Recent Developments in Mathematical General Relativity I: Cosmic Censorship

Yakov Shlapentokh-Rothman

University of Toronto

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2. Explain some of the progress towards answering these questions that has been made.

3. Give some examples of how the study of low-regularity solutions can be useful to understand these problems.

(Review of) The Initial Value Problem

Theorem (Choquet-Bruhat 1952 and Choquet-Bruhat–Geroch 1969)

Given a (suitably regular) Riemannian manifold $(\mathcal{N}, h)$ *and symmetric* 2*-tensor* k_{ab} *along* $\mathcal{N}$ *which satisfy the "constraint equations," there exists a unique maximal globally hyperbolic Lorentzian manifold* $(\mathcal{M}, g)$ *which solves the Einstein vacuum equations*

$$\mathrm{Ric}\,(g) = 0$$

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- The constraint equations for h and k are consequences of the Gauss and Codazzi equations and $\mathrm{Ric}\,(g) = 0$.
- Proof uses that in a suitable coordinate system, $\mathrm{Ric}\,(g) = 0$ becomes a system of quasilinear wave equations for the metric g.

Part I: Crash Course in Penrose Diagrams

Null Coordinates on Spherically Symmetric Spacetimes

Fact: Every spherically symmetric globally hyperbolic spacetime $(\mathcal{M}, g)$ may locally be equipped with coordinates $\left(u, v, \theta^A\right) \in \mathcal{U} \times \mathbb{S}^2$ for suitable $\mathcal{U} \subset \mathbb{R}^2$, so that

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Quick Example: Minkowski space is

$$(\mathbb{R}^4, m = -dt^2 + dx^2 + dy^2 + dz^2).$$

In spherical coordinates the metric becomes

$$m = -dt^2 + dr^2 + r^2 d\mathbb{S}^2 = -dudv + r^2 d\mathbb{S}^2,$$

for $u = t - r$ and $v = t + r$.

Penrose Diagram

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1. Since m and $\tilde{g}$ are conformal to each other, they have the same light cones, which moreover lift to null hypersurfaces in the original spacetimes.
2. Since $\tilde{U}$ is finite, you can easily depict it visually.
3. Causal relations in $\tilde{U}$ are respected by their lifts to the original spacetime.

Penrose Diagram of Minkowski Space

After quotienting we have that $m = -dudv$ where

$$\mathcal{U} = \{(u, v) : v - u \geq 0\}.$$

We then set $\tilde{u} \doteq \arctan(u)$ and $\tilde{v} \doteq \arctan(v)$, so that

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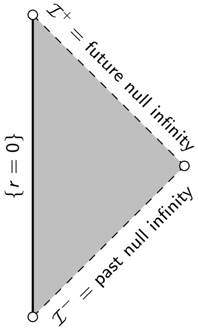

Part II: The Schwarzschild Black Hole, Gravitational Collapse, and Weak Cosmic Censorship

Schwarzschild Metric

The Schwarzschild metric, discovered in 1916(!), is the following metric in coordinates $(t, r, \theta, \phi) \in \mathbb{R} \times (2|M|, \infty) \times \mathbb{S}^2$, for any $M \in \mathbb{R}$:

$$g = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

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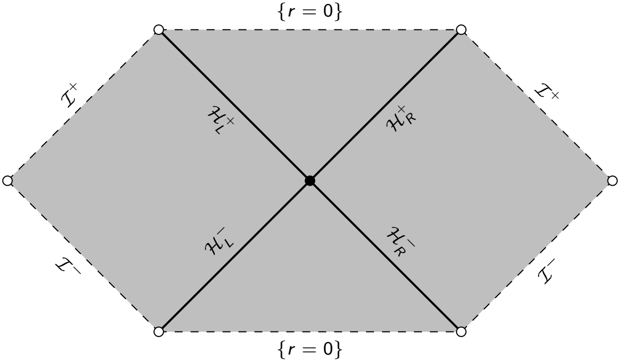
The metric has a timelike Killing field ∂_t and is *asymptotically flat* as $r \to \infty$. This metric expression breaks down at $r = \min(2M, 0)$, but in the case when $M > 0$, it can be extended to a larger manifold. Instead of describing this explicitly, let us see the Penrose diagram:

