Local Langlands for GL_2

Milan Lopuhaä

January 15, 2015

1 Προλεγομένα

After a semester of hard work, we have now come to the Local Langlands correspondence for GL_2 for odd p (the characteristic of the local field). The proof of the theorem, however, is a rather technical affair that relies on many properties of L-functions and ϵ-factors that have not been discussed in this seminar so far, and doing so today would cost us far more time. Therefore we will have to content ourselves with a proof of the non-cuspidal case, and the construction in the cuspidal case.

The main statement is as follows. Let F be a local field, and let $\mathrm{G}_2(F)$ be the set of equivalence classes of two-dimensional semisimple Deligne representations of the Weil group $\mathcal{W}_F$. Also, let $\mathcal{A}_2(F)$ be the set of equivalence classes of irreducible smooth representations of $G = \mathrm{GL}_2(F)$. Also, we identify a character χ of $F^\times$ with both $\chi \circ a_F$ (on the Galois side) and with $\chi \circ \det$ (on the automorphic side), where $a_F : \mathcal{W}_F^{\mathrm{ab}} \xrightarrow{\sim} F^\times$ is the isomorphism from local class field theory. For convenience, we write $R = (\rho_R, V_R, \mathfrak{n}_R)$ for an element of $\mathcal{G}_2(F)$, and we denote $\chi \otimes R$ for $(\chi \otimes \rho_R, V_R, \mathfrak{n}_R)$.

Theorem 1.1 (Langlands Correspondence). *Let $\psi \in \hat{F} \setminus \{1\}$ be given. Then there is a unique map*

$$\pi : \mathcal{G}_2(F) \longrightarrow \mathcal{A}_2(F)$$

such that for all $R \in \mathcal{G}_2(F)$ and all $\chi \in \widehat{F^\times}$ one has

$$\begin{aligned} L(\chi\pi(R), s) &= L(\chi \otimes R, s), \\ \epsilon(\chi\pi(R), s, \psi) &= \epsilon(\chi \otimes R, s, \psi). \end{aligned}$$

This map is a bijection and independent of the choice of ψ.

Note that the L-function and the ϵ-factor of a Weil-Deligne representation R have so far not been defined in our seminar; they are defined as follows (see 31.3[1]). From the definition of a Deligne representation R one can see that $\ker \mathfrak{n}_R$ is itself a semisimple smooth representation σ_R of $\mathcal{W}_F$. We define

$$\begin{aligned} L(R,s) &= L(\sigma_R,s), \\ \epsilon(R,s,\psi) &= \epsilon(\rho_R,s,\psi)\frac{L(\rho_R^\vee,1-s)}{L(\rho_R,s)}\frac{L(\sigma_R,s)}{L(\sigma_{R^\vee},1-s)}. \end{aligned}$$

Recall from Johan's talk that $R^\vee = (\rho_R^\vee, V^\vee, -\mathfrak{n}^\vee)$. Note also that in general the L-function and ϵ-factor of R is not the same as those of the underlying smooth representation ρ_R!

Now for the proof of the theorem we start off with some preliminary remarks. First, the uniqueness of the map is guaranteed by theorem 27.2, which states that elements of $\mathcal{A}_2(F)$ are uniquely determined by their L-functions and ϵ-factors. Second, we have a natural partition of $\mathcal{G}_2(F)$ into $\mathcal{G}_2^1(F)$ and $\mathcal{G}_2^0(F)$, consisting of the Deligne representations R for which ρ_R is respectively irreducible and reducible. Similarly, we can partition $\mathcal{A}_2(F)$ into $\mathcal{A}_2^1(F)$, the cuspidal representations, and $\mathcal{A}_2^0(F)$, the non-cuspidal representations. The use of these partitions is made clear by the following proposition.

Proposition 1.2. *Let $R \in \mathcal{G}_2(F)$. Then $R \in \mathcal{G}_2^0(F)$ if and only if $L(\chi \otimes R, s) = 1$ for all characters χ of $F^\times$. Furthermore, let $\tau \in \mathcal{A}_2(F)$. Then $\tau \in \mathcal{A}_2^0(F)$ if and only if $L(\chi \otimes \tau, s) = 1$ for all characters χ of $F^\times$.*

Proof. The automorphic statement is precisely proposition 27.2. On the Galois side, we know that R is irreducible if and only if $\chi \otimes R$ is irreducible, and $L(R,s) = 1$ by definition for irreducible R (see 29.3). On the other hand, if $R = \chi_1 \oplus \chi_2$ for some characters χ_1, χ_2 of $F^\times$, then $L(\chi \otimes R, s) = L(\chi \otimes \chi_1, s)L(\chi \otimes \chi_2, s)$, which shows that we may choose χ such that this is not equal to 0.

