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Application of BSR BILU algorithm for 3D simulation of pile foundations.

FEM package. Linear problems. Benchmark 2.

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Abstract

The recently introduced (B)lock-(S)ize (R)eduction (B)lock-ILU preconditioning algorithm is implemented for 3D simulation of a doubled pile system in a multilayer soil media (Benchmark 2). The problem is described by a coupled system of elliptic partial differential equations with strongly varying coefficients. The emphasis is on some specific items related to the computer simulation of the foundation elements of bridge structures. Based on the performance analysis some practical rules to specify the parameters of the iterative algorithm are formulated. At the end a short engineering analysis of the numerical results is presented.

1 Introduction

The foundations of the engineering constructions transfer and distribute the total loading on the foundation bed soil. The spatial work of the construction results to a composed stressed–strained state, in particular, in the zones of active interactions.

The design of bridge constructions with a sufficient reliability requires the determining of the parameters of this stressed–stained state. The typical cases include multilayer media with strongly varying material characteristics.

We consider the 3D linear elasticity problem under the following general assumptions:

- The displacements are small;
- The material properties are isotropic.

Then the stressed–strained state is described by the Navier system of differential equations (see, e.g. in [?], [?]).

The finite element formulation of the considered 3D problem leads to a symmetric positive definite block system with sparse block–entries

$$A = \begin{pmatrix} A_{11} & A_{12} & & & 0 \\ A_{21} & A_{22} & A_{23} & & \\ & \ddots & \ddots & \ddots & \\ & & A_{n-1,n-2} & A_{n-1,n-1} & A_{n-1,n} \\ 0 & & & A_{n,n-1} & A_{nn} \end{pmatrix}$$

and therefore is amenable to iterative solution by block schemes [?], [?]. Here we apply the block–ILU method proposed in Chan and Vassilevski [?] (see also in [?], [?],[?]).

The next theorem for the convergence rate of the block-size reduction BILU algorithm holds:

Theorem 1. The BSR BILU factorization preconditioner C has relative condition number with respect to the stiffness matrix A of order $O((\frac{H}{h})^2)$, where $\frac{H}{h}$ is the maximal size reduction factor.

The proof follows from the general theory as presented in [?], and from the Korn's inequality (see for a detailed proof in a 2D case in [?]).

2 Benchmark 2

Doubled pile system in a multilayer soil media is considered. The computational domain is a parallelogram determined by $x_1^{max} = 31m$; $x_2^{max} = x_3^{max} = 37.2m$. Note that here x_1 is the coordinate ox in the vertical direction. The boundary conditions are as follows: homogeneous Dirichlet boundary conditions on the bottom side $x_1 = 0$ (the displacements are equal to zero); homogeneous Neumann boundary conditions on the vertical sides of Ω (the stresses are equal to zero); Neumann boundary conditions on the top side $x_1 = x_1^{max}$, where the load (V, H) from the upper construction is applied. The cross section $x_3 = 18.6$ of Ω is presented in Fig.1. The cross section size of the piles is the same as in Benchmark 1, i.e., $D = 1.2m$. The distance between the piles is $a = 2.4m$. The length of the piles and the size of the active zone under the top

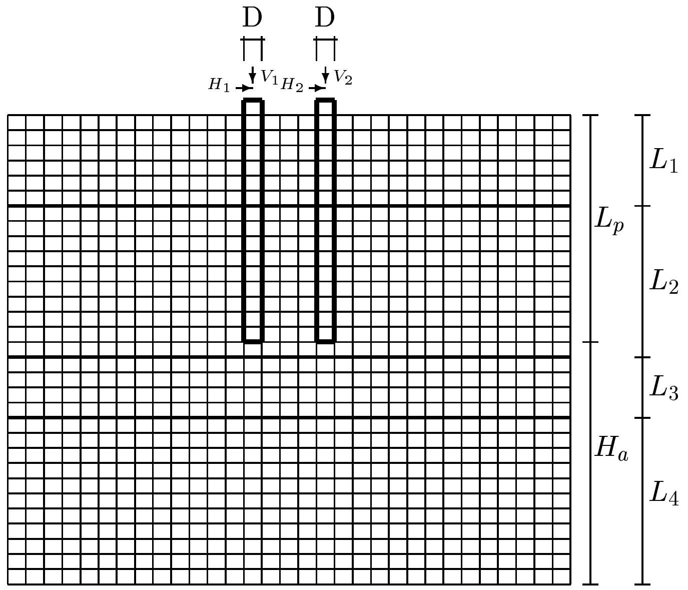

Figure 1: *Benchmark 2; Cross section of the computational domain* Ω $\alpha : x_3 = 18.6$*;*

of the pile, and the mechanical characteristics of the piles are as follows $L_P = 15m$, $H_{act} = 12m$, and $E_P = 31500MPa$, and $\nu_P = 0.2$.

