Benchmarking for HIPERGEOS

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Report GANMN 97-03

November 1997

[1]Supported by the "Fonds National de la Recherche Scientifique", Chercheur qualifié.

1 Introduction

In this report, we present the numerical results obtained with the solver DRBILU for a set of discrete linear elasticity problems that involved characteristic features of real-life applications met by the industrial partners of the project HIPERGEOS (Copernicus Grant No CP 94-00820).

Letting

$$A\mathbf{u} = \mathbf{b} \tag{1}$$

be the system to solve, this solver reads A and $\mathbf{b}$ from disk and computes an approximate solution vector $\mathbf{u}$ using the preconditioned Conjugate Gradient algorithm. The zero vector is used as initial approximation and iterations are stopped when the 2-norm of the relative residual error has been reduced by a factor 10^{-8}.

The preconditioner is based on the 3×3 block structure of the stiffness matrix induced by the separate ordering of the different components of the unknown displacements. More precisely, it is blockwise incomplete factorization of the block diagonal part D of this 3×3 block structure, previously reduced to an M-matrix by a diagonal compensation technique [1, 6] in cases the used discretization scheme does not directly lead to an M-matrix. The considered factorization algorithm is the one from [3], which is a block extension of the DRIC method [4] (the parameter was chosen so that the resulting upper bound equals $\frac{2}{3}\sqrt{n_y n_z}$, where n_y, n_z are the number of nodes in directions y and z respectively).

The considered data structure to store the upper triangular part of the (symmetric) matrix A is the "diagonal" format as described in [5]. This means that nonzero diagonals are stored in a rectangular array, and their offset with respect to the main diagonal in a companion integer vector. This scheme reproduces the performances of a direct (stencil based) addressing scheme, but is more versatile with respect to the use of different discretizations. Separate arrays are used to store D and the remaining part of A, which allows to keep minimal the storage requirements for the preconditioner.

Section 2 contains a synthetic description of the benchmark problems, and Section 3 the numerical results.

2 Benchmarks

2.1 Square footing problem

Description : Problem deals with the influence of flexible footing represented by localized pressure to the stress development in a soil medium. The computational domain is a parallelepiped 50x50x40 m which represents one quarter of the overall situation, see Figure below.

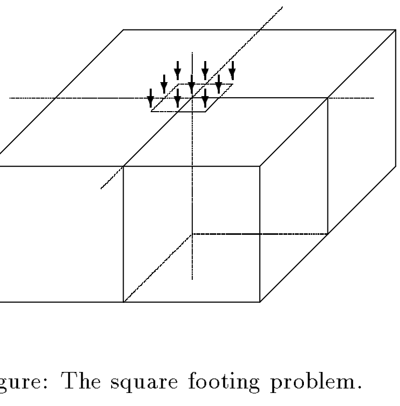

Figure: The square footing problem.

Equations used : equations of elasticity.

Material data : one isotropic material

- soil (E = 14 MPa, $\nu = 0.3$, $\rho = 2.5\, gcm^{-3}$)

Boundary conditions : zero normal displacements and zero tangential stresses on all sides except the top side. Stress on the top side.

Loading : pressure of 2.4 MPa on the square area 5x5 m at the left back corner of the top face (one quarter of footing).

Relevant user output : stresses.

Discretization : rectangular grid 40x40x40 nodes with a grid refinement under the footing. Maximal dimension of hexahedral bricks: 2.5x2.5x1.5 m, minimal dimension: 0.5x0.5x0.5 m. The discretization is made by the linear tetrahedral finite elements and gives the linear system of 192000 unknowns. The discretization is made by TGRID preprocessor and the linear system is created by FEMSYS code, see [1].

References: [2] R. Blaheta and R. Kohut: *Benchmarks and programs for testing iterative solvers for 3D problems of geomechanics,* Report 9502 (Dept. of Applied Mathematics, Institute of Geonics, Ostrava 1995)

Contact: R. Kohut (kohut@ugn.cas.cz), A. Kolcun (kolcun@ugn.cas.cz)

2.2 Stope stability problem

Description : Problem concerns mining in the coal seam: the computational domain is a cube of 150x150x150 m which contains the characteristic stope mining situation.