The proposition shows that we in fact have to construct maps $\pi^0 : \mathcal{G}_2^0(F) \longrightarrow \mathcal{A}_2^0(F)$ and $\pi^1 : \mathcal{G}_2^1(F) \longrightarrow \mathcal{A}_2^1 F$. We start with the second one as it is considerably easier.

[1] All references are, of course, to [1].

2 The noncuspidal correspondence

Theorem 2.1. *There is a unique map*

$$\pi^1 : \mathcal{G}_2^1(F) \longrightarrow \mathcal{A}_2^1(F)$$

such that for all $R \in \mathcal{G}_2^1(F)$ *and all characters* χ *of* $F^\times$ *one has*

$$L(\chi \otimes \pi^1(R), s) = L(\chi \otimes R, s).$$

This map is bijective and satisfies

$$\begin{aligned} \pi^1(\chi \otimes R) &= \chi \otimes \pi^1(R) \\ \epsilon(\pi^1(R), s, \psi) &= \epsilon(R, s, \psi) \end{aligned}$$

for all $R \in \mathcal{G}_2^1(F)$, $\chi \in \hat{F^\times}$, *and* $\psi \in \hat{F} \setminus \{1\}$.

Before we start the proof, it is useful to recall the classification of both sides of the map.

Proposition 2.2. *Let* $R \in \mathcal{G}_2^1(F)$. *Then* R *can be written in one of the two following forms:*

- $R = \left(\begin{pmatrix} \chi_1 & 0 \\ 0 & \chi_2 \end{pmatrix}, F^2, 0 \right)$ *for some* $\chi_1, \chi_2 \in \widehat{F^\times}$. *In this case* $L(R, s) = L(\chi_1, s)L(\chi_2, s)$ *and* $\epsilon(R, s, \psi) = \epsilon(\chi_1, s, \psi)\epsilon(\chi_2, s, \psi)$ *for all* $\psi \in \widehat{F^\times}$.
- $R = \left(\begin{pmatrix} \varphi||\cdot||^{\frac{1}{2}} & 0 \\ 0 & \varphi||\cdot||^{-\frac{1}{2}} \end{pmatrix}, F^2, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right)$ *for some* $\varphi \in \widehat{F^\times}$. *In this case* $L(R, s) = L(\varphi, s + \frac{1}{2})$ *and* $\epsilon(R, s, \psi) = -\epsilon(\varphi, s, \psi)$.

Furthermore, this is unique up to the permutation of χ_1 *and* χ_2 *in the first option.*

Proof. The L-functions and ϵ-factors can be calculated via the definitions in 31.3 and 29.3, although it is more complicated than the book suggests. As for the rest, since ρ_R is reducible, we may write $\rho_R = \chi_1 \oplus \chi_2$ for some characters χ_1, χ_2 of $F\times$. Since $\ker \mathfrak{n}_R$ is a subrepresentation of ρ_R, it is either $\chi_1 \oplus \chi_2$ (hence $\mathfrak{n}_R = 0$) or χ_2, so that after scaling we may assume $\mathfrak{n} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. If we check for the property of $\mathfrak{n}_R$, we see

$$\rho_R(x)\mathfrak{n}_R\rho_R(x)^{-1} = \begin{pmatrix} 0 & \chi_1\chi_2^{-1}(x) \\ 0 & 0 \end{pmatrix},$$

which shows that $\chi_1\chi_2^{-1}(x) = ||x||$, or equivalently, there exists a $\varphi \in \widehat{F^\times}$ such that $\chi_1 = \varphi|| \cdot ||^{\frac{1}{2}}$ and $\chi_2 = \varphi|| \cdot ||^{-\frac{1}{2}}$.