The scheme of the loading (the vertical loads V_1, V_2 and the horizontal load $H = H_1 = H_2$) are shown in Fig.1, where $V_1 = 2000kN$, $V_2 = 4000kN$ and $H = 150kN$.

The mechanical characteristics in the linearized case are as follows:

1. Pile: Concrete;
   $E_P = 31500MPa$; $\nu_P = 0.2$;

2. Layer 1: $L_1 = 6m$; Organic clay;
   $E_{L_1} = 5.2MPa$; $\nu_{L_1} = 0.4$;

3. Layer 2: $L_2 = 10m$;
   Softly plastic clay;
   $E_{L_2} = 9.4MPa$; $\nu_{L_2} = 0.35$;

4. Layer 3: $L_3 = 4m$; Clay sand;
   $E_{L_3} = 14.0MPa$; $\nu_{L_3} = 0.25$;

5. Layer 4: $L_4 = 14m$; Moderate sand;
   $E_{L_4} = 21.4MPa$; $\nu_{L_4} = 0.2$;

The finite element method with three-linear finite elements is used for the numerical solution of the problem (see, e.g., in [?]). The size of the discrete problem corresponding to the basic discretization (Fig.1) is $N = 98304$.

3 Performance analysis

In this section, we analyze the performance rate of the BSR BILU PCG iterative method, varying the parameters (m_p, m_l) (see below) of the block–size reduction BILU algorithm. The computations are done on a SUN SPARC station Ultra-1.

The third column of Table 1 shows the number of iterations as a measure of the convergence rate of the preconditioner. The iteration stopping criterion is $(C^{-1}r^{N_{it}}, r^{N_{it}}) < 10^{-6}$, where r^j stands for the residual at the jth iteration step of the preconditioned conjugate gradient method. Here m_p stands for the coarse–block size with respect to the *plane* blocks, as well as, m_l is the coarse–block size related to the *line* blocks.

TABLE 1

Performance of the 3D block-size reduction BILU solver.
Benchmark 2: Doubled pile system in a multilayer media;
$n_{x_1} = n_{x_2} = n_{x_3} = 32$; $N = 98304$.

m_p	m_l	number of iterations	user time in minutes
1	1	221	37
1	2	162	30
2	1	217	39
2	2	151	30
1	3	177	37
3	1	216	41
2	3	167	37
3	2	149	32
1	4	169	43
4	1	216	45
2	4	156	41
4	2	148	33

In general, the presented data are in a good agreement with the theoretical estimate (Theorem 1), where

$$\frac{H}{h} = \max\{\frac{n_{x_1}}{m_p}, \frac{n_{x_2}}{m_l}\}.$$

They illustrate the flexibility of the iterative algorithm. One can see the stronger influence of the reduction parameter m_l on the number of iterations.

The proper choice of the factorization parameters m_p and m_l, provides the opportunity to control effectively the convergence rate of the iterative solver. The number of the iterations is relatively large, that is a result from the coefficients jump. We recall that $E_{pile} = 31500$ MPa, $E_{soil} \in \{5.2, 9.4, 14.0, 21.0\}$ MPa. Note, that this situation is typical for the engineering applications under consideration. Compare to the similar test data concerning Benchmark 1, we also see an increase of the number of iterations (60% approximately) as a result of the more complicated problem (stressed–strained state of the media).

