SCGR2023

Workshop "Singularities and Curvature in General Relativity"

19-23 June 2023 - Nijmegen, The Netherlands

Schedule

Place: ground floor of the Huygens building, Faculty of Science, Radboud University Nijmegen

Address: Heyendaalseweg 135, 6525 AJ Nijmegen, The Netherlands

Day	Time	Activity	Room	Speaker	Affiliation	Title
Monday 6/19	9:00-9:15	Registration/Welcome	HG00.303			
	9:15-9:30	Opening Remarks				
	9:30-10:30	Keynote Lecture		M. Kiessling	Rutgers University	The Einstein-Infeld-Hoffmann legacy in mathematical relativity
	10:30-11:00	Coffee break				
	11:00-12:00	Keynote Lecture		M. Kunzinger	University of Vienna	Ricci curvature bounds: synthetic versus analytic
	12:15-13:45	Lunch				
	13:45-14:45	Minicourse A	HG00.071	Y. Shlapentokh-Rothman	University of Toronto	Recent Developments in Mathematical General Relativity I
	14:50-15:40	Invited Address		Q. Wang	University of Oxford	Rough solutions of the 3-D compressible Euler equations
	15:40-17:00	Coffee and Poster Session				
	18:30--	Conference Dinner	Arsenaal 1824			
Tuesday 6/20	9:00-10:00	Minicourse B	HG00.303	A. Bernig	Goethe University Frankfurt	Curvature measures of singular spaces in the Riemannian and pseudo-Riemannian setting I
	10:00-10:30	Coffee break				
	10:30-11:20	Invited Address		M. Sánchez	Universidad de Granada	The geometry of globally hyperbolic spacetimes with timelike-boundary
	11:25-12:15	Invited Address		R. Perales	CONACyT-UNAM	Intrinsic flat stability of the positive mass theorem for graphical manifolds
	12:15-12:25	Group Photo				
	12:25-13:45	Lunch				
	13:45-14:45	Minicourse A	HG00.307	Y. Shlapentokh-Rothman	University of Toronto	Recent Developments in Mathematical General Relativity II
	14:50-15:40	Invited Address		I. Stavrov	Lewis & Clark College	On representing relativistic objects as aggregates of point-sources
	15:40-16:10	Coffee break				
	16:10-17:00	Invited Address		S. Suhr	Ruhr-Universität Bochum	Gromov-Hausdorff Theory for Lorentzian Spaces

Day	Time	Activity		Speaker	Affiliation	Title
Wednesday 6/21	9:00-10:00	Minicourse B	HG00.303	A. Bernig	Goethe University Frankfurt	Curvature measures of singular spaces in the Riemannian and pseudo-Riemannian setting II
	10:00-10:30	Coffee break				
	10:30-11:00	Contributed Talks Session I		C. Sämann	University of Oxford	Lorentzian Hausdorff measures, doubling and more
	11:05-11:35			E. Toprak	Yale University	Dirac operators under the influence of general-relativistic gravity: self-adjointness and spectrum
	11:40-12:10			E. De Amorim	University of Cologne	The Maxwell-Bopp-Lande-Thomas-Podolsky-Einstein system for a static point source
	12:15-13:45	Lunch				
		Free Afternoon / Excursion				
Thursday 6/22	9:00-10:00	Minicourse B	HG00.303	A. Bernig	Goethe University Frankfurt	Curvature measures of singular spaces in the Riemannian and pseudo-Riemannian setting III
	10:00-10:30	Coffee break				
	10:30-11:20	Invited Address		D. Faifman	Tel Aviv University	Lipschitz-Killing curvatures of generic subsets of pseudo-Riemannian manifolds
	11:25-12:15	Invited Address		N. Gigli	SISSA	Hyperbolic nonsmooth calculus
	12:15-13:45	Lunch				
	13:45-14:45	Minicourse A	HG00.303	Y. Shlapentokh-Rothman	University of Toronto	Recent Developments in Mathematical General Relativity III
	14:50-15:40	Invited Address		M. Van de Moortel	Rutgers University	Singularities in the interior of black holes
	15:40-16:10	Coffee break				
	16:10-17:00	Invited Address		G. Holzegel	Universität Münster	Quasilinear wave equations on black hole backgrounds
Friday 6/23	9:00-10:00	Keynote Lecture	HG00.303	R. McCann	University of Toronto	A synthetic null energy condition
	10:00-10:30	Coffee break				
	10:30-11:00	Contributed Talks Session II		M. Braun	Fields Institute	Gromov's reconstruction theorem and measured Gromov-Hausdorff convergence in Lorentzian geometry
	11:05-11:35			M. Flaim	Universität Bonn	An isoperimetric-type inequality in Lorentzian spacetimes using Geometric flows
	11:40-12:10			M. Reintjes	City University of Hong Kong	On the regularity implied by the assumptions of geometry
	12:15-13:45	Lunch				
	13:45-14:15	Contributed Talks Session III	HG00.303	L. García-Heveling	Radboud University	Finite causal volumes
	14:20-14:50			I. Kadar	University of Cambridge	Small data global existence for systems of wave equations: The role of asymptotics
	14:55-15:25			G. Hood	Imperial College London	A scattering construction for the wave equation on spacetimes with negative cosmological constant
	15:25-15:55	Coffee break				
	15:55-16:55	Keynote Lecture		K. Landsman	Radboud University	Singularities in GR: From Einstein to Hawking
	16:55-17:00	Closing Remarks				

