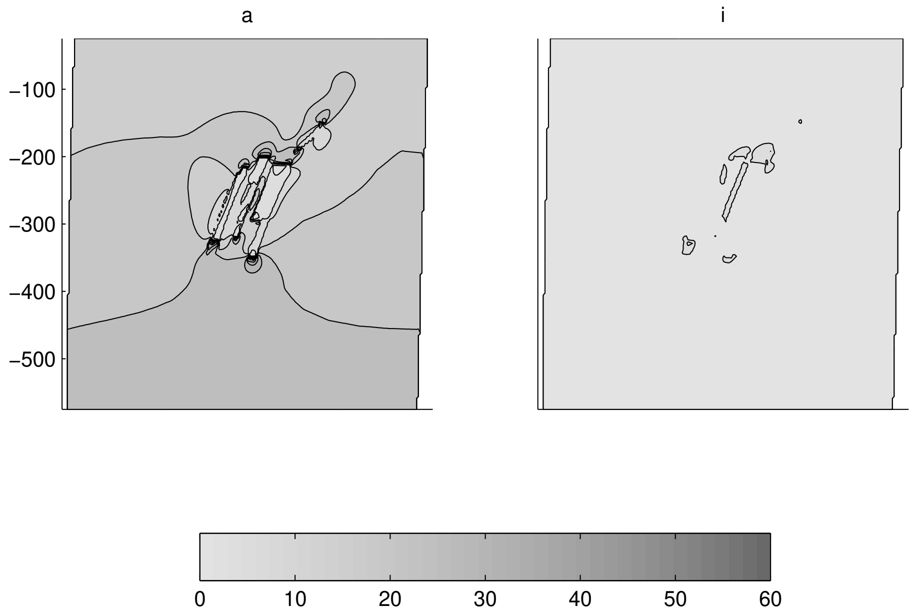

Figure 9: The maximal shear stress for the last stage of mining and both anisotropic (a) and isotropic (i) case of the pre–mining stress in the cross section $I = 35$.

Figure 9 shows the maximal shear stress for the last stage of mining and both anisotropic and isotropic case of the pre–mining stress. It shows typical local stress concentrations around the narrow edges of the mined–out block and absolutely substantial difference between the cases of anisotropic and isotropic pre–mining stress. It shows the the importance of measurements of the pre–mining stress as well as necessity to consider the worst possible scenario.

Figure 9 turns our attention to the questions of simplifications and uncertainty in the performed modelling. Various types of simplifications are assumed: linear elastic behaviour of rocks and the goaf material, use of average material parameters, use of history independent modelling procedure for the mining with caving or filling. Some uncertainly is involved in the material parameters, initial stress and strength parameters. These facts has been considered in interpretation of the results of modelling which was based on the worst possible scenarios, comparison of results from variants of the modelling as well as comparison of the results of modelling with real observed effects whenever it was possible.

6 Conclusions

We present en example of large scale mathematical modelling which was possible to perform due to development of numerical methods and software tools for the high performance computing (finite element software GEM3/HPC).

The computed results could be interpretated with a special care to the simplifications and uncertainly involved in the modelling. This interpretation makes finally possible to formulate answers to the main questions which are very important for the economy and safety of mining. These answers, formulated in [10], are

- mining in the depths under 1000 m below the surface induces changes of the stress fields in the far field zone which are relatively small. This indicates no reasons for the increased rock burst hazard.
- there is remarkable differences between the considered mining technologies. Overhand stoping method should not be recommended for using in greater depths due to extreme high concentrations of stresses which can occur in ore pilar above the advancing drive (opening). It is quite sure that this can cause opening being unstable and dangerous for mining operations and people. Underhand stoping method can be recommended as the mining method through the next 3–4 levels (150-200 m deeper) in zone no. 4 of the Rožínka mine.
- the forthcoming mining in the sub–vein of the zone no. 4 should be planned carefully with respect of the influence of mining in the neighbouring sub–veins. New mining face should not be located in the zone of the increased stresses induced by mining in the other sub–vein.

