<u>PROJECT REPORT HG CP 94–0820–MOST–4</u>

HIPERGEOS Benchmark Problems Related to Bridge Engineering Applications

Asen Georgiev
MOST XXI, Sofia, Bulgaria,
Angel Baltov and Svetozar Margenov
BAS, Sofia, Bulgaria

Abstract

The final version of the benchmark problems related to bridge engineering applications of the HIPERGEOS codes are presented in this report. The goal of the benchmarking is to test the abilities of the developed 3D solvers for real live engineering problems. The structure of the presentation follows the research plan of the HIPERGEOS CP 94–0820 Project. Two modifications of the first benchmark problem including a strengthened nonlinear elasticity and plasticity are given in the attached appendix.

The presented real technical data (including the drafts and the diagrams) are selected from the MOST XXI archives.

1 Mathematical formulation of the problem

We consider the 3D problem of elasto–plasticity under the following general assumptions:

- The displacements are small;
- The material properties are isotropic.

Then the stressed–strained state is described by a coupled elliptic system of three differential equations, which can be written in the form

$$D\underline{\sigma} + \underline{p} = \quad 0 \qquad \forall (x_1, x_2, x_3) \in \Omega \tag{1}$$

$$\begin{aligned} \underline{u} = & \quad \underline{u}_D \qquad \forall (x_1, x_2, x_3) \in \Gamma_D \\ \sum_{i=1}^{3} \sigma_{ij} n_i = & \quad p_{Nj} \qquad \forall (x_1, x_2, x_3) \in \Gamma_N, \quad j = 1 \div 3, \end{aligned}$$

where $\underline{u}^T = (u_1, u_2, u_3)$ is the vector of the unknown displacements, Ω is the computational domain (in our considerations Ω is assumed to be a logical cube), and Γ_D and Γ_N are the parts of the boundary of Ω with respectively Dirichlet and Neumann boundary conditions; $\underline{p}$ is the vector of the volume forces, and $\underline{u}_D$ and $\underline{\sigma}_N$ are the given boundary data, respectively the vector of the given displacements on Γ_D and the vector of the given stresses on Γ_N.

The strains $\underline{\epsilon}^T = (\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, 2\epsilon_{12}, 2\epsilon_{23}, 2\epsilon_{31})$ and the stresses $\underline{\sigma}^T = (\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31})$ are determined as follows

$$D = \begin{pmatrix} \frac{\partial}{\partial x_1} & 0 & 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_3} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} & 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{pmatrix}, \tag{2}$$

$$\underline{\epsilon} = D^T \underline{u}, \tag{3}$$

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix} = E^* \begin{pmatrix} (1-\nu) & \nu & \nu & & & \\ \nu & (1-\nu) & \nu & & & \\ \nu & \nu & (1-\nu) & & & \\ & & & (1-2\nu)/2 & & \\ & & & & (1-2\nu)/2 & \\ & & & & & (1-2\nu)/2 \end{pmatrix} \begin{pmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{23} \\ 2\epsilon_{31} \end{pmatrix}, \tag{4}$$

where $\nu = \nu(\epsilon)$ and $E = E(\epsilon)$ stand respectively for the Poisson ratio and for the deformation module, and $E^* = \frac{E}{(1+\nu)(1-2\nu)}$.

The last relations represent the nonlinear nature of the generalized Hooke's law. There are various approaches to describe the nonlinearity of the elasto–plastic materials. Now we will specify the generalized Hooke's law under the following additional assumptions:

- The Poisson ratio $\nu \in (0, \frac{1}{2})$ is a constant for a given material (soil layer or constructive element);

- The strain–stress diagrams presenting the dependence between the normal stains and stresses $\sigma_{ii} = \varphi(\epsilon_{ii})$ for the nonlinear materials are given either as a function or as a set of experimental data.

This assumptions are natural for the considered engineering problems, where the soil layers are typically nonlinear with their specific elasto–plastic characteristics, while the piles are usually treated as linear elastic solids. Obviously, this means that the coefficients in the boundary value problem (1) are piece–wise continuous with jumps through the inner boundaries between the different soil layers as well as between the soil and the piles.