Abstracts

Minicourse A

Yakov Shlapentokh-Rothman (University of Toronto)

Recent Developments in Mathematical General Relativity

Lecture 1 (Cosmic Censorship): In the first lecture we will give a review of/crash course in enough General Relativity to understand the motivations for and the statements of the famous weak and strong cosmic censorship conjectures.

Lecture 2 (Black Holes Inside and Out): In the second lecture we will give an overview of developments around the interrelated topics of the stability of black hole exteriors and the stability/instability of the corresponding interiors.

Lecture 3 (Low-Regularity Solutions and Singularities): In the final lecture we will describe some situations where one can successfully study the dynamics of the Einstein vacuum equations in a low-regularity setting and the related problem of understanding singularity formation.

Minicourse B

Andreas Bernig (Goethe University Frankfurt)

Curvature measures of singular spaces in the Riemannian and pseudo-Riemannian setting

The general aim of this mini-course is to show how the modern theory of valuations can be used in order to extend classical curvature notions (in particular scalar curvature) from (pseudo-)Riemannian manifolds to certain singular, but tame sets.

The course consists of three lectures. In the first lecture, we introduce several classes of singular subsets of Euclidean space or of manifolds: convex sets, sets of positive reach, manifolds with corners, differentiable polyhedra, semialgebraic and subanalytic sets, sets definable in some o-minimal structure. The main point is that such sets, although being singular, are tame enough to admit an Euler characteristic. We also introduce the normal cycle of such sets.

In the second lecture, we introduce intrinsic volumes in the Euclidean/Riemannian setting. The tube formulas by Steiner and Weyl will be discussed, as well as the modern valuation-theoretic approach to Weyl's principle. A special emphasis will be given to the total scalar curvature (Einstein-Hilbert functional), which is one of the intrinsic volumes. It can be linked to curvature bounds in the sense of metric geometry (Alexandrov spaces). Moreover, the classical first variation formula for the total scalar curvature can be extended to the singular setting and yields a distributional Einstein tensor of singular spaces.

In the third lecture, we explore how much of this theory carries over to the pseudo-Euclidean and pseudo-Riemannian case. In these cases, one needs generalized valuations, and I will spend some time explaining this notion. It turns out that one can still define intrinsic volumes, which are complex-valued generalized valuations. They satisfy a pseudo-Riemannian version of the Weyl principle. This lecture will be followed by D. Faifman's talk, where an extension to generic submanifolds of pseudo-Riemannian manifolds is discussed.

Monday – 19 June 2023

Michael Kiessling (Rutgers University)

The Einstein-Infeld-Hoffmann legacy in mathematical relativity

In their celebrated 1938 paper ``The Gravitational Equations and the Problem of Motion,'' Einstein, Infeld, and Hoffmann claimed that Einstein's vacuum field equations, respectively the Einstein-Maxwell system, would determine the equations of motion of (charged) point particles whose ``world lines'' were replaced with timelike singularities of the spacetime. Alas, their mathematical manipulations do not support their claims. This talk surveys the state of affairs in the pursuit of a well-posed relativistic joint initial value problem for charged point particles, treated as singularities, and their electromagnetic fields and the curved spacelike punctured slices they interact with.

