

ADDENDUM

In Section 9 of the paper, I apply the main result to some concrete classes of surfaces. It should be pointed out that for surfaces S with $p_g = q = 1$ and $K^2 = 3$ we obtain the Tate conjecture and the Mumford–Tate conjecture in all cases, not only if the general Albanese fiber has genus 3 (which is case (e) in Theorem 9.4). Namely, in the paper [11] by Catanese and Ciliberto, it is shown that (for $p_g = q = 1$ and $K^2 = 3$) either the general Albanese fiber has genus 3, or there exists a curve A of genus 1 and a dominant rational map $S \dashrightarrow A^{(2)}$ to the second symmetric square of A . This implies that the motive $\mathbf{H}^2(S)$ is isomorphic to the sum of $\mathbf{H}^2(A^{(2)})$ and some copies of $\mathbf{Q}(-1)$. As the Mumford–Tate conjecture is true for $A^{(2)}$, this gives what we want.