To specify the stressed–strained state of the elasto–plastic media by a scalar function we introduce the *strain intensity* ϵ_0 defined by the formula

$$\epsilon_0 = \sqrt{\frac{2}{3}}\sqrt{(\epsilon_{11}-\epsilon_{22})^2+(\epsilon_{22}-\epsilon_{33})^2+(\epsilon_{33}-\epsilon_{11})^2+6(\epsilon_{12}^2+\epsilon_{23}^2+\epsilon_{13}^2)}.$$

Now if the function φ is known then the deformation module $E = E(\underline{\epsilon})$ is determined by the relation

$$E(\underline{\epsilon}) = \varphi'(\frac{3}{2(1+\nu)}\epsilon_0). \tag{5}$$

Note that the function $\varphi(\epsilon)$ is always strictly increasing which guarantees the ellipticity of the problem (1).

2 Benchmark problems

Two benchmark problems are presented in this section. They illustrate some typical requirements to the HIPERGEOS code related to the bridge engineering applications. The problems are described by the differential equations (1). The considered foundation elements are filling piles. It is assumed that the construction technology guarantees the ideal contact between the piles and the surrounding soil layers.

The computational domain is the parallelogram $\Omega = (0, x_1^{max}) \times (0, x_2^{max}) \times (0, x_3^{max})$. The boundary conditions are as follows: homogeneous Dirichlet boundary conditions on the bottom side $x_3 = 0$ (the displacements are equal to zero); homogeneous Neumann boundary conditions on the vertical sides of Ω (the stresses are equal to zero); Neumann boundary conditions on the top side $x_3 = x_3^{max}$, where the load (V, H) from the upper construction is applied. The volume force has the form $\underline{p} = (0, 0, -\gamma g)$ representing the proper weight of the continuous media.

- **Benchmark 1.** Single pile in a homogeneous sandy clay soil layer is considered. The computational domain is determined by $x_1^{max} = x_2^{max} = 37.2m; x_3^{max} = 31.0m$. The vertical cross section $x_2 = 18.6$ (the same as with $x_1 = 18.6$) of Ω is presented in Fig.1. The cross section size of the pile is $D = 1.2m$. The length of the pile and the size of the active zone under the top of the pile are respectively $L_P = 15m$ and $H_{act} = 12m$. The mechanical characteristics of the pile are as follows: specific volume weight $\gamma_P = 25kN/m^3$; $E_P = 31500MPa$; $\nu_P = 0.2$.

  The scheme of loading (the vertical load V and the horizontal load H oriented along the x_1 direction) of the pile is shown in Fig.1, where $V = 3000kN$ and

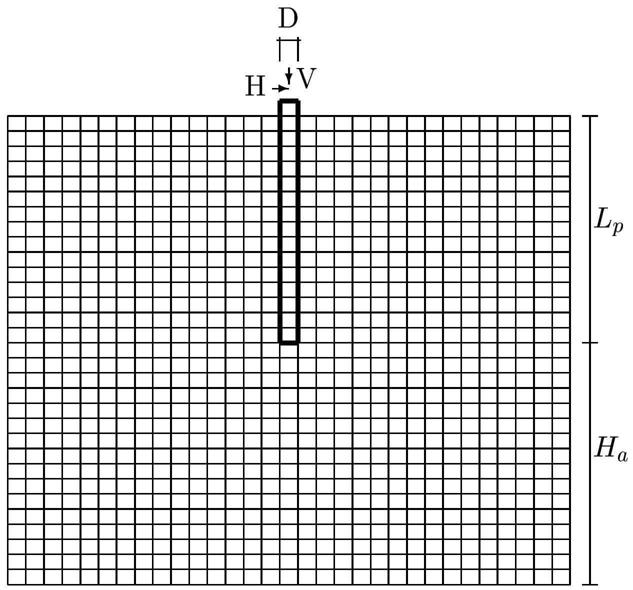


Figure 1: *Benchmark 1; Cross section of the computational domain Ω; Basic uniform discretization mesh.*