Michael Kunzinger (University of Vienna)

Ricci curvature bounds: synthetic versus analytic

We compare and relate the two main approaches to defining Ricci curvature bounds for Riemannian manifolds with metrics of regularity below C^2. On the one hand, weak derivatives and the notion of positive distributions can be applied to metrics of low regularity. On the other hand, optimal transport theory gives a characterization of Ricci bounds via displacement convexity entropy functionals, which carries over even to the general setting of metric measure spaces. Both approaches are compatible with the classical definition via the Ricci tensor (and hence with each other) in the case of a smooth metric. We show that distributional bounds imply entropy bounds for C^1 metrics and that the converse is true for metrics of regularity $C^{1,1}$ under an additional condition on convergence of regularizations. We also give an outlook on ongoing work relating analytic and synthetic approaches to the singularity theorems of General Relativity.

Qian Wang (University of Oxford)

Rough solutions of the 3-D compressible Euler equations

I will talk about my work on the compressible Euler equations. We prove the local-in-time existence the solution of the compressible Euler equations in 3-D, for the Cauchy data of the velocity, density and vorticity $(v, \rho, \omega) \in H^s \times H^s \times H^{s'}$, $2<s'<s$. The result extends the sharp result of Smith-Tataru and Wang, established in the irrotational case, i.e $\omega = 0$, which is known to be optimal for $s>2$. At the opposite extreme, in the incompressible case, i.e. with a constant density, the result is known to hold for $\omega \in H^s$, $s>3/2$ and fails for $s< 3/2$, see the work of Bourgain-Li. It is thus natural to conjecture that the optimal result should be $(v, \rho, \omega) \in H^s \times H^s \times H^{s'}$, $s>2$, $s'>3/2$. We view our work as an important step in proving the conjecture. The main difficulty in establishing sharp well-posedness results for general compressible Euler flow is due to the highly nontrivial interaction between the sound waves, governed by quasilinear wave equations, and vorticity which is transported by the flow. To overcome this difficulty, we separate the dispersive part of sound wave from the transported part, and gain regularity significantly by exploiting the nonlinear structure of the system and the geometric structures of the acoustic spacetime.

(The abstracts for the posters can be found at the end)

Tuesday – 20 June 2023

Miguel Sánchez (Universidad de Granada)

The geometry of globally hyperbolic spacetimes-with-timelike-boundary

Globally hyperbolic spacetimes with-timelike-boundary are useful to model both deterministic spacetimes with a cut-off and conformal completions of spacetimes with naked singularities distributed on a timelike hypersurface. In this talk, mainly based on [1], some of their properties will be reviewed, including those related to causality, the existence of global orthogonal splittings, the relation between properties of the boundary and the interior of spacetime and the Cauchy problem.

[1] L.A. Aké, J.L. Flores, M. Sánchez: Structure of globally hyperbolic spacetimes-with-timelike-boundary. Rev. Mat. Iberoam. 37 (2021), no. 1, pp. 45–94.

Raquel Perales (National Autonomous University of Mexico, Oaxaca)

Intrinsic flat stability of the positive mass theorem for graphical manifolds

The rigidity of the Riemannian positive mass theorem for asymptotically flat manifolds states that the total mass of such a manifold is zero if and only if the manifold is isometric to the Euclidean space. This leads us to ask us whether a stability result holds. We will provide a positive answer for a class of asymptotically flat graphical manifolds described by Huang-Lee-Sormani by using the intrinsic flat distance.

Iva Stavrov (Lewis & Clark College)

On representing relativistic objects as aggregates of point-sources

An earlier work of the speaker points to a possibility that ``macroscopic" relativistic objects can be viewed as aggregates of idealized relativistic point-sources; specifically, one employs intrinsic flat limits of Brill-Lindquist-Riemann sums. The latter, however, implicitly assume (asymptotically) Euclidean background, and as such they do not automatically extend to other settings. Following a brief overview of the prior work the speaker will discuss the adjustments needed for the compact setting, focusing primarily on the role played by the Green's function of the conformal Laplacian.

Stefan Suhr (Ruhr-Universität Bochum)

Gromov-Hausdorff Theory for Lorentzian Spaces

Motivated by longstanding open question in the theory of the Einstein equations such as the strong cosmic censorship conjecture the talk will introduce a notion of Gromov-Hausdorff distance for a class of abstract Lorentzian spaces, called bounded Lorentz-metric spaces. I will discuss the definition and key examples as well as first stability results for natural geometric conditions under this notion of convergence. This is joint work with Ettore Minguzzi (Florence).