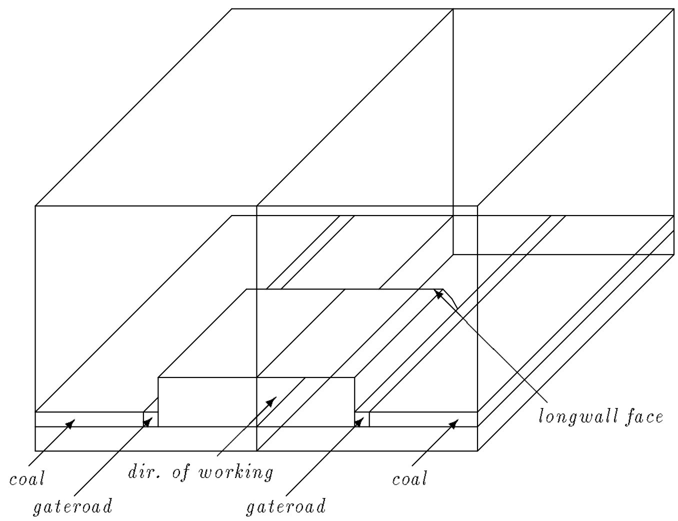

Figure: The stope mining problem.

Equations used : equations of elasticity.

Material data : three isotropic materials:

- rock (E = 8000 MPa, $\nu = 0.2$, $\rho = 2.5\,gcm^{-3}$)
- coal (E = 800 MPa, $\nu = 0.3$, $\rho = 1.3\,gcm^{-3}$)
- goaf (E = 200 MPa, $\nu = 0.4$, $\rho = 2.2\,gcm^{-3}$)

Boundary conditions : zero normal displacements and zero tangential stresses on all sides except the top side. Stress on the top side.

Loading : pressure of 12.5 MPa on the whole top side.

Relevant user output : stresses.

Discretization : rectangular grid of 40x40x40 nodes with the grid refinement in the area of mining. Maximal dimension of hexahedral bricks: 10x15x10 m, minimal dimension: 1x1.5x1.5 m.
The discretization is made by the linear tetrahedral finite elements and gives the linear system of 192000 unknowns. The discretization is made by TGRID preprocessor and the linear system is created by FEMSYS code, see [1].

References: [2] R. Blaheta and R. Kohut: *Benchmarks and programs for testing iterative solvers for 3D problems of geomechanics,* Report 9502 (Dept. of Applied Mathematics, Institute of Geonics, Ostrava 1995)

Contact: R. Kohut (kohut@ugn.cas.cz), A. Kolcun (kolcun@ugn.cas.cz)

2.3 Headgate problems

Description : Problems concern the stability of coal pillars which are periodically distributed in the corridor passing throw a coal seam. Beside the corridor, the situation include a mined out area excavated by the longwall mining technology. Computational domain is a parallelepiped 1800x237x86 m.

We shall consider three variants of the described problem:
A – the whole situation,
B – restricted problem situated in 2/3 of the whole computational domain,
C – restricted problem situated in 1/3 of the whole computational domain.

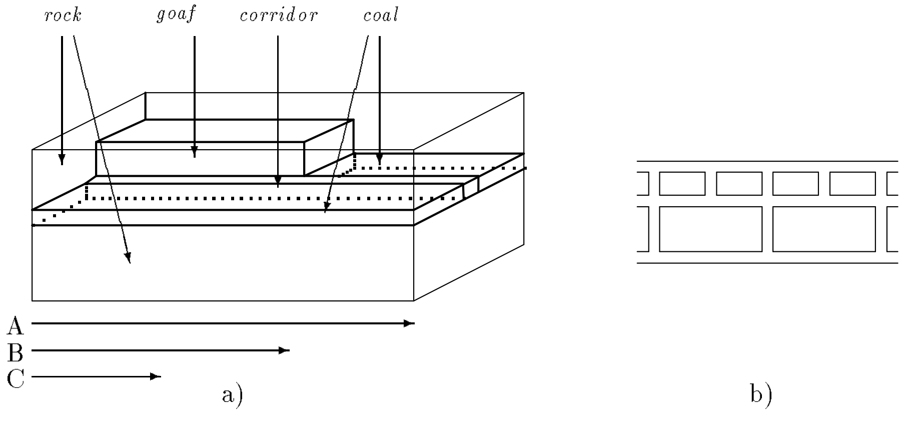

Figure: (a) Headgate problems with extent of the domain for the considered variants A, B and C, (b) top view on the detail of distribution of the coal pillars in the corridor.

Equations used : equations of elasticity.