To classify the non-cuspidal irreducible representations of $\mathrm{GL}_2(F)$, let us first recall some notation. For a character $\chi = \chi_1 \otimes \chi_2$ of T (hence of B), we write $\iota_B^G(\chi)$ for $\mathrm{Ind}_B^G(\delta_B^{-\frac{1}{2}} \otimes \chi)$. Also, for $\varphi \in \widehat{F^\times}$, we write $\varphi \cdot \mathrm{St}_G$ (the twisted Steinberg representation) for the quotient fitting in the following short exact sequence:

$$0 \longrightarrow \varphi_G \longrightarrow \mathrm{Ind}_B^G \varphi_T \longrightarrow \varphi \cdot \mathrm{St}_G \longrightarrow 0$$

By theorems 9.11 and 26.1 we now have the following classification.

Proposition 2.3. *Let $\tau \in \mathcal{A}_2^1(F)$. Then τ can be written in one of the three following forms:*

- *$\tau = \iota_B^G\chi$ for a character $\chi = \chi_1 \otimes \chi_2$ of T such that $\chi \neq \varphi \cdot \delta_B^{\pm\frac{1}{2}}$ for any character φ of $F^\times$. In this case, $L(\tau, s) = L(\chi_1, s)L(\chi_2, s)$ and $\epsilon(\tau, s, \psi) = \epsilon(\chi_1, s, \psi)\epsilon(\chi_2, s, \psi)$ for all $\psi \in \widehat{F^\times}$.*
- *$\tau = \varphi \circ \det$ for some character φ of $F^\times$. In this case $L(\tau, s) = L(\varphi, s + \frac{1}{2})L(\varphi, s - \frac{1}{2})$ and $\epsilon(\tau, s, \psi) = \epsilon(\varphi, s - \frac{1}{2}, \psi)\epsilon(\varphi, s + \frac{1}{2}, \psi)$ for all $\psi \in \widehat{F^\times}$.*
- *$\tau = \varphi \cdot \mathrm{St}_G$ for some character φ of $F^\times$. In this case $L(\tau, s) = L(\varphi, s + \frac{1}{2})$ and $\epsilon(\tau, s, \psi) = -\epsilon(\varphi, s, \psi)$.*

Furthermore, this is unique up to the permutation of χ_1 and χ_2 in the first option.

Proof of 2.1. The proof of the theorem is now rather obvious. Let R be a Deligne representation with $\mathfrak{n}_R = 0$. If $\chi_1\chi_2^{-1} \neq || \cdot ||^{\pm 1}$, we set $\pi_F^1(R) = \iota_B^G(\chi_1 \otimes \chi_2)$, which is irreducible. Otherwise, let φ be such that $\{\chi_1, \chi_2\} = \{\varphi|| \cdot ||^{\pm\frac{1}{2}}\}$, and set $\pi_F^1(R) = \varphi \circ \det$. Finally, if we are in the second option, we take $\pi_F^1(R) = \varphi \cdot \mathrm{St}_G$. It is now easily verified that this map indeed a bijection and retains L-functions and ϵ-factors. Furthermore by 27.3 a non-cuspidal representation is uniquely determined by its L-function.

3 The unramified correspondence

In this section, we will define the correspondence between unramified Deligne representations and unramified automorphic representations. This proves the Langlands correspondence for $p \neq 2$, as in that case all representations are unramified; to finish the proof for $p = 2$ we would need another semester.

Again, we start with some preliminary remarks. By 27.2 every irreducible smooth representation of G is uniquely determined by its L-function and ϵ-factor. Therefore, if a map π^0 exists, it is automatically unique. Also, the way to express $\epsilon(\rho, s, a\psi)$ in terms of $\epsilon(\rho, s, \psi)$ is the same on both sides see (29.4(2) and 24.3). Therefore, if a map suffices for a single ψ, it suffices for all ψ. Therefore, we may take ψ to be of level 1, which makes calculations slightly easier. Of course, we will not actually perform these calculations.

A representation $\tau \in \mathcal{A}_2^0(F)$ is called *unramified* if there exists an unramified character $\varphi \in \widehat{F^\times} \setminus \{1\}$ such that $\varphi \otimes \tau \cong \tau$. We denote the set of unramified representations by $\mathcal{A}_2^{\mathrm{nr}}(F)$. Completely analogous we can define $\mathcal{G}_2^{\mathrm{nr}}(F)$.