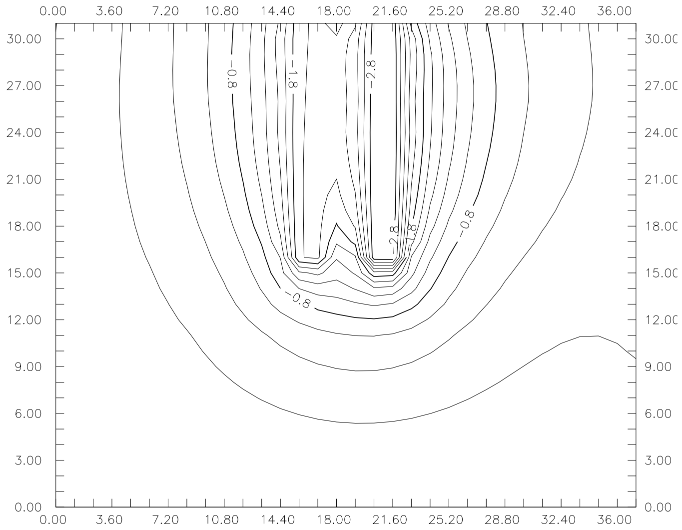

Figure 2: *3D simulation of a doubled pile system in a multilayer media; Cross section of the computational domain* Ω $\alpha : x_3 = 18.6$*; Isolines of the vertical displacements* u_1 *in* $[cm]$.

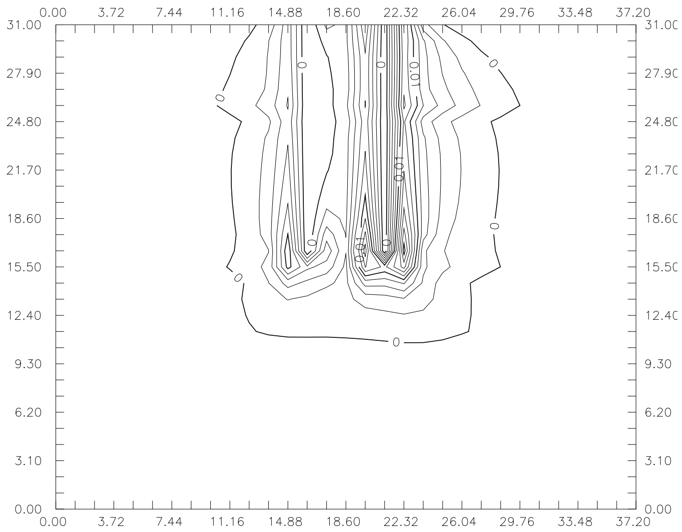

Figure 3: *3D simulation of a doubled pile system in a multilayer media; Cross section of the computational domain* Ω $\alpha : x_3 = 18.6$*; Isolines of the strain intensity* ϵ_0.

Finally, we examine the *real time* performance, measured by the *user time*. The numerical data show, that the computational times slightly vary when $1 \leq m_p \leq 4$ and $m_l = 2$ More precisely, the minimal time is achieved for $(m_p, m_l) \in \{(1, 2), (2, 2)\}$.

4 Engineering analysis

We analyze in this section some engineering aspects of the performed numerical tests with Benchmark 2 as described in §2. The test data correspond to a real upper bridge structure, that transfers the external loads to the foundation bed soil, through the pile system.

The computed vertical displacements u_3 related to the cross section $\alpha : x_3 = 18.6$ are presented by isolines in Fig. 2. The scale of the isolines is 0.2 cm. The maximal value $u_3^{max} = 2.963cm$ is in a good agreement with some measurements for similar soil layers. The analysis of the field of the displacements confirms that the size of the computational domain is well determined having in mind the considered loads. This means that the boundaries of the domain Ω are far enough from the zone of active interactions between the piles and the surrounding soil layers.

The isolines of the strain intensity ϵ_0 and of the vertical stress σ_{11} are given in Figs. 3-4. The scales of the isolines in these figures are as follows: a) for the strain intensity: 0.002; b) for the stresses: 0.001 kN/cm^2. The strains and the stresses are computed in the centers of the finite elements.The stresses only in the soil media are shown in Fig. 4. The maximal strain intensity is $\epsilon_0^{max} = 0.02362$. The zones with a big density of the isolines in Fig. 3 are these where a more considerable redistribution of the stresses can be expected if a nonlinear elasticity model is applied. Quadratic extrapolation of the normal stresses in the concrete bodies as well as in the soil layers and a successive averaging of the computed values are applied to obtain the normal stresses in the contact nodes between the top of the piles and the soil layers. In such a way we get the maximal vertical stress in the soil media $\sigma_{11}^{max}(soil) = 0.03252$ kN/cm^2. A more detailed analysis shows that approximately 12% of the vertical external load is transfered to the soil spots under the top of the piles. The figures also show that the zones under the tops of the piles require a local mesh refinement to achieve better accuracy of the strains/stresses. The last Fig. 5 presents the isolines of the sheare stresses σ_{12} in a scale of 0.001 kN/cm^2. The maximal value for this stress is $\sigma_{12}^{max} = 0.01238$ kN/cm^2.