Material data : three isotropic materials:

- rock (E = 8000 MPa, $\nu = 0.2$, $\rho = 2.5\,gcm^{-3}$)
- coal (E = 800 MPa, $\nu = 0.3$, $\rho = 1.3\,gcm^{-3}$)
- goaf (E = 80 MPa, $\nu = 0.4$, $\rho = 2.2\,gcm^{-3}$)

Boundary conditions : zero normal displacements and zero tangential stresses on all sides except the top side. Stress on the top side.

Loading : pressure of 12.5 MPa on the whole top side.

Relevant user output : stresses.

Discretization : rectangular grids of 302x26x33, 202x26x33, 102x26x33 nodes are used for the variants A, B and C, respectively. The grids are refined in the vicinity of the corridor. Maximal dimension of hexahedral bricks is: 6.1x16x6 m, minimal dimension is: 6.1x5.25x0.6 m.

The discretization is made by the linear tetrahedral finite elements and gives the linear system of 777348, 519948 and 262548 unknowns for the variants A, B and C, respectively. The discretization is made by TGRID preprocessor and the linear system is created by FEMSYS code, see [1].

References: [2] R. Blaheta and R. Kohut: *Benchmarks and programs for testing iterative solvers for 3D problems of geomechanics,* Report 9502 (Dept. of Applied Mathematics, Institute of Geonics, Ostrava 1995)

[2] R. Blaheta: *HIPERGEOS related benchmarks, software and testing,* HIPERGEOS Workshop, Rožnov p. R. 1996

Contact: R. Kohut (kohut@ugn.cas.cz), A. Kolcun (kolcun@ugn.cas.cz)

3 Numerical results

The results given below have been obtained by running DRBILU in double precision (FORTRAN 77) on a SUN SPARC20 (50 MHz, 128 Mb RAM) workstation. Input data files have been generated by running a double precision version of TGRID [2].

In the listings, under the item `DIAGONALS`, we give statistics about the nonzero diagonals (`%nonzero` is the proportion of entries treated as nonzero that are effectively nonzero). On the other hand, the extremal "roots of the associated polynomials" may be used as estimates of the extremal eigenvalues of the preconditioned system, and give thus information about the conditioning. Finally, note that the "time for preprocessing" is essentially the time needed to read the matrix and right hand side from disk.

3.1 Square footing problem

```
DIAGONALS :
 Coupling same equation:
            ioff:       0       1      40    1600
         nonzero:  192000  179320  179320  179360
  positive nonzero:  192000       0       0       0
        %nonzero:  100.00   93.40   93.40   93.42
 Coupling different equations:
            ioff:       0       1      40      41    1600    1601    1640    1641
         nonzero:  176320  349676  349676  346712  346712  343786  343786  340860
        %nonzero:   91.83   91.06   91.06   90.29   90.29   89.53   89.53   88.77

Memory needed (matrix & preconditioner):  32256000 bytes  (  30.76 Mb)
Memory needed for the iteration vectors:   9446400 bytes  (   9.01 Mb)

CONVERGENCE HISTORY
```

```
   0 0.10000E+01
  10 0.48264E+00
  20 0.17451E-01
  30 0.70757E-03
  40 0.39170E-04
  50 0.89560E-06
  60 0.14970E-07
  61 0.97125E-08

ROOTS OF THE ASSOCIATED POLYNOMIAL

  SMALLEST ROOTS at iteration   61  (at iteration   60):
  0.17900E+00    0.20200E+00    0.27973E+00    0.33895E+00    0.35543E+00
  (.17900E+00)   (.20200E+00)   (.27973E+00)   (.33906E+00)   (.35562E+00)

   LARGEST ROOTS at iteration   61  (at iteration   60):
   9.9523E+00     9.9522E+00     9.2298E+00     9.0100E+00     8.9153E+00
  (9.9523E+00)   (9.9522E+00)   (9.2298E+00)   (9.0100E+00)   (8.9153E+00)


time for preprocessing : real =  4.00E+01  CPU =  3.93E+01
time for factorization : real =  1.00E+00  CPU =  1.62E+00
time for solution :      real =  2.12E+02  CPU =  2.11E+02


DOF     Value (solution)        Value (r.h.s.)
  1 0.000000000000000D+00 0.000000000000000D+00
  2 0.000000000000000D+00 0.000000000000000D+00
  3 0.256184594370735D+01 0.222500241471060D+00
  4-0.239060247321954D-01 0.000000000000000D+00
  5 0.000000000000000D+00 0.000000000000000D+00
  6 0.255353083390605D+01 0.333601429994654D+00
  7-0.471164083881461D-01 0.000000000000000D+00
  8 0.000000000000000D+00 0.000000000000000D+00
  9 0.253904585092349D+01 0.333601429994654D+00
 10-0.702587373706160D-01 0.000000000000000D+00
 11 0.000000000000000D+00 0.000000000000000D+00
 12 0.251620100351005D+01 0.333601429994654D+00
 13-0.934977450132110D-01 0.000000000000000D+00
 14 0.000000000000000D+00 0.000000000000000D+00
 15 0.248342731000986D+01 0.333601429994654D+00
```