Recall from Gert's talk that we have the set of admissible pairs $\mathbf{P}_2(F)$, consisting of pairs $(E/F, \chi)$ such that E/F is a tamely ramified quadratic field extension, and χ is a character of $E^\times$ such that χ does not factor through the norm map $\mathrm{N}_{E/F}$ and if $\chi | U_E^1$ does factor through $\mathrm{N}_{E/F}$, then E/F is unramified. The use for this set is given by the following theorem.

Theorem 3.1. *There is a bijection*

$$\begin{aligned} \Gamma : \mathbf{P}_2(F) &\xrightarrow{\sim} \mathcal{A}_2^{\mathrm{nr}} \text{ for } p = 2, \\ \mathbf{P}_2(F) &\xrightarrow{\sim} \mathcal{A}_2^0 \text{ for } p \neq 2. \end{aligned}$$

This suggests that we want to find a bijection $\mathbf{P}_2(F) \xrightarrow{\sim} \mathcal{G}_2^{\mathrm{nr}}(F)$. There is a straightforward choice for such a bijection:

Theorem 3.2. *The map $\Xi : (E/F, \chi) \mapsto \mathrm{Ind}_{\mathcal{W}_E}^{\mathcal{W}_F} \chi$ induces a bijection*

$$\begin{aligned} \Xi : \mathbf{P}_2(F) &\xrightarrow{\sim} \mathcal{G}_2^{\mathrm{nr}} \text{ for } p = 2, \\ \mathbf{P}_2(F) &\xrightarrow{\sim} \mathcal{G}_2^0 \text{ for } p \neq 2. \end{aligned}$$

This gives us a bijection $\Gamma \circ \Xi^{-1} : \mathcal{G}_2^0(F) \longrightarrow \mathcal{A}_2^0(F)$ for odd p. However, this map does *not* retain ϵ-factors! We need to twist every admissible pair $(E/F, \chi)$ by a character Δ_χ of $E^\times$ to ensure this. The definition of this character is easy if E/F is unramified; we then define Δ_χ to be the unique unramified character of $E^\times$ of order 2.

To define this character for a totally ramified extension, first note that $\mathrm{N}_{E/F}(E^\times)$ is of order 2 in $F^\times$, so there is a unique non-trivial character $\kappa_{E/F}$ of $F^\times$ that is trivial on $\mathrm{N}_{E/F}(E^\times)$. Let μ_F be the group of roots of unity in F with order prime to p, let ω be a prime element of E, and let β be any element of $E^\times$. We know that $U_E = \mu_E U_E^1 = \mu_F U_E^1$. As a consequence, there is a unique root of unity $\zeta(\beta,\omega) \in \mu_F$ such that $\beta\omega^{-v_E(\beta)} \equiv \zeta(\beta,\omega)$ mod U_E^1. Recall that an admissible pair $(E/F,\chi)$ is *minimal* if $\chi|U_E^n$ does not factor through $\mathrm{N}_{E/F}$, where n is the level of χ. Any admissible pair can be gained from tensoring a minimal pair with a character of $F^\times$. Also, recall the definition of the Langlands constant (independent of s by 30.4)

$$\lambda_{E/F}(\psi) = \frac{\epsilon(\mathrm{Ind}_{\mathcal{W}_E}^{\mathcal{W}_F} 1_E, s, \psi)}{\epsilon(1_E, s, \psi_E)}.$$

We now have the notation to state the following proposition:

Proposition 3.3. *The following statements hold, which we use as the definition of a character Δ_χ of $E^\times$:*

- *Let $(E/F,\chi)$ be a minimal pair such that E/F is totally ramified. Let n be the level of χ and let $\alpha \in \mathfrak{p}_E^{-n}$ satisfy $\chi(1+x) = \psi_E(\alpha x)$ for all $x \in \mathfrak{p}_E^n$. Also, let ψ be a character of $F^\times$. Then there is a unique character Δ_χ of $E^\times$ such that the following hold:*

$$\begin{aligned} \Delta_\chi|U_E^1 &= 1, \\ \Delta_\chi|F^\times &= \kappa_{E/F}, \\ \Delta_\chi(\omega) &= \kappa_{E/F}(\zeta(\alpha,\omega))\lambda_{E/F}(\psi)^n, \end{aligned}$$

  for any prime element ω of E. Furthermore, Δ_χ is independent of ψ and α.