Wednesday – 21 June 2023

Clemens Sämann (University of Oxford)

Lorentzian Hausdorff measures, doubling and more

We define a one-parameter family of canonical volume measures on Lorentzian length spaces. In the Lorentzian setting, this allows us to define a geometric dimension - akin to the Hausdorff dimension for metric spaces - that distinguishes between e.g. spacelike and null subspaces of Minkowski spacetime. The volume measure corresponding to its geometric dimension gives a natural reference measure on a synthetic or limiting spacetime, and allows us to define what it means for such a spacetime to be collapsed (in analogy with metric measure geometry and the theory of Riemannian Ricci limit spaces). As a crucial tool we introduce a doubling condition for causal diamonds and a notion of causal doubling measures. Moreover, applications to continuous spacetimes and connections to synthetic timelike curvature bounds are given. This is joint work with Robert J. McCann and the article is available at arXiv:2110.04386 [math-ph]. If time permits we will also discuss recent work on causally doubling measures.

Ebru Toprak (Yale University)

Dirac operators under the influence of general-relativistic gravity: self-adjointness and spectrum

In this talk, I will speak about how the static non-linear electromagnetic-vacuum spacetime of a point nucleus with negative bare mass affects the self-adjointness, and the spectrum of the general relativistic Dirac Hamiltonian for a test electron, without and with an anomalous magnetic moment. I will first discuss the Reissner–Weyl–Nordström (RWN) spacetime both inside and outside of the event horizon. Then I will present a study that interpolates between the extreme cases of a test electron in a) RWN, which sports a very strong curvature singularity with negative infinite bare mass, and b) the Hoffmann spacetime (Born or Born–Infeld's electromagnetic vacuum) with vanishing bare mass, which features the mildest possible curvature singularity. These are joint work with Michael K.-H Kiessling and A.Shadi Tahvildar-Zadeh.

Erik De Amorim (University of Cologne)

The Maxwell-Bopp-Lande-Thomas-Podolsky-Einstein system for a static point source

We discuss the existence of a static, spherically symmetric spacetime that is the solution of the Einstein field equations coupled with an electric field obeying the equations of electromagnetism of Maxwell-Bopp-Lande-Thomas-Podolsky for a static point charge. Contrary to what happens with the Reissner-Weyl-Nordstrom spacetime, the electric-field energy is finite, just as for this same theory on a background flat spacetime. We also discuss the behavior of certain spacetime-curvature invariants around the singularity.

Thursday – 22 June 2023

Dmitry Faifman (Tel Aviv University)

Lipschitz-Killing curvatures of generic subsets of pseudo-Riemannian manifolds

In the mini-course of A. Bernig, intrinsic volumes and the corresponding Lipschitz-Killing curvature measures on general pseudo-Riemannian manifolds are introduced. Those are generalized valuations and curvatures, which therefore cannot be applied to arbitrary submanifolds. We will introduce a regularity condition for submanifolds, under which restriction and evaluation are possible. We will then see that this is in fact an intrinsic notion, allowing to extend intrinsic volumes to a generic class of manifolds with signature-changing metric. As a simple application, we will derive a Chern-Gauss-Bonnet theorem for generic domains in pseudo-Riemannian manifolds. Time permitting, we will touch upon the integral-geometric side of the intrinsic volumes. Based on a joint work with A. Bernig and G. Solanes.

Nicola Gigli (SISSA)

Hyperbolic nonsmooth calculus

In the last 25 years, tremendous progresses have been made in the field of non-smooth analysis: after the pioneering works of the Finnish school and Cheeger's seminal contribution, interest on the topic has been revamped by Lott-Sturm-Villani's papers on weak lower Ricci curvature bounds.
More recently, there has been a surging interest in non-smooth 'hyperbolic' geometry, i.e. in spaces whose smooth counterpart are Lorentzian manifolds rather than 'elliptic' Riemannian ones. Motivations come both from geometry and physics and concern in particular, after works of Cavalletti, McCann, Mondino, Suhr, genuinely non-smooth theories of gravity. These new geometries require new calculus tools: in this talk I will present some partial, but promising, results.
Based on joint works with Beran, Braun, Calisti, McCann, Ohanyan, Rott, Saemann.