3.2 Stope stability problem

```
DIAGONALS :
 Coupling same equation:
             ioff:       0       1      40    1600
          nonzero:  192000  179031  179093  179012
 positive nonzero:  192000       0       0       0
         %nonzero:  100.00   93.25   93.28   93.24
 Coupling different equations:
             ioff:       0       1      40      41    1600    1601    1640    1641
          nonzero:  176148  349106  349226  346040  346024  342646  342886  339516
```

```
        %nonzero:   91.74   90.91   90.94   90.11   90.11   89.23   89.29   88.42

Memory needed (matrix & preconditioner):  32256000 bytes  (  30.76 Mb)
Memory needed for the iteration vectors:   9446400 bytes  (   9.01 Mb)

CONVERGENCE HISTORY

   0 0.10000E+01
  10 0.43213E+00
  20 0.70393E-01
  30 0.52753E-02
  40 0.51766E-03
  50 0.39992E-04
  60 0.18248E-05
  70 0.15059E-06
  80 0.10736E-07
  81 0.82346E-08

ROOTS OF THE ASSOCIATED POLYNOMIAL

  SMALLEST ROOTS at iteration   81  (at iteration   80):
  0.17372E+00    0.26903E+00    0.29355E+00    0.34668E+00    0.44556E+00
  (.17372E+00)   (.26903E+00)   (.29356E+00)   (.34877E+00)   (.45323E+00)

   LARGEST ROOTS at iteration   81  (at iteration   80):
   2.0779E+01     2.0779E+01     2.0658E+01     2.0658E+01     1.9764E+01
  (2.0779E+01)   (2.0779E+01)   (2.0658E+01)   (2.0658E+01)   (1.9764E+01)


time for preprocessing : real =  4.00E+01  CPU =  3.98E+01
time for factorization : real =  2.00E+00  CPU =  1.63E+00
time for solution :      real =  2.87E+02  CPU =  2.86E+02


DOF     Value (solution)        Value (r.h.s.)
  1 0.000000000000000D+00 0.000000000000000D+00
  2 0.000000000000000D+00 0.000000000000000D+00
  3 0.333166968562669D+00 0.391086136197917D+03
  4-0.318735720683315D-02 0.000000000000000D+00
  5 0.000000000000000D+00 0.000000000000000D+00
  6 0.332635108212496D+00 0.585684292708333D+03
  7-0.618119227091537D-02 0.000000000000000D+00
  8 0.000000000000000D+00 0.000000000000000D+00
  9 0.331510776273835D+00 0.585684292708333D+03
 10-0.890779896385224D-02 0.000000000000000D+00
 11 0.000000000000000D+00 0.000000000000000D+00
 12 0.329754761146194D+00 0.585684292708333D+03
 13-0.112742491677371D-01 0.000000000000000D+00
 14 0.000000000000000D+00 0.000000000000000D+00
 15 0.327400060503619D+00 0.363716485677083D+03
```