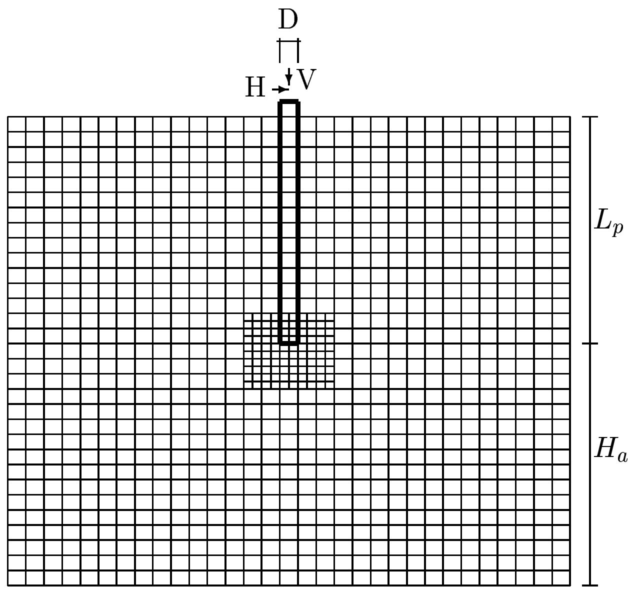
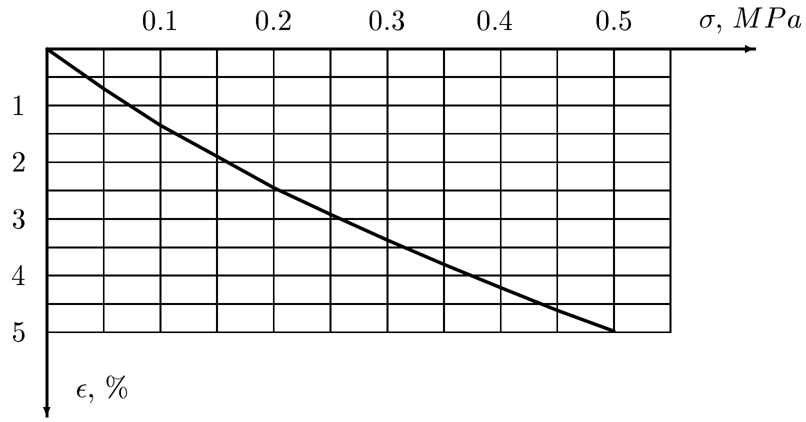


Figure 2: *Benchmark 1; Cross section of the computational domain Ω; One level local refinement composite grid.*



σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	0.70	1.35	1.90	2.45	2.92	3.37	3.80	4.21	4.61	4.98

$$\gamma = 18kN/m^3, \ \nu = 0.3$$

Figure 3: *Benchmark 1; Homogeneous soil layer; Sandy clay; Stress–strain diagram.*

$H = 250kN$. It is assumed that the loads are uniformly distributed on the cross section of the pile.

The stress–strain diagram $\sigma = \varphi(\epsilon)$ and the other needed characteristics of the soil layer are given in Fig.3.

A possible basic scheme of discretization is shown in Fig.1. The mesh sizes are $h_{x1} = h_{x2} = 1.2m$ and $h_{x3} = 1m$. An idea of a composite grid with one level local mesh refinement is shown in Fig.2. The local refinement area covers the zone of active interaction between the top of the pile and the surrounding soil media.

Let us assume that the finite element method with trilinear finite elements is used for the numerical solution of the problem. Then the size of the discrete problem corresponding to the basic discretization (Fig.1) is $N = 98304$.

- **Benchmark 2.** Doubled pile system in a multilayer soil media is considered in this test problem.

  The computational domain is determined by $x_1^{max} = x_2^{max} = 37.2m; x_3^{max} = 31m$. The vertical cross section $x_2 = 18.6$ (the same as with $x_1 = 18.6$) of Ω is presented in Fig.4. The cross section size of the piles is the same as in the previous problem, i.e., $D = 1.2m$. The distance between the piles is $a = 2.4m$. The length of the pile and the size of the active zone under the top of the pile, and the mechanical characteristics of the piles are as follows $L_P = 15m$, $H_{act} = 12m$, and $\gamma_P = 25kN/m^3$, $E_P = 31500MPa$, and $\nu_P = 0.2$.

