Euler's family of semi-magic squares of squares of order three

In 1770 Euler published a family of semi-magic squares of order 3 consisting of squares of rational numbers, namely

A^2	B^2	C^2
D^2	E^2	F^2
G^2	H^2	I^2

where $A = (p^2+q^2-r^2-s^2)/u$, $B = [2(qr+ps)]/u$, $C = [2(qs-pr)]/u$, $D = [2(qr-ps)]/u$, $E = (p^2-q^2+r^2-s^2)/u$, $F = [2(rs+pq)]/u$, $G = [2(qs+pr)]/u$, $H = [2(rs-pq)]/u$, $I = (p^2-q^2-r^2+s^2)/u$ and $u = p^2+q^2+r^2+s^2$.

Euler remarks that the sum of the squares in each column and each row equals 1. So $A^2+B^2+C^2 = 1$, $D^2+E^2+F^2 = 1$ etc. Hence multiplying by u^2 we get the following family of semi-magic squares of squares (of integers), which magic sum equals $u^2 = (p^2+q^2+r^2+s^2)^2$

$(p^2+q^2-r^2-s^2)^2$	$[2(qr+ps)]^2$	$[2(qs-pr)]^2$
$[2(qr-ps)]^2$	$(p^2-q^2+r^2-s^2)^2$	$[2(rs+pq)]^2$
$[2(qs+pr)]^2$	$[2(rs-pq)]^2$	$(p^2-q^2-r^2+s^2)^2$

Some hundred years later Edouard Lucas started to study the squares of squares problem (for order 3) by showing that the family above does not contain a magic square of squares. In other words, in this family there does not occur one single square such that also the two diagonals have the same magic sum!

In 1996 Lee Sallows found the first order 3 semi-magic square of squares where only one diagonal fails to have the magic sum, namely

127^2	46^2	58^2
2^2	113^2	94^2
74^2	82^2	97^2

Recently (2005) Christian Boyer observed (*The Mathematical Intelligencer*, Vol. 27, no. 2, Spring 2005, pp 52-64) that Sallow's square is a member of Euler's family: take $p = 1$, $q = 3$, $r = 4$ and $s = 11$!