3.3 Headgate problem

3.3.1 version C

```
DIAGONALS :
 Coupling same equation:
             ioff:       0       1     102    2652
          nonzero:  262548  247614  237774  243588
  positive nonzero:  262548       0       0       0
         %nonzero:  100.00   94.31   90.56   92.78
 Coupling different equations:
             ioff:       0       1     102     103    2652    2653    2754    2755
          nonzero:  240432  477232  467728  464344  470864  466732  456144  452322
         %nonzero:   91.58   90.88   89.07   88.43   89.67   88.89   86.87   86.14

Memory needed (matrix & preconditioner):  44108064 bytes  (  42.06 Mb)
Memory needed for the iteration vectors:  12984192 bytes  (  12.38 Mb)

CONVERGENCE HISTORY

   0 0.10000E+01
  10 0.23509E+00
  20 0.14483E-01
  30 0.42007E-03
  40 0.30501E-04
  50 0.59443E-05
  60 0.94895E-06
  70 0.50421E-07
  78 0.85796E-08

ROOTS OF THE ASSOCIATED POLYNOMIAL

  SMALLEST ROOTS at iteration   78  (at iteration   77):
  0.10401E+00    0.17169E+00    0.17376E+00    0.24181E+00    0.26797E+00
  (.10402E+00)   (.17186E+00)   (.17376E+00)   (.24270E+00)   (.28730E+00)

   LARGEST ROOTS at iteration   78  (at iteration   77):
   1.1816E+01     1.1739E+01     1.1693E+01     1.1654E+01     1.1596E+01
  (1.1816E+01)   (1.1739E+01)   (1.1693E+01)   (1.1654E+01)   (1.1596E+01)


time for preprocessing : real =  4.90E+01  CPU =  4.89E+01
time for factorization : real =  3.00E+00  CPU =  2.19E+00
time for solution :      real =  3.68E+02  CPU =  3.68E+02


DOF     Value (solution)       Value (r.h.s.)
  1 0.000000000000000D+00 0.000000000000000D+00
  2 0.000000000000000D+00 0.000000000000000D+00
  3 0.660833771588015D+00 0.410255900566667D+03
  4-0.138016573202486D-03 0.000000000000000D+00
  5 0.000000000000000D+00 0.000000000000000D+00
  6 0.660749603039118D+00 0.614785645200000D+03
  7-0.259236390346380D-03 0.000000000000000D+00
```

```
  8 0.000000000000000D+00 0.000000000000000D+00
  9 0.660608139155049D+00 0.614785645200000D+03
 10-0.356662827470457D-03 0.000000000000000D+00
 11 0.000000000000000D+00 0.000000000000000D+00
 12 0.660432477943304D+00 0.614785645200000D+03
 13-0.430604365000265D-03 0.000000000000000D+00
 14 0.000000000000000D+00 0.000000000000000D+00
 15 0.660240901565590D+00 0.614785645200000D+03
```

3.3.2 version B

```
DIAGONALS :
 Coupling same equation:
             ioff:       0       1     202    5252
          nonzero:  519948  494494  472574  484188
 positive nonzero:  519948       0       0       0
         %nonzero:  100.00   95.10   90.89   93.12
 Coupling different equations:
             ioff:       0       1     202     203    5252    5253    5454    5455
          nonzero:  479392  952992  932528  927224  938864  932032  909444  903222
         %nonzero:   92.20   91.64   89.68   89.17   90.28   89.63   87.46   86.86

Memory needed (matrix & preconditioner):  87351264 bytes  (  83.30 Mb)
Memory needed for the iteration vectors:  25713792 bytes  (  24.52 Mb)

CONVERGENCE HISTORY

   0 0.10000E+01
  10 0.22819E+00
  20 0.14290E-01
  30 0.13693E-02
  40 0.41638E-03
  50 0.22545E-03
  60 0.50983E-04
  70 0.56260E-05
  80 0.48656E-06
  90 0.43630E-07
  97 0.97342E-08

ROOTS OF THE ASSOCIATED POLYNOMIAL

  SMALLEST ROOTS at iteration   97  (at iteration   96):
  0.28434E-01    0.10555E+00    0.16856E+00    0.17383E+00    0.19014E+00
  (.28434E-01)   (.10555E+00)   (.16877E+00)   (.17386E+00)   (.19088E+00)

   LARGEST ROOTS at iteration   97  (at iteration   96):
   1.1816E+01     1.1716E+01     1.1705E+01     1.1682E+01     1.1643E+01
  (1.1816E+01)   (1.1716E+01)   (1.1705E+01)   (1.1682E+01)   (1.1642E+01)


time for preprocessing : real =  1.23E+02  CPU =  1.14E+02
time for factorization : real =  7.00E+00  CPU =  5.05E+00
time for solution :      real =  9.67E+02  CPU =  9.28E+02
```

```
DOF     Value (solution)        Value (r.h.s.)
  1 0.000000000000000D+00 0.000000000000000D+00
  2 0.000000000000000D+00 0.000000000000000D+00
  3 0.660852167144282D+00 0.410255900566667D+03
  4-0.134566627603180D-03 0.000000000000000D+00
  5 0.000000000000000D+00 0.000000000000000D+00
  6 0.660768008375652D+00 0.614785645200000D+03
  7-0.252338770674502D-03 0.000000000000000D+00
  8 0.000000000000000D+00 0.000000000000000D+00
  9 0.660626551705483D+00 0.614785645200000D+03
 10-0.346319353799412D-03 0.000000000000000D+00
 11 0.000000000000000D+00 0.000000000000000D+00
 12 0.660450894935753D+00 0.614785645200000D+03
 13-0.416816799033761D-03 0.000000000000000D+00
 14 0.000000000000000D+00 0.000000000000000D+00
 15 0.660259321202252D+00 0.614785645200000D+03
```