Maxime Van de Moortel (Rutgers University)

Singularities in the interior of black holes

The analysis of the interior of a dynamical black hole, namely the region beyond the event horizon, is intimately connected to spacetime singularities and constitutes an essential step in the proof of the celebrated Strong Cosmic Censorship Conjecture. I will survey the recent mathematical results in the black hole interior, most of which under spherical symmetry, and explain how the treatment of these black hole interior singularities also unveils rich developments in Lorentzian Geometry.

Gustav Holzegel (Universität Münster)

Quasilinear wave equations on black hole backgrounds

After a brief introduction to the geometric features of black holes, I will discuss recent joint work with Dafermos, Rodnianski and Taylor introducing a new scheme to prove small data global existence results for quasilinear wave equations on black hole backgrounds (see arXiv:2212.14093). A key ingredient is a new physical space estimate at the top order which avoids understanding the detailed structure of trapped geodesics. The relation to the stability problem for black holes will also be discussed.

Friday – 23 June 2023

Robert McCann (University of Toronto)

A synthetic null energy condition

While Einstein's theory of gravity is formulated in a smooth setting, the celebrated singularity theorems of Hawking and Penrose describe many physical situations in which this smoothness must eventually breakdown. In positive-definite signature, there is a highly successful theory of metric and metric-measure geometry which includes Riemannian manifolds as a special case, but permits the extraction of nonsmooth limits under curvature and dimension bounds analogous to the energy conditions in relativity: here sectional curvature is reformulated through triangle comparison, while Ricci curvature is reformulated using entropic convexity along geodesics of probability measures. This lecture explores recent progress in the development of an analogous theory in Lorentzian signature, whose ultimate goal is to provide a nonsmooth theory of gravity. We highlight how the null energy condition of Penrose admits a nonsmooth formulation as a variable lower bound on timelike Ricci curvature.

Mathias Braun (Fields Institute)

Gromov's reconstruction theorem and measured Gromov-Hausdorff convergence in Lorentzian geometry

An important current topic in nonsmooth general relativity is to find a good notion of convergence of Lorentzian spaces. While recent works have introduced promising analogues to Gromov-Hausdorff convergence, in this talk we concentrate on its measured counterpart. We first prove a Lorentzian Gromov reconstruction theorem, which indicates a good notion of isomorphy of measured Lorentzian spaces. Based on that, we propose different definitions of measured Lorentz-Gromov-Hausdorff convergence. Finally, we outline their mutual relation as well as possible applications. In collaboration with Clemens Sämann (University of Oxford).

Marco Flaim (Universität Bonn)

An isoperimetric-type inequality in Lorentzian spacetimes using geometric flows

Thanks to their regularizing properties, geometric flows can be used to prove geometric inequalities in certain Riemannian manifolds. An example is the mean curvature flow for the isoperimetric inequality on Ricci-nonnegative, asymptotically Euclidean Riemannian manifolds. Here we want to mimic this technique in certain Lorentzian manifolds, on the model of Friedmann universes. The tool will be again geometric flows and we will obtain a reversed isoperimetric inequality for space-like hypersurfaces. While in the Riemannian setting a deep study of weak solutions of the flows was needed, here the causal structure allows us to work with smooth, immortal solutions.

Moritz Reintjes (City University of Hong Kong)

On the regularity implied by the assumptions of geometry

We establish that curvature alone controls the derivatives of a connection, including the gravitational metrics of General Relativity, and the Yang-Mills connections of Particle Physics. Specifically, we prove that the regularity of L^p connections can be lifted by coordinate/gauge transformation to one derivative above their L^p bounded Riemann curvature, (i.e., to *optimal regularity*), thereby removing apparent singularities in the underlying geometry. As an application in General Relativity, our optimal regularity result implies that the Lipschitz continuous metrics of shock wave solutions of the Einstein-Euler equations are non-singular, (geodesic curves, locally inertial coordinates and the Newtonian limit all exist in a classical sense). Moreover, by the extra connection derivative, we extend *Uhlenbeck compactness* from Riemannian to Lorentzian geometry, and from compact to non-compact gauge groups. The proofs are based on our discovery of, and existence theory for, a novel system of non-linear partial differential equations, (the RT-equations), non-invariant equations which are *elliptic* independent of metric signature, and which provide a general procedure for constructing coordinate and gauge transformations that regularize spacetime connections.