Penrose Diagram of Schwarzschild



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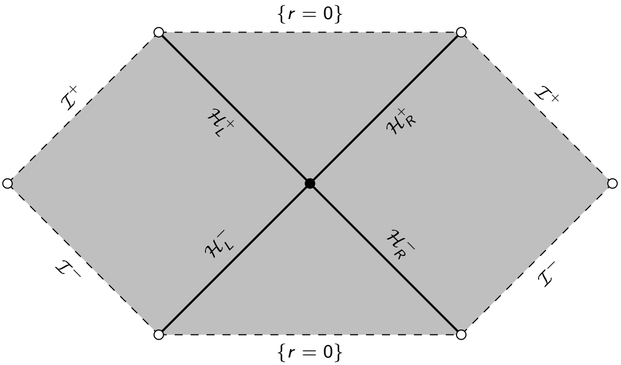

The rightmost diamond corresponds to the original coordinate system, with r converging to $2M$ on $\mathcal{H}_A^+ \cup \mathcal{H}_A^-$. The metric approaches the $\mathcal{I}^\pm$ of Minkowski space as one approaches $\mathcal{I}^\pm$. The leftmost diamond is a isometric copy of the rightmost diamond.

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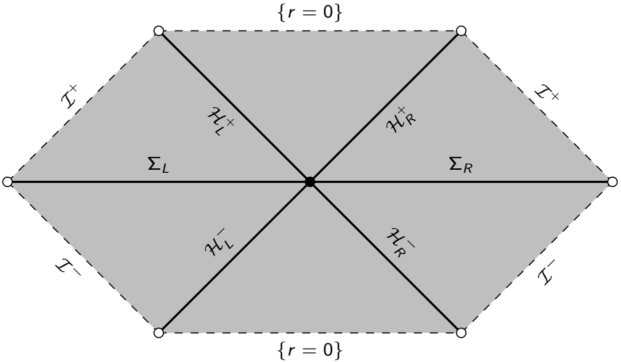

The hypersurface Σ_R corresponds to the original $\{t = 0\}$. The hypersurface $\Sigma = \Sigma_L \cup \Sigma_R$ is a Cauchy hypersurface which is diffeomorphic to $\mathbb{R} \times \mathbb{S}^2$ (not $\mathbb{R}^3$!).

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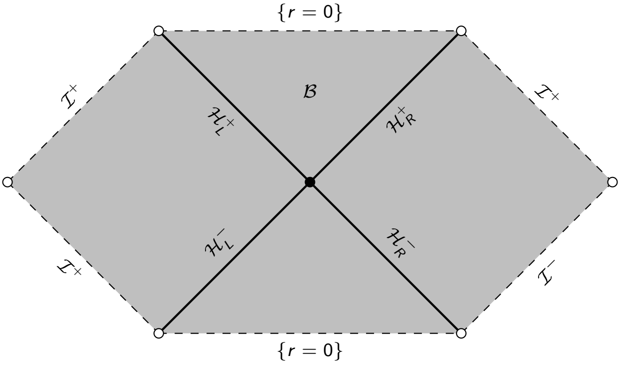

Every geodesic which enters the top region $\mathcal{B}$ ends in finite time at the $\{r = 0\}$ boundary. A computation shows that curvature blows along these geodesics. In fact, the spacetime is inextendible as even a C^0 manifold (Sbierski 2018)!

