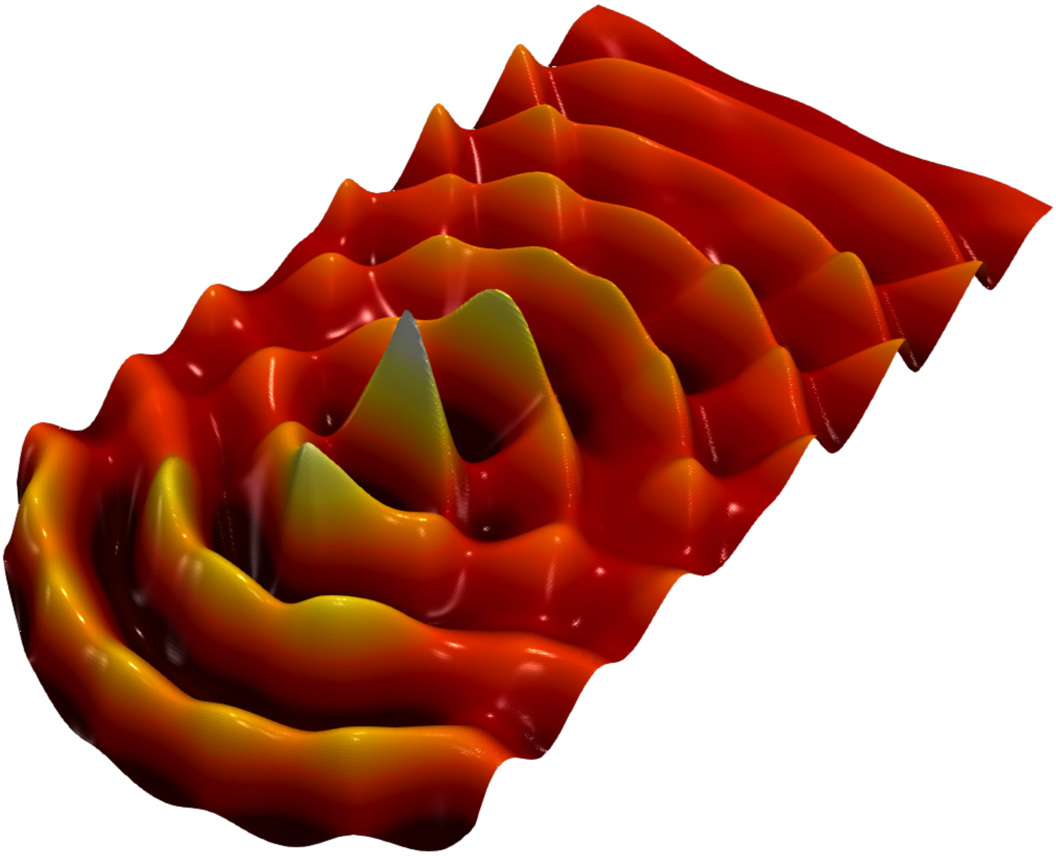
Nonlinear PDEs:
Analysis & Simulation



Abstracts

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Radboud University

Karen Veroy-Grepl
A Reduced Basis Ensemble Kalman Method for Inverse Problems:

In the process of estimating the state dynamics of distributed-parameter systems, data from physical measurements can be incorporated into the mathematical model to reduce the uncertainty in the parameter estimates and, consequently, improve the state prediction. This process of data assimilation must deal with the data and model misfit arising from experimental noise as well as from model inaccuracies. In our study, we focus on the ensemble Kalman method (EnKM) [1], an iterative Monte Carlo method for the solution of inverse problems. The method is gradient-free and, just like the ensemble Kalman filter, relies on an ensemble of “particles” (here, a sample of parameter values) to identify the state that better reproduces the physical observations, while preserving the physics of the system as described by the model.

In this talk, we show how model order reduction can be combined with the EnKM to greatly accelerate the EnKM solution of inverse problems.
In addition, we experimentally study the latter’s performance with respect to different levels of noise and model error. Such numerical experiments, e.g., involving unknown distributed parameters in two or more spatial dimensions, can be very expensive and are (here) enabled only by the computational efficiency of the surrogate models. For a physical problem governed by non-linear parabolic partial differential equations, we investigate the role of the ensemble size on the reconstruction error and extend the method by introducing a measurement bias correction to improve the parameter estimate.

This is work in collaboration with Francesco Silva, Cecilia Pagliantini, Martin Grepl, and Nicole Aretz.

[1] M. Iglesias, K. Law and A. Stuart,Ensemble Kalman Methods for Inverse Problems, InverseProblems, 29, (2013).

Martina Chirilus-Bruckner
New localized solutions for reaction-diffusion systems

Localized structures such as fronts or pulses play an important role in unlocking the complicated dynamics exhibited by reaction-diffusion systems. Much of the past research in the nonlinear waves community has been dedicated to study existence, stability, bifurcations and interaction of localized structures. This talk will explain how new types of these important special solutions, namely, fronts and pulses with heterogeneous tails can be studied. These do not necessarily approach constant or periodic states, but arbitrary bounded solutions. As such they can be viewed as a generalization of conventional fronts and pulses. From an application point of view, these new solutions naturally arise in PDEs with spatially varying coefficients that are used to improve the modelling of phenomena in heterogenous environments.