Leonardo García-Heveling (Radboud University)

Finite causal volumes

In this talk, I will argue that spacetimes containing a point whose causal future has finite volume should be regarded as singular or incomplete. Then I will discuss this idea in the context of the singularity theorems, which in their usual formulation predict geodesic incompleteness. Regarding Hawking's singularity theorem, I will explain how an alternative proof of it due to Treude and Grant shows that the spacetime is also ``volume incomplete". Turning to Penrose's singularity theorem, I will show that a volume version thereof, which is still open, would imply the existence of an event horizon, thus supporting Penrose's weak cosmic censorship conjecture.

Istvan Kadar (University of Cambridge)

Small data global existence for systems of wave equations: The role of asymptotics

Systems of wave equations may fail to be globally well-posed, even for small initial data. Attempts to classify systems into well-, and ill-posed categories work by identifying structural properties of the equations that can work as indicators of well-posedness. The most famous of these are the null and weak null conditions. As noted by Keir, related formulations may fail to properly capture the effect of undifferentiated terms in systems of wave equations. We show that this is because null conditions are only good for categorising behaviour close to null infinity and propose an alternative condition for semilinear equations that work for undifferentiated non-linearities as well. Furthermore, we give an example of a system satisfying the weak null condition with global ill-posedness due to undifferentiated terms.

Gemma Hood (Imperial College London)

A scattering construction for the wave equation on spacetimes with a negative cosmological constant

Given the sharp logarithmic decay of linear waves on the Kerr-AdS black hole (Holzegel, Smulevici '13), it is expected that the Kerr-AdS spacetime is unstable as a solution of the Einstein vacuum equations. However, the scattering construction presented here for exponentially decaying nonlinear waves on a fixed Kerr-AdS background serves as a first step to confronting the scattering problem for the full Einstein system. In this context, one may hope to derive a class of perturbations of Kerr-AdS which remain 'close' and dissipate sufficiently fast.

Klaas Landsman (Radboud University Nijmegen)

Singularities in GR: From Einstein to Hawking

This is a historical talk on the search for the "right" definition of a singular space-time in general relativity. It is also a conceptual analysis of what it means to give a good definition in mathematical physics. I will start with the early struggles of Einstein (and even Hilbert and Weyl, who were as confused as Einstein), and work towards the 1960s with crucial contributions by Misner, Penrose, and Hawking.

Posters

Tobias Beran (University of Vienna)

Bonnet-Myers Rigidity for Lorentzian length spaces

I will prove that nice Lorentzian length spaces with a global curvature bound below by K<0 (triangle comparison) and a maximizing timelike geodesic of length $\pi/\sqrt{-K}$ splits as a Lorentzian warped product with warping function cosine. (The Lorentzian Bonnet-Myers theorem states that if global curvature bound below by K<0 and one can find non-degenerate triangles, maximizing geodesics can have length at most $\pi/\sqrt{-K}$.)

Saúl Burgos (Universidad de Granada)

The c-completion of Lorentzian metric spaces

Inspired by some Lorentzian versions of the notion of metric and length space introduced by Kunzinger and Sämann, and more recently, by Müller, and Minguzzi and Suhr, it is possible to construct the c-completion of these models by previously discussing the notion of Lorentzian metric space.

It is shown that not only this construction is applicable to very general Lorentzian metric spaces, including low regularity spacetimes, but also endow the c-completion with a structure of Lorentzian metric space by itself. Moreover, the c-completion constitutes a well-suited extension of the original space, which is sensible to certain causal properties of that space. This talk is based on joint work with José L. Flores (Málaga) and Jónatan Herrera (Córdoba).

Matteo Calisti (University of Vienna)

The timelike measure contraction property on generalized cones

We define the curvature condition ”timelike measure contraction property in the sense of Ohta” TMCPO on a Lorentzian pre-length space following the work of Ohta [Oht07]. We first check the compatibility with its entropic versions defined by Cavalletti-Mondino [CM20] and Braun [Bra22] and also its compatibility with the smooth case. We also investigate if doubling properties of causal diamonds [MS22] can be obtained in this context. Then, as in the positive definite counterpart the Bonnet-Myers theorem follows. We then prove that, in a Minkowski cone over a Polish metric measure space, the curvature information encoded in the MCPO of the base space can be passed on the Minkowski cone over it through the TMCPO. Finally we prove stability properties of TMCPO via the convergence introduce by Cavalletti and Mondino. Joint work with Clemens Sämann and Christian Ketterer.