  The scheme of the loading (the vertical loads V_1, V_2 and the horizontal load $H = H_1 = H_2$) are shown in Fig.4, where $V_1 = 2000kN$, $V_2 = 4000kN$ and

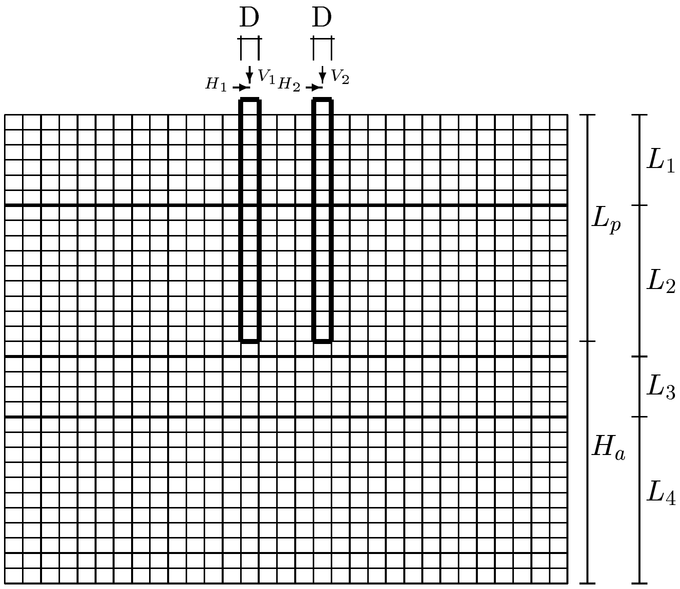


Figure 4: *Benchmark 2; Cross section of the computational domain* Ω*;*

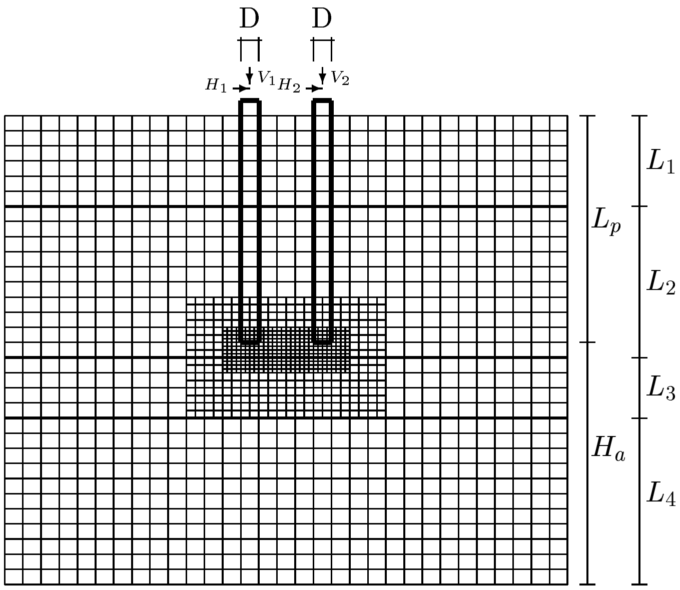
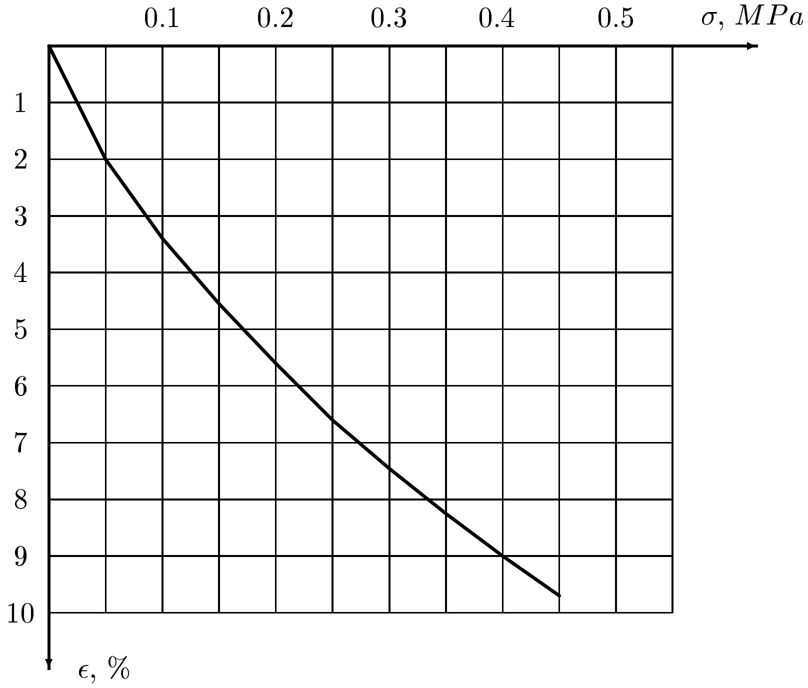