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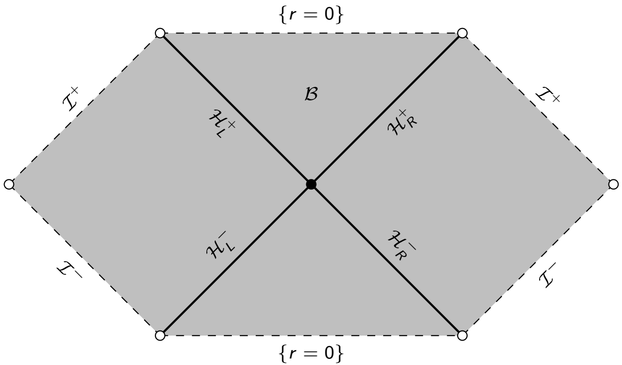

The region $\mathcal{B}$ is the complement in $\mathcal{M}$ of the past of $\mathcal{I}^+$. Since one imagines that a far away observer "lives" on $\mathcal{I}^+$ we call $\mathcal{B}$ the black hole region in that it cannot communicate with the far away observers. The nasty singularity is thus completely hidden inside $\mathcal{B}$. The boundaries of $\mathcal{B}$ (and it's time reversed image) and denoted by $\mathcal{H}^\pm$ and called the future and past event horizons.

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1. Schwarzschild has a Cauchy hypersurface which is diffeomorphic to $\mathbb{R} \times \mathbb{S}^2$, while we want $\mathbb{R}^3$...

2. All spherically solutions to the Einstein vacuum equations are isometric locally to Schwarzschild! So we need to either drop spherical symmetry or change the equations to have a guide for dynamics.

Gravitational Collapse

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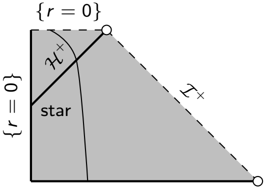

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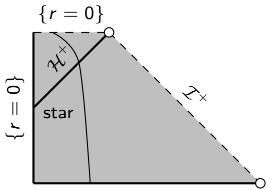

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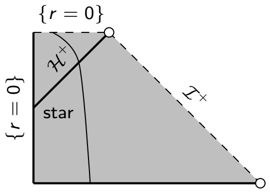

- Outside of the star, the metric is isometric to Schwarzschild.
- Again there is a nasty singularity at $\{r = 0\}$ which is hidden from $\mathcal{I}^+$ by the event horizon.

Completeness of $\mathcal{I}^+$

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Definition

Let $(\mathcal{M}, g)$ be a maximal development associated to an asymptotically flat initial data set. Let $\mathcal{N}$ be a null hypersurface which is complete in the outgoing direction and converges to a Minkowski cone.*

Then we say that $\mathcal{I}^+$ is complete if the following holds: Let $v(s)$ be a 1-parameter family of null vectors transverse to $\mathcal{I}^+$ which are parallel transported to the asymptotically flat end of $\mathcal{N}$. We then require that the maximal affine length of the null geodesics starting at $v(s)$ must converge to infinity as $s \to \infty$.

*You can always find such a cone by the stability of Minkowski space (Christodoulou–Klainerman 93).

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- It is not a priori clear why the word "generic" is necessary.
- A version of this conjecture has been understood in (Christodoulou 99) for the spherically symmetric Einstein-scalar field model.
- One expects a resolution of this to involve, as a preliminary step, the identification of all stationary black hole solutions and an understanding of their stability.

Part III: The Kerr Black Hole and Strong Cosmic Censorship

Penrose Diagrams Outside Spherical Symmetry

It is very useful to have a version of Penrose diagrams which works outside of spherical symmetry. Key point is that any spacetime $(\mathcal{M}, g)$ may be locally equipped with a coordinate system $(u, v, \theta^A) \subset \mathcal{U} \times \mathbb{S}^2$ for suitable $\mathcal{U} \subset \mathbb{R}^2$ so that

$$g = -2\Omega^2 (du \otimes dv + dv \otimes du) + \not{g}_{AB} \left(d\theta^A - b^A du\right) \otimes \left(d\theta^B - b^B du\right).$$

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The hypersurfaces of constant u and constant v in such a coordinate system will be null hypersurfaces. Like before, we can rescale u and v to produce a new functions $\tilde{u}(u)$ and $\tilde{v}(v)$ so that the new coordinates $(\tilde{u}, \tilde{v}, \theta^A) \subset \tilde{U} \times \mathbb{S}^2$ for $\tilde{U}$ lying in a finite subset of $\mathbb{R}^2$. Then, just as before we may consider the region $\tilde{\mathcal{U}} \subset \mathbb{R}^2$ equipped with the Minkowski metric. Again, causal relations in the Penrose diagram can be lifted to the actual spacetime.