The modelling of the effects of the mining at the Rožná deposit can be continued according to the further requirements of the GEAM Dolní Rožínka mine. The quality of the modelling can be then further enhanced by measuring the pre–mining stress and stress–strain behaviour of the rocks and goaf materials. From the side of the numerical solution a further progress could be achieved by further enhancement of the efficiency of solvers, aposteriori analysis of the discretization error and adaptation of the computational grid ([13]), sensitivity analysis with respect to parameters of both the pre–mining stress and material behaviour as well as using mathematical methods of finding reliable solution under uncertainly in the input data. The corresponding research work will be held in the Department of Applied Mathematics of the Institute of Geonics AS CR.

References

[1] R. Beauwens and Y. Notay. Efficient block preconditioners for elasticity problems. Technical Report GANMN 98-01, Université Libre de Bruxelles, Brussels, Belgium, 1998.

[2] R. Blaheta: Displacement decomposition – incomplete factorization preconditioning techniques for linear elasticity problems, Numer. Lin. Alg. Appl. 1(1994), 107-126

[3] R. Blaheta, O. Jakl, J. Starý: A parallel CG solver for FE analysis of 3D problems in geomechanics, in GEOMECHANICS'96, A.A. Balkema, Rotterdam 1997.

[4] R. Blaheta, O. Jakl and J. Starý, PVM-implementation of the PCG method with Displacement Decomposition. The Fourth European PVM-MPI Users' Group Meeting in Krakow. In, M. Bubak, J. Dongarra, J. Wasniewski (eds.) Recent Advances in Parallel Virtual Machine and Message Passing Interface, Lecture Notes in Computer Science 1332, Springer Verlag 1997.

[5] R. Blaheta, O. Jakl, J. Starý: An application of parallel high performance computing in geomechanics, submitted In: Proc. 7th Seminar Contemporary mathematical methods in engineering, J. Doležalová ed., Technical University of Ostrava, 1998

[6] R. Blaheta, O. Jakl, R. Kohut and A. Kolcun: An application of large scale mathematical modelling in geomechanics, In: Proc. 6th Seminar Contemporary mathematical methods in engineering, J. Doležalová ed., Technical University of Ostrava, 1997

[7] R. Blaheta, O. Jakl, R. Kohut, A. Kolcun, J.Malk, Z. Rakowski and A.Hájek: Mathematical Modelling of the Effects of Mining at the Deposit Dolní Rožínka, HIPERGEOS Report, IGAS Ostrava, August 1997

[8] R. Blaheta, R. Kohut: Iterative local refinement methods for nonlinear problems. Software for iterative local refinement methods, HIPERGEOS Report, IGAS Ostrava, April 1998

[9] DIAMO state enterprise. Branch plant GEAM Dolní Rožínka. Information booklet.

[10] A. Hájek and Z. Rakowski, Role of large scale mathematical models in development of new mining technology. HIPERGEOS Report, GEAM D. Rožínka, March 1998.

[11] E.Hoek: Strength of jointed rock masses, Geotechnique 33, 1983

[12] R. Kohut: Solving problems on composite grids. In: Proc. 7th Seminar Contemporary mathematical methods in engineering, J. Doležalová ed., Technical University of Ostrava, 1998

[13] A. Kolcun: Aposteriori error estimates for 3D elasticity problems. In: Proc. 7th Seminar Contemporary mathematical methods in engineering, J. Doležalová ed., Technical University of Ostrava, 1998

[14] Z. Rakowski: Mining of large ore bodies by underhand slicing with caving under artificial roof without limitation of worked height in Rožná deposit. Technical report, IGAS Ostrava, 1994 (In Czech)

[15] L. Staš et al.: In situ measurements of the horizontal stresses at the Rožná deposit by the method of hydrofracturing. Technical Report, IGAS Ostrava, April 1997