- *Let $(E/F,\chi)$ be any totally ramified admissible pair. Write $\chi = \chi' \otimes \xi_E$ for a minimal pair $(E/F,\chi')$ and a character ξ of F'. Then $\Delta_\chi := \Delta_{\chi'}$ is independent of the decomposition $\chi = \chi' \otimes \xi_E$.*

This character Δ_χ has the following useful property.

Lemma 3.4. *The map*

$$\begin{aligned} \Delta : \mathbf{P}_2(F) &\longrightarrow \mathbf{P}_2(F) \\ (E/F,\chi) &\mapsto (E/F,\Delta_\chi\chi) \end{aligned}$$

is a bijection.

Proof. The reason for this is that $\Delta_{\Delta_\chi \chi} = \Delta_\chi$, as can be proven from the definition. Also, one can prove that the Langlands constant is a (not necessarily primitive) fourth power root of unity, so $\Delta_\chi^4 = 1$ for all χ. This together shows that the map is a bijection.

This finally gives us the map we need for the tame Langlands correspondence. Unfortunately, the proof relies on quite some technical details of the previous chapters that were not discussed in their respective talks. Therefore I will omit the proof.

Theorem 3.5 (Tame Langlands correspondence)**.** *The maps*

$$\begin{aligned} \pi^{\mathrm{nr}} &= \Gamma \circ \Delta \circ \Xi^{-1} : \mathcal{G}_2^{\mathrm{nr}}(F) \to \mathcal{A}_2^{\mathrm{nr}}(F) \text{ for } p = 2, \\ \pi^{0} &= \Gamma \circ \Delta \circ \Xi^{-1} : \mathcal{G}_2^{0}(F) \to \mathcal{A}_2^{0}(F) \text{ for } p \neq 2, \end{aligned}$$

are the unique maps with the property that for all characters $\chi \in \widehat{F^\times}, \psi \in \hat{F} \setminus \{1\}$ *one has*

$$\begin{aligned} \epsilon(\chi\pi^{\mathrm{nr}}(R), s, \psi) &= \epsilon(\chi \otimes R, s, \psi) \text{ for } p = 2, \\ \epsilon(\chi\pi^{0}(R), s, \psi) &= \epsilon(\chi \otimes R, s, \psi) \text{ for } p \neq 2. \end{aligned}$$

Furthermore, π^{nr} *and* π^0 *are bijections and preserve duals and twisting by characters of* $F^\times$*.*

As was mentioned before, this completes the local Langlands correspondence for odd p.

4 The l-adic Langlands correspondence

Although in this seminar we have only worked with representations over $\mathbf{C}$, it would be nice to have a Langlands correspondence for representations over other fields as well. In this section we will discuss the Langlands correspondence for representations over $\overline{\mathbf{Q}}_l$ (The assumption $l \neq p$ is irrelevant at this point), assuming we know its existence over the field $\mathbf{C}$. Let us use the notations $\mathcal{G}_2(F, \overline{\mathbf{Q}}_l)$ and $\mathcal{A}_2(F, \overline{\mathbf{Q}}_l)$ for the relevant sets of Deligne and automorphic representations, respectively. Since algebraically closed fields of the same characteristic and infinite transcendence degree are isomorphic, one may non-canonically pick an isomorphism $\iota : \mathbf{C} \xrightarrow{\sim} \overline{\mathbf{Q}}_l$. This induces isomorphisms $\iota_* : \mathcal{G}_2(F) \xrightarrow{\sim} \mathcal{G}_2(F, \overline{\mathbf{Q}}_l)$ and $\iota_* : \mathcal{A}_2(F) \xrightarrow{\sim} \mathcal{A}_2(F, \overline{\mathbf{Q}}_l)$. By the Langlands correspondence this gives a bijection

$$^{\iota}\pi : \mathcal{G}_2(F, \overline{\mathbf{Q}}_l) \xrightarrow{\sim} \mathcal{A}_2(F, \overline{\mathbf{Q}}_l).$$

However, this bijection is not canonical, in the sense that it is not equivariant under the action of $\operatorname{Aut}\overline{\mathbf{Q}}_l$ on both sides. So ideally, we would want a bijection π_l between $\mathcal{G}_2(F,\overline{\mathbf{Q}}_l)$ and $\mathcal{A}_2(F,\overline{\mathbf{Q}}_l)$ with the following (not well-defined) properties:

- π_l is equivariant under the action of $\operatorname{Aut}\overline{\mathbf{Q}}_l$;
- π_l should preserve some information in terms of L-functions and ϵ-factors;
- π_l should be unique with respect to the previous two properties.

