Some highlights in my own research on polynomial automorphisms

- *A criterion to decide if a polynomial map is invertible and to compute the inverse*, Comm. in Algebra 18 (10), 1990, 3183-3186.

In this paper I introduce Gröbner basis theory into the field of polynomial automorphisms, in order to give an algorithmic solution of the problem: how to decide if a given polynomial map $F : k^n \to k^n$ has a polynomial inverse. The algorithm also computes the inverse, in case it exists. The algorithm works for all fields.
This result is now considered "classical" and appeared in text books on Gröbner basis such as for example T. Becker and V. Weispfenning (1993), Gröbner Bases.
In Theorem 3.2.1 of my book [E] the above result is generalised to morphisms between finitely generated k-algebras, where k is any field. The original paper formed the starting point of much follow-up research.

- *An algorithm to compute the invariant ring of a G_a-action on an affine variety*, Journal of Symbolic Computation (1993), 16, 551-555.

In this paper I give an algorithm which computes the kernel of a locally nilpotent derivation on any finitely generated k-domain A, where k is a field of characteristic zero, in case the kernel is a finitely generated k-algebra. This result is often referred to as Van den Essen's algorithm (see [F] and [N]) and is used in various research papers to compute kernels of locally nilpotent derivations. Furthermore, the algorithm can also be used to *prove* that the kernel of a given locally nilpotent derivation is finitely generated. Namely if the algorithm terminates, it guarantees that the kernel indeed is finitely generated!

- (with Hubbers) *A new class of invertible polynomial maps*, Journal of Algebra, 187 (1997), 214-226.

In this paper we introduce a large class of polynomial automorphisms over arbitrary commutative rings.
Members of this new class of automorphisms are used to give counterexamples to various open problems/conjectures. In particular it led to counterexamples of the Markus-Yamabe Conjecture (see the next paper).

- (with Cima, Gasull, Hubbers and Mañosas) *A polynomial counterexample to the Markus-Yamabe Conjecture*, Advances in Mathematics, 131, no. 2 (1997), 453-457.

In this paper we give counterexamples to one of the most famous conjectures in the theory of dynamical systems: the Markus-Yamabe Conjecture, also known as the Global Asymptotic Stability Jacobian conjecture.
We show that the conjecture is false in all dimensions greater or equal to three (by giving explicit counterexamples). For the dimensions one and two the conjecture was proven to be true by Fessler, Gutierrez and Glutsuk in 1993-1994. The Markus-Yamabe conjecture was open for 35 years!

Also we give a negative answer, in all dimensions greater or equal to three, to a discrete version of the Markus-Yamabe Conjecture, the so-called La Salle problem, which was open since 1976.
Our paper appeared in the Featured Review Section of the Mathematical Reviews.

- (with van Rossum) *Triangular derivations related to problems on affine n-space*, Proc. of the A.M.S., Vol. 130, no. 5 (2001), 1311-1322.

In this paper we introduce a special class of triangular derivations and show how embeddings of k^r in k^n can be characterized by these derivations. This enables us to establish a relation between the Cancellation problem and the embedding problem and leads us to candidate counterexamples for both the Cancellation problem and the Linearization problem in dimension five.

- (with de Bondt) *A reduction of the Jacobian conjecture to the symmetric case*, Proc. of the A.M.S., Vol. 133, no. 8 (2005), 2201-2205.

It is well-known since the classical papers of Bass, Connell, Wright/Yagzhev (1980), that it suffices to investigate the Jacobian Conjecture for polynomial mappings of the form $F = x + H$ with JH nilpotent and H homogeneous of degree three. In this paper we show that we may *additionally* assume that JH is symmetric. This implies that H is the gradient of a homogeneous polynomial of degree four.
In a remarkable follow-up paper W. Zhao, *Hesse nilpotent polynomials and the Jacobian Conjecture*, Trans. AMS 359 (2007), 249-274.

this result is used to show the following: let Δ be the Laplace operator i.e. $\Delta = \partial_1^2 + \cdots + \partial_n^2$, where $\partial_i = \frac{\partial}{\partial x_i}$, then the Jacobian Conjecture is equivalent to the so-called Vanishing Conjecture:

Vanishing Conjecture For every $n \geq 1$ we have: if f is a homogeneous polynomial of degree four in n variables over the complex numbers such that $\Delta^m f^m = 0$ for all $m \geq 1$, then $\Delta^{m-1} f^m = 0$ for all sufficiently large m.

- (with de Bondt) *The Jacobian Conjecture, Linear triangularization for homogeneous polynomial maps in dimension three*, J. of Algebra 294, no. 1 (2005), 294-306.

In this paper we completely solve the three-dimensional Jacobian Conjecture for all homogeneous polynomial maps of the form $x+H$, where H is homogeneous of degree greater or equal to two.

- *A simple solution of Hilbert's fourteenth problem*, Colloquium Mathematicum 105 (2006), 167-170.

In this paper I give a very short proof that the kernel of a locally nilpotent derivation in dimension five, due to Daigle and Freudenburg, is not-finitely generated, thereby giving a simple solution of Hilbert's fourteenth problem.

- (with Adjamagbo) *A short proof of the equivalence of the Dixmier, Jacobian and Poisson Conjectures* (to appear).

In this paper we give a short, purely algebraic, proof of the equivalence of the Jacobian Conjecture and the Dixmier Conjecture. This result was obtained earlier by Tsuchimoto (2005) and Belov and Kontsevich (2005).
Our proof is much simpler and also introduces one more equivalent conjecture, the Poisson Conjecture.