Kerr Spacetime

Let $M > 0$ and $a \in (-M, M)$.* In Boyer–Lindquist coordinates

$$(t, r, \theta, \phi) \in \mathbb{R} \times (r_+, \infty) \times \mathbb{S}^2, \qquad r_+ \doteq M + \sqrt{M^2 - a^2},$$

the Kerr metric (discovered in 1963) is given by

$$g = -\frac{\Delta}{\rho^2}\left(dt - a^2 \sin^2 d\phi\right)^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \left(r^2 + a^2\right) d\phi\right)^2,$$

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*The case $|a| = M$ is the "extremal case." It is extremely interesting, but I won't talk about it here.

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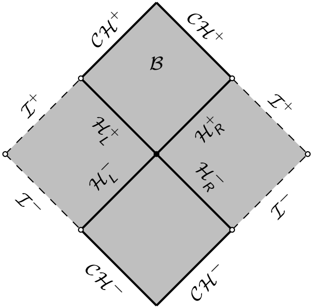

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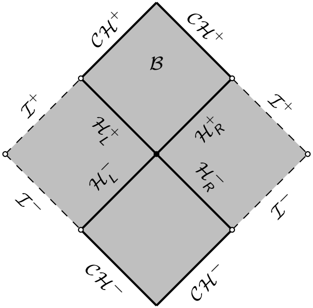
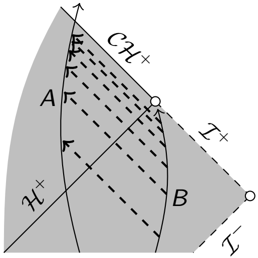


- There is no singularity in the black hole region! Instead the maximal globally hyperbolic development ends in a *smooth* Cauchy horizon $\mathcal{CH}^+$.

- There are many non-unique ways to smoothly extend the spacetime as a solution; thus the theory loses its predictive power in this example even though nothing appears to go wrong with the regularity.

Penrose's Blue Shift Instability

Consider the situation where a free falling observer A falls into the black hole and crosses the Cauchy horizon and a free falling observer B stays outside the black hole. Penrose observed that if B sends a null geodesic towards A at time t (measured by B's proper time), then the energies* of the null geodesics when they intersect A will diverge as $t \to \infty$. This is the famous *blue-shift* instability along $\mathcal{CH}^+$.



*The energy of a future oriented null geodesic γ is $g(\dot{\gamma}, N)$ where N is some choice of a gloablly defined timelike vector field.

Strong Cosmic Censorship

Since null geodesics are related to the high frequency behavior of waves, it is natural to conjecture that there is some type of instability for solutions to the wave equation near the Cauchy horizon. In turn, it is then natural to conjecture that a gravitational perturbation of Kerr would create some type of instability. Extrapolating this lead Penrose to the following:*

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- *This is a bit revisionist. The original expectation was that for generic solutions, some type of Schwarzschild like (i.e. spacelike) singularity should emerge. We will see later that this is incorrect.

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Conjecture (Strong Cosmic Censorship)

The maximal development of a generic solution to the Einstein vacuum equations should be inextendible as a suitably regular solution.

- *This is a bit revisionist. The original expectation was that for generic solutions, some type of Schwarzschild like (i.e. spacelike) singularity should emerge. We will see later that this is incorrect.
- It is unclear what the correct notion of genericity and regularity should be. A popular proposal (due to Christodoulou) for the regularity is "Christoffel symbols in L^2_{loc}." This is motivated by the fact that such a condition is useful for naive weak formulations of the Einstein equations.