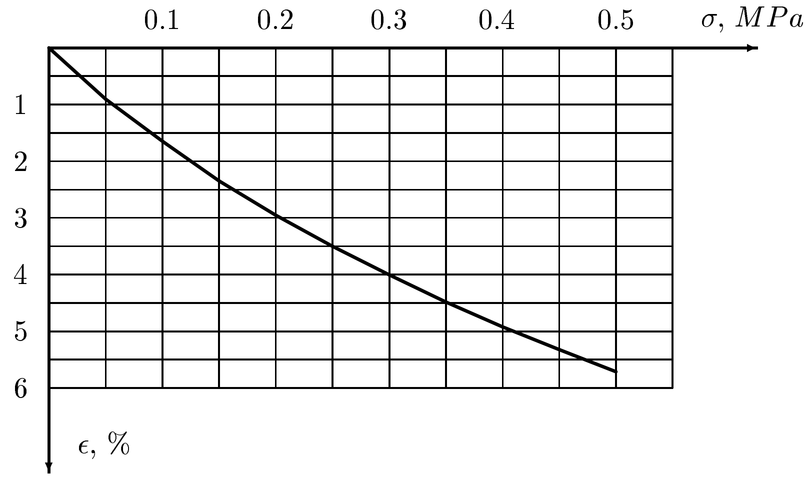
Figure 5: *Benchmark 2; Cross section of the computational domain* Ω*; Two level local refinement grid.*



σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
ϵ, %	2.00	3.40	4.55	5.60	6.60	7.45	8.25	9.00	9.70

$$\gamma = 19kN/m^3, \ \nu = 0.4$$

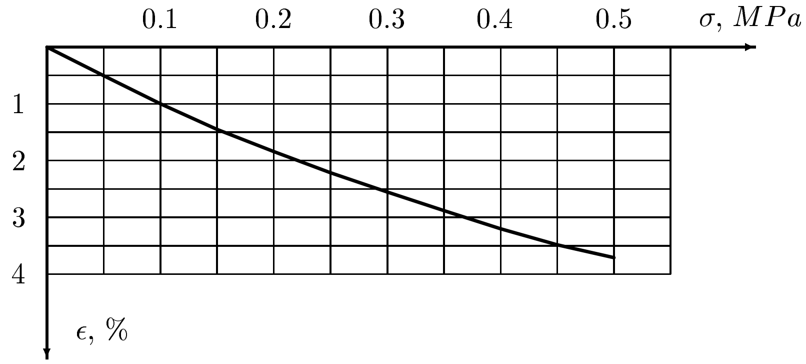
Figure 6: *Benchmark 2; Layer 1,* $H_{L1} = 6m$*; Organic clay; Stress–strain diagram.*



σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	0.90	1.65	2.35	2.95	3.50	4.00	4.48	4.92	5.32	5.71

$$\gamma = 19kN/m^3, \ \nu = 0.35$$

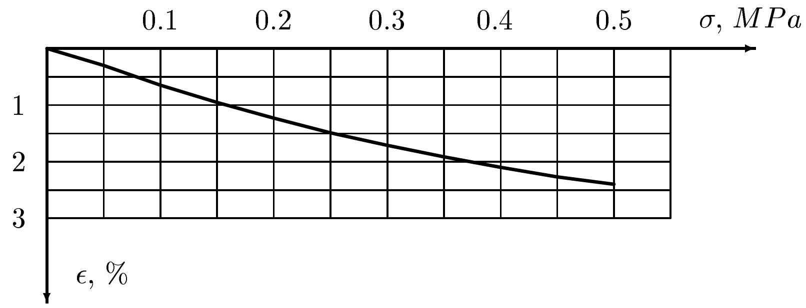
Figure 7: *Benchmark 2; Layer 2,* $H_{L2} = 10m$*; Softly plastic clay; Stress–stain diagram.*



σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	0.50	1.00	1.45	1.84	2.21	2.55	2.88	3.20	3.48	3.71

$$\gamma = 17kN/m^3, \ \nu = 0.25$$

Figure 8: *Benchmark 2; Layer 3, $H_{L3} = 4m$; Clay sand; Stress–strain diagram.*



σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	0.30	0.65	0.95	1.23	1.49	1.71	1.91	2.10	2.27	2.40

$$\gamma = 16kN/m^3, \ \nu = 0.20$$

Figure 9: *Benchmark 2; Layer 4, $H_{L4} = 14m$; Moderate sand; Stress–strain diagram.*

$H = 150kN$.