3.3.3 version A

```
DIAGONALS :
 Coupling same equation:
             ioff:       0       1     302    7852
          nonzero:  777348  741374  707374  724788
  positive nonzero:  777348       0       0       0
         %nonzero:  100.00   95.37   91.00   93.24
 Coupling different equations:
             ioff:       0       1     302     303    7852    7853    8154    8155
          nonzero:  718352 1428752 1397328 1390104 1406864 1397332 1362744 1354122
         %nonzero:   92.41   91.90   89.88   89.41   90.49   89.88   87.65   87.10

Memory needed (matrix & preconditioner): 130594464 bytes  ( 124.54 Mb)
Memory needed for the iteration vectors:  38443392 bytes  (  36.66 Mb)

CONVERGENCE HISTORY

   0 0.10000E+01
  10 0.20977E+00
  20 0.14394E-01
  30 0.59789E-02
  40 0.29940E-02
  50 0.35105E-02
  60 0.71670E-03
  70 0.13931E-03
  80 0.22323E-04
  90 0.23250E-05
 100 0.19328E-06
 110 0.19115E-07
 113 0.96083E-08

ROOTS OF THE ASSOCIATED POLYNOMIAL
```

```
  SMALLEST ROOTS at iteration  113  (at iteration  112):
  0.12927E-01    0.47959E-01    0.10417E+00    0.16020E+00    0.17339E+00
  (.12927E-01)   (.47959E-01)   (.10417E+00)   (.16024E+00)   (.17341E+00)

   LARGEST ROOTS at iteration  113  (at iteration  112):
   1.1816E+01     1.1706E+01     1.1698E+01     1.1687E+01     1.1670E+01
  (1.1816E+01)   (1.1706E+01)   (1.1698E+01)   (1.1687E+01)   (1.1670E+01)


time for preprocessing : real =  2.01E+02  CPU =  1.89E+02
time for factorization : real =  4.60E+01  CPU =  8.77E+00
time for solution :      real =  1.79E+03  CPU =  1.63E+03


DOF     Value (solution)        Value (r.h.s.)
  1 0.000000000000000D+00 0.000000000000000D+00
  2 0.000000000000000D+00 0.000000000000000D+00
  3 0.661068239639349D+00 0.410255900566667D+03
  4-0.922277362789388D-04 0.000000000000000D+00
  5 0.000000000000000D+00 0.000000000000000D+00
  6 0.660984194223684D+00 0.614785645200000D+03
  7-0.167681310437670D-03 0.000000000000000D+00
  8 0.000000000000000D+00 0.000000000000000D+00
  9 0.660842814466193D+00 0.614785645200000D+03
 10-0.219351871718064D-03 0.000000000000000D+00
 11 0.000000000000000D+00 0.000000000000000D+00
 12 0.660667195757284D+00 0.614785645200000D+03
 13-0.247540602988567D-03 0.000000000000000D+00
 14 0.000000000000000D+00 0.000000000000000D+00
 15 0.660475632894262D+00 0.614785645200000D+03
```

References

[1] O. Axelsson: *Iterative Solution Methods*, University Press, Cambridge, 1994.

[2] R. Blaheta, R. Kohut: Benchmarks and programs for testing iterative solvers for 3D problems of geomechanics, TR – DAM – 9502, IGAS Ostrava, 1995.

[3] Magolu Monga-made and Y. Notay: Dynamically relaxed block incomplete factorizations, TR – 985, Department of Mathematics, University of Utrecht, 1996.

[4] Y. Notay: DRIC: a dynamic version of the RIC method, *J. Num. Lin. Alg. with Appl.*, 1 (1994), 511–532.

[5] Y. Saad: SPARSKIT: a basic tool kit for sparse matrix computations, TR, University of Minnesota, Minneapolis, 1994.

[6] P. Saint-Georges, G. Warzee, R. Beauwens, and Y. Notay: High performance PCG solvers for FEM structural analyses, *Int. J. Numer. Meth. Eng.*, 39 (1996), 1313–1340.