References

[Bra22] M. Braun. Rényi’s entropy on Lorentzian spaces. timelike curvature-dimension conditions. 2022, 2206.13005.

[CM20] F. Cavalletti and A. Mondino. Optimal transport in Lorentzian synthetic spaces, synthetic timelike Ricci curvature lower bounds and applications. preprint, arXiv:2004.08934 [math.MG], 2020. doi:10.48550/arXiv.2004.08934.
[MS22] R. J. McCann and C. Sämann. A Lorentzian analog for Hausdorff dimension and measure. Pure and Applied Analysis, 4(2):367–400, 2022. doi:10.2140/paa.2022.4.367.
[Oht07] S.-I. Ohta. On the measure contraction property of metric measure spaces. Comment. Math. Helv., pages 805–828, 2007.

Lawrence Frolov (Rutgers University)

Joint evolution of point-charge sources and their fields in one space dimension

We present a proof for the conjecture in classical field theory that perturbed scalar particles asymptotically return towards rest in 1 + 1 dimensions, due to the process of back-reaction. Taking the approach of Kiessling, we derive an expression for the force on a charged particle in one space dimension when it is coupled to a massless scalar field and an electromagnetic vector potential. The force law derived is the unique law which locally conserves the energy and momentum of the total particle-field system. With this law we provide a simple, closed form expression for the self-force resulting from the process of back-reaction and show that it is restorative, proportional to negative velocity, and causes isolated charges to asymptotically return to rest after being perturbed by radiation. We cement these results by studying the joint evolution problem for the particle-fields system and prove its global well-posedness in the single particle case, and its local well-posedness in the multi-particle case, for all time until any two particles collide. This work contributes to our understanding of the dynamics of particle-field systems and may have implications for the study of charged particles in higher dimensions.

Leonardo García-Heveling (Radboud University Nijmegen)

Volume singularities

In General Relativity, the notion of singularity is usually defined via either geodesic incompleteness or curvature blow-up. I propose a new definition of singularity using the volume measure on spacetime, and motivate it from a quantum gravity perspective. I will also discuss its pros and cons compared to the usual notions of singularity, both in the Big Bang and in the black hole setting.

Alejandro Penuela Diaz (University of Potsdam)

Local foliations by critical surfaces of the Hawking energy and small sphere limit

The Hawking energy is one of the most famous local energies in general relativity, by using a Lyapunov-Schmidt reduction procedure we construct unique local foliations of critical surfaces of the Hawking energy on initial data sets. Any quasilocal energy should satisfy the so-called small sphere limit, therefore we also discuss the relation between these surfaces and the small sphere limit. In particular, we discuss some discrepancies on the small sphere limit, so when approaching a point with these foliations and when approaching as in the small sphere limit.

Felix Rott (University of Vienna)

Alexandrov's Patchwork and the Bonnet-Myers theorem for Lorentzian length spaces

We present two results in the study of global timelike curvature bounds within the Lorentzian length space framework. On the one hand, we construct a Lorentzian analogue to Alexandrov's Patchwork from metric geometry, thus proving that suitably nice Lorentzian length spaces with local upper timelike curvature bound also satisfy a corresponding global upper bound. On the other hand, for spaces with global and negative lower timelike curvature bounds, we provide a Bonnet–Myers style result, constraining their diameter (with respect to the time separation function). Joint work with Tobias Beran and Lewis Napper. based on https://arxiv.org/abs/2302.11615

Quinten Rutgers (MPI Leipzig)

Late-time asymptotics for spherically symmetric charged black hole spacetimes

Understanding the late-time asymptotics of wave equations on black hole spacetimes is important for many reasons. For example, in the context of the cosmic censorship conjectures, the late-time behaviour on the event horizon can give information about the extendibility properties of the interior. In this project we study the late-time asymptotics of black hole solutions to the Einstein-Maxwell-scalar field (EMSF) system in spherical symmetry, extending methods introduced by Angelopoulos, Aretakis and Gajic for the linear wave equation to a nonlinear setting. The goal of the project is to show that for a generic class of small initial data for the EMSF system, the scalar field decays inverse polynomially through a characteristic foliation, the leading order tail can be identified, and the decay rates are the same as for the linear wave equation on a Reissner-Nordström spacetime, where the rates are given by Price's law.