The stress–strain diagrams $\sigma = \varphi(\epsilon)$ and the other needed characteristics of the different soil layers are given in Figs. 6-9.

A basic scheme of discretization for this problem is shown in Fig.4. The mesh sizes are the same as before, i.e., $h_{x1} = h_{x2} = 1.2m$ and $h_{x3} = 1m$. Fig 5 presents a possible composite mesh with two levels of local mesh refinement.

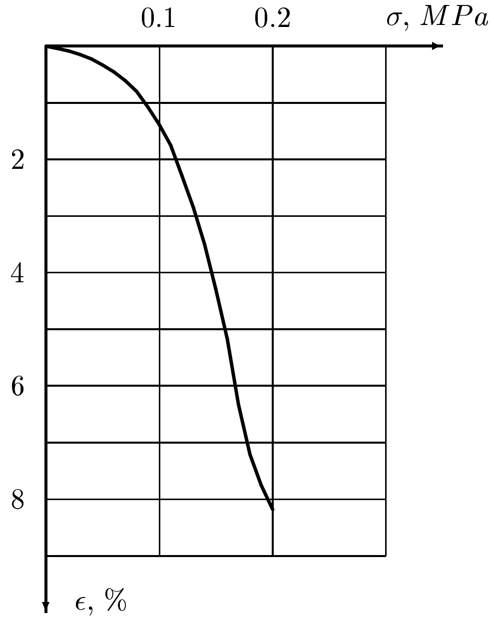
The size of the basic discrete problem for this benchmark is the same as for the previous, i.e. $N = 98304$.

APPENDIX

Two additional test problems (A1 and A2) with a strengthened nonlinear behaviour of the soil layers are presented in this Appendix. The geometry of the problems and the mechanical characteristics of the pile are the same as in Benchmark 1. The changes relate to the external loading and the soil layers strain–stress diagrams.

- **Test problem A1.** The scheme of the external loading of the pile is shown in Fig.1. Here $V = 15\ 000kN$ and $H = 250kN$. The loads are uniformly distributed on the cross section of the pile.

  The stress–strain diagram $\sigma = \varphi(\epsilon)$ and the other needed characteristics of the soil layer are given in Fig.10.

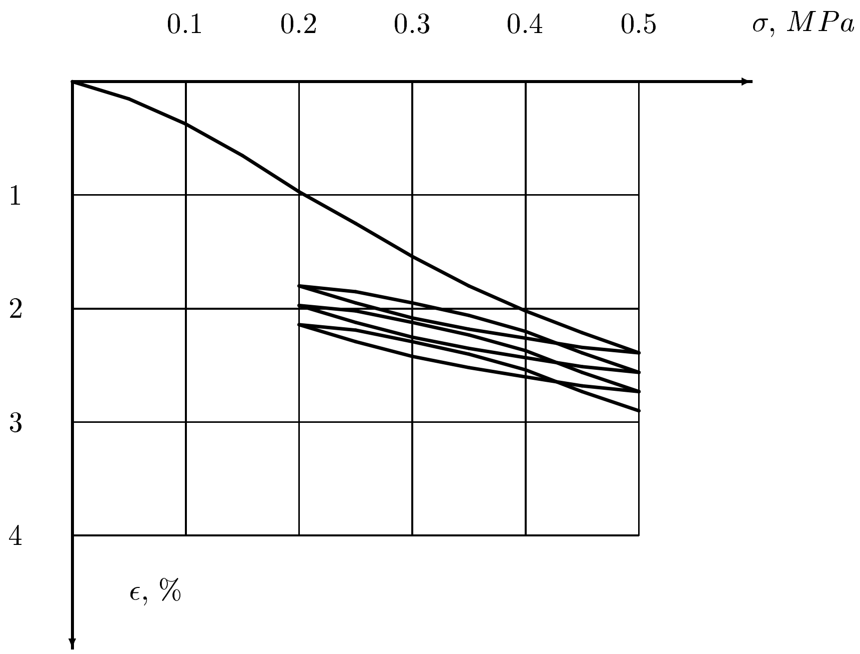


σ, MPa	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
ϵ, %	0.041	0.086	0.150	0.225	0.333	0.458	0.613	0.800	1.081	1.382
σ, MPa	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
ϵ, %	1.750	2.290	2.850	3.500	4.290	5.170	6.330	7.210	7.750	8.180