Since L-functions and ϵ-factors are only defined for representations over $\mathbf{C}$, we will first construct an isomorphism $\Pi_{\mathbf{C}} : \mathcal{G}_2(F) \xrightarrow{\sim} \mathcal{A}_2(F)$ that is equivariant under the action of $\operatorname{Aut}\mathbf{C}$ and which will have some nice properties with respect to L-functions and ϵ-factors. The construction is as follows. For $R \in \mathcal{G}_2(F)$, let $\tilde{R} = (\rho_R||\cdot||^{-\frac{1}{2}}, V_R, \mathfrak{n}_R)$.

Proposition 4.1. *The map*

$$\begin{array}{rcl} \Pi_{\mathbf{C}} : \mathcal{G}_2(F) & \to & \mathcal{A}_2(F) \\ R & \mapsto & \pi(\tilde{R}) \end{array}$$

is an isomorphism of $\operatorname{Aut}\mathbf{C}$*-sets. Furthermore, it is the unique map* $\mathcal{G}_2(F) \longrightarrow \mathcal{A}_2(F)$ *satisfying*

$$\begin{array}{rcl} L(\chi\Pi_{\mathbf{C}}(R), s) & = & L(\chi\otimes R, s-\frac{1}{2}), \\ \epsilon(\chi\Pi_{\mathbf{C}}(R), s, \psi) & = & \epsilon(\chi\otimes R, s-\frac{1}{2}, \psi) \end{array}$$

For all characters χ of $F^\times$ and nontrivial characters ψ of F.

Sketch of proof. The properties with respect to the L-functions and ϵ-factors follow from the properties of the map π; this also shows the unicity and the bijectivity. Also, since on both sides representations are uniquely determined by the L-functions and ϵ-factors of their twists, it is enough to know the action of $\operatorname{Aut}\mathbf{C}$ on this level. By diving into the definitions of these one can show that on the Galois side one has

$$\begin{array}{rcl} L({}^\eta R, s) & = & {}^\eta L(R, s) \\ \epsilon({}^\eta R, s, \eta\circ\psi) & = & {}^\eta\epsilon(R, s, \eta\circ\psi) \end{array}$$

for all $R \in \mathcal{G}_2(F)$, $\eta \in \operatorname{Aut}\mathbf{C}$ and $\psi \in \hat{F}\setminus\{1\}$, whereas on the automorphic side one has

$$\begin{array}{rcl} L({}^\eta\tau, s-\frac{1}{2}) & = & {}^\eta L(\tau, s-\frac{1}{2}) \\ \epsilon({}^\eta\tau, s-\frac{1}{2}, \eta\circ\psi) & = & {}^\eta\epsilon(\tau, s-\frac{1}{2}, \eta\circ\psi) \end{array}$$

for all $\tau \in \mathcal{A}_2(F)$, $\eta \in \operatorname{Aut} \mathbf{C}$ and $\psi \in \hat{F} \setminus \{1\}$. This shows that the map $\Pi_{\mathbf{C}}$ is $\operatorname{Aut} \mathbf{C}$-equivariant, which completes the "proof".

The l-adic analogon to the Langlands correspondence is now an easy consequence. Of course such an analogon holds for any field isomorphic to $\mathbf{C}$.

Theorem 4.2. *Let K be an algebraically closed field of transcendence degree $2^{\aleph_0}$. Then there is a unique bijection*

$$\Pi_K : \mathcal{G}_2(F, K) \xrightarrow{\sim} \mathcal{A}_2(F, K)$$

with the property that $\Pi_K({}^{\iota}R) = {}^{\iota}\Pi_{\mathbf{C}}(R)$ for all $R \in \mathcal{G}_2(F)$ and all $\iota : \mathbf{C} \xrightarrow{\sim} K$.

References

[1] Colin J Bushnell and Guy Henniart. *The local Langlands conjecture for* GL_2, volume 335 of *Grundlehren der mathematischen Wissenschaften.* Springer, 2006.