Fred J. Vermolen
A numerical method for morpho-poroelasticity to simulate mechanical deformations in growing and shrinking media

Soft tissues are often subject to mechanical deformations and growth or shrinking processes. In this talk, we consider a morpho-elastic model for the combination of these processes.
Furthermore, since many tissues are essentially porous, we enrich the classical poroelastic Biot model with morpho-elasticity. During this talk, we consider hybrid models that incorporate cellular forces as well as fully continuum-based models. Next to this, we consider finite element methods and Bayesian-inspired quantification of uncertainty to accomodate for patient-dependent values of input parameters.
Furthermore, machine learning techniques for the fast approximation of solutions will be discussed as an implementation in a clinical setting.
The presentation will be mathematical and illustrative with applications from burn injuries and cancer development.

Arjen Doelman
Spatial Ecology & Singularly Perturbed Reaction-Diffusion Equations

Pattern formation in ecological systems is driven by counteracting feedback mechanisms on widely different spatial scales. Moreover, ecosystem models typically have the nature of reaction-diffusion systems: the dynamics of ecological patterns can be studied by the methods (geometric) singular perturbation theory. In this talk we give an overview of the surprisingly rich cross-fertilization between ecology, the physics of pattern formation and the mathematics of singular perturbations. We show how a mathematical approach uncovers mechanisms by which real-life ecosystems may evade (catastrophic) tipping under slowly varying climatological circumstances. This insight is based on two crucial ingredients: the careful numerical study of Busse balloons (in (parameter, wavenumber)-space) associated to spatially periodic patterns and the validation of the analytical and numerical model predictions by field observations. Vice versa, ecosystem models motivate the study of classes of singularly perturbed reaction diffusion equations that exhibit much more complex behavior than the models so far studied by mathematicians: we present sveral novel mathematical and numerical research directions initiated by ecology.

Barry Koren
Machine learning and reduced order modeling for uncertainty quantification in nonlinear PDE problems

A multigrid method for uncertainty quantification in nonlinear PDE problems is proposed. The principle behind the method is that the relative solution error between grid levels has a spatial structure that is by good approximation independent of the actual grid level. The method learns this structure by employing a sequence of convolutional neural networks, that are well-suited to automatically detect local truncation errors as latent quantities of the solution. By using transfer learning, the information of coarse grid levels is reused on fine grid levels to minimize the required number of samples on fine grid levels. The method outperforms an existing multi-level method for uncertainty quantification, for a relevant test case.

As an alternative for machine learning in uncertainty quantification, the potentials of the use of a reduced order model approach in uncertainty quantification are quickly explored.

Advantages and challenges of the machine learning approach and the reduced order modeling approach are compared. Combination of both approaches in uncertainty quantification may be an interesting future research topic.

Mark Peletier
Convergence of a gradient flow to a non-gradient-flow

In recent years a range of methods has been developed to prove the convergence of a sequence of gradient systems to a limit gradient system, using for instance Minimizing Movement, the Evolution Variational Inequality, or the Energy-Dissipation Inequality.

In this paper we study a sequence of gradient systems that converges, but to a limit that is _not_ a gradient system. The system is given by a Fokker-Planck equation in one dimension, and describes the evolution of the law of a Brownian particle in a two-well energy landscape. This setup is well known from the so-called 'Kramers limit', in which the limit of vanishing noise causes the system to collapse onto just two states, corresponding to the bottom of the wells.

In the system that we study the wells have different depth, and as a result the behaviour in the small-noise limit is markedly different from the classical case: the limit is a `one-way' transfer from the higher well to the lower one.

In this setup all existing gradient-flow-convergence methods fail, and indeed they should fail, because the gradient-flow structure is lost in this limit. We show that variational convergence does hold for a generalized variational description of the gradient flow. Consistent with the loss of gradient-flow structure, the limit of this generalized variational formulation does not describe a gradient flow.

Rob Stevenson
First order least squares methods for parabolic and instationary Stokes equations

This presentation is about the numerical solution of parabolic PDEs. nstead of the common time marching schemes we will consider a monolithic approach based on simultaneous space-time variational formulation of the PDE. Advantages of this approach are that they produce numerical approximations from the employed trial spaces that are quasi-best, allow for local refinements simultaneously in space and time, and their much better suitability for a massively parallel implementation. Moreover, as we will illustrate, they are superior in applications where the full time evolution is needed at the same time, as with problems of optimal control or data assimilation. If time permits, then we discuss the generalisation of this approach to the instationary Stokes equations.

Anna Geyer
Nonlinear wave-current interactions in stratified flows

This talk concerns nonlinear mathematical models for water waves in stratified fluids subject to underlying currents. A prominent example of this scenario are equatorial ocean dynamics which are characterised by the propagation of internal waves along a thermocline due to strong density stratification and the presence of a depth-dependent current, the Equatorial Undercurrent. As a first step towards a better understanding of this wave-current interaction, we derived KdV type equations from the Euler equations, taking into account an arbitrary depth dependent density distribution and steady current. In the first part of my talk, the focus is on continuous density variations, where we obtain a different model equation at each water depth. In the second part, we look at discontinuously stratified two-layer fluids and derive equations modeling the propagation of weakly nonlinear waves along the interface and the free surface, which are interacting with an arbitrary prescribed steady current. This is joint work with Ronald Quirchmayr.