Number Fields

Errata, version 2023-1
update September 16, 2025

- page 52 line -14: $p = 2$ and $m \equiv 3 \pmod 4$, in which case p ramifies.
- page 64 exercise 18: with $p \nmid \operatorname{disc}(f)$. Let $K = \mathbb{Q}(\alpha)$. Show that
- page 64 exercise 19: and $p^2 \nmid \operatorname{disc}(f)$. Let $K = \mathbb{Q}(\alpha)$. Prove that the ideal
- page 86 line 16: $b^2 + 4a(-c) = D$
- page 127 line 13: $(\psi(\varepsilon_1, \ldots, \psi(\varepsilon