$$\gamma = 18kN/m^3, \ \nu = 0.42$$

Figure 10: *Test problem A1; Clay; Stress–strain diagram.*

- **Test problem A2.** The external loading in this test problem consists of two components: steady load (V_S, H_S), and temporary load (V_T, H_T). The scheme of loading contains: a) first step of a total loading with $V = V_S + V_T$ and $H = H_S + H_T$; b) three unloading–loading cycles corresponding to the temporary component (V_T, H_T). Here $V_S = 800kN$, $H_S = 0kN$, $V_T = 700kN$ and $H_T =$



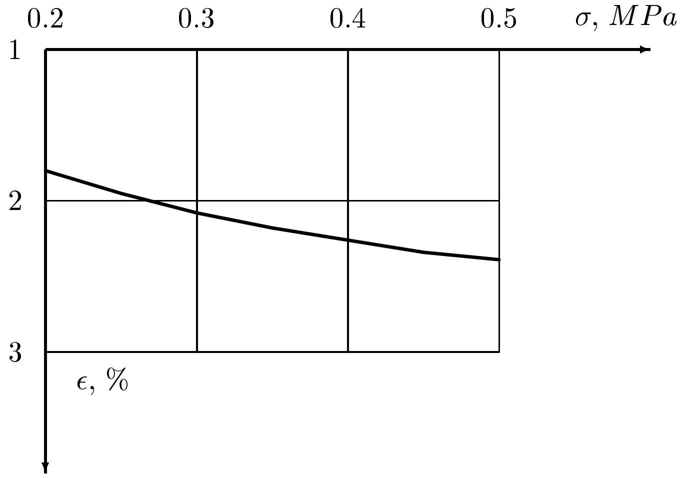
σ, MPa	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	0.15	0.37	0.65	0.97	1.25	1.54	1.80	2.02	2.21	2.39

$$\gamma = 18kN/m^3, \ \nu = 0.19$$

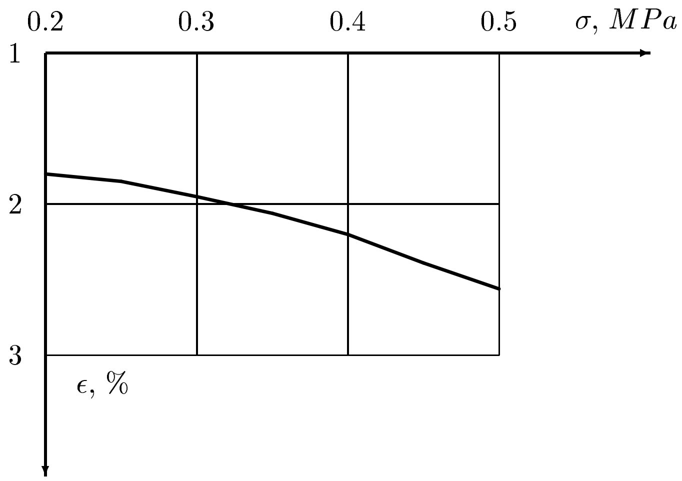
Figure 11: *Test problem A2; Clay; Stress–strain diagram. Three unloading–loading cycles.*

$250kN$. The loading–unloading cycles leads to a typical plastic behaviour of the soil layer. It is assumed that after three unloading–loading cycles the behaviour of the soil layer can be considered as elastic.

The composed loading–unloading stress–strain diagram $\sigma = \varphi(\epsilon)$ and the other needed characteristics of the soil layer are given in Fig.11. A more detailed diagrams corresponding to the unloading and loading phases are shown in Fig. 12.



σ, MPa	0.50	0.45	0.40	0.35	0.30	0.25	0.20
ϵ, %	2.39	2.34	2.26	2.18	2.08	1.95	1.80



σ, MPa	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ϵ, %	1.80	1.85	1.95	2.06	2.20	2.39	2.56

$$\gamma = 18kN/m^3, \ \nu = 0.19$$

Figure 12: *Test problem A2; Clay; Unloading and loading stress–strain diagrams.*