ERRATUM

In Section 7.1 of the paper, I write that we may choose the CM type $\Phi$ to be primitive. This is not true, but fortunately also not needed. Let us explain this.

It is not true that any CM field admits a primitive CM type. Example: let $E = kE_0$ be the compositum of a real quadratic field $E_0$ and an imaginary quadratic field $k$. Then besides $E_0$ and $k$, there is a third quadratic subfield $k' \subset E$, which is again imaginary quadratic. Let $\sigma$, $\bar{\sigma}$, $\tau$ and $\bar{\tau}$ be the complex embeddings of $E$, chosen such that $\sigma$ and $\tau$ are the same on $k$. Up to a renaming of these elements, the only possibly CM types are $\{\sigma, \tau\}$, which is induced from $k$, and $\{\sigma, \bar{\tau}\}$, which is induced from $k'$.

Choosing $\Phi$ to be primitive would have as consequence that $\operatorname{End}^0(A) = E$. This, however, does not play a role in the rest of the argument. All that matters is that $E$ is its own commutant in $\operatorname{End}^0(A)$, which implies that $\operatorname{End}_k(\mathbb{H}^1(A_S)) = E$ (used in (7.1.1)) and $\operatorname{End}_E(\mathbb{H}^1(A)) = E$ (in Section 7.7). This is true regardless of how we choose $\Phi$. 