5 Conclusions and remarks

As a general conclusion, the qualitative and the quantitative analysis of the presented test data show that the results obtained are realistic. We evaluate these results as a prospective next step in the 3D large–scale simulation of pile systems in a multilayer soil media.

The results presented in this report confirms the general conclusions, remarks, and practical requirements to the HIPERGEOS code, based on the previous analysis of results related to the Benchmark 1.

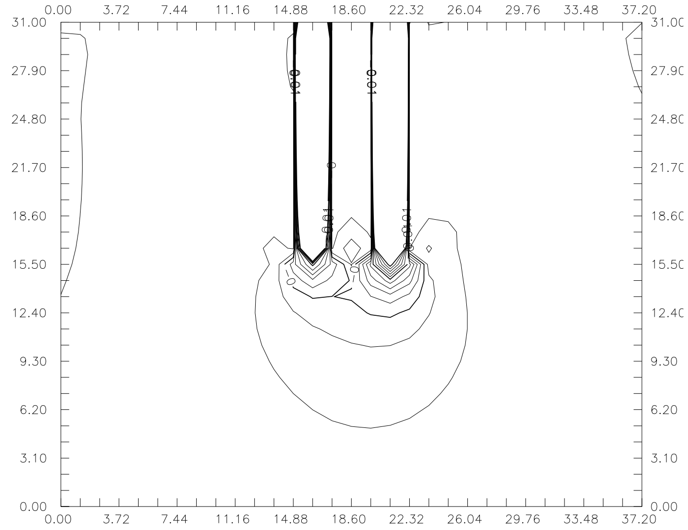

Figure 4: *3D simulation of a doubled pile system in a multilayer media; Cross section of the computational domain* Ω $\alpha : x_3 = 18.6$*; Isolines of the vertical stresses* σ_{11} *in* $[\frac{kN}{sm^2}]$.

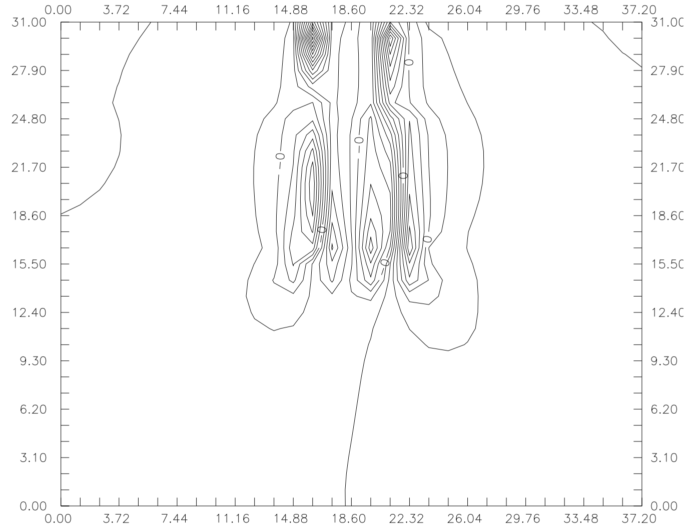

Figure 5: *3D simulation of a doubled pile system in a multilayer media; Cross section of the computational domain* Ω $\alpha : x_3 = 18.6$*; Isolines of the shear stresses* σ_{12} *in* $[\frac{kN}{sm^2}]$.

- The sizes of the computational domain are properly chosen, so that the influence of the boundary conditions is negligible with respect to the stressed–strained state in the zones of active interaction between the piles and the soil media.
- The local mesh refinement and the nonlinearity of the soil layers are naturally required to increase the accuracy of the strains/ stresses in the zones under the tops of the piles. The high density of the isolines indicates a concentration of the strains/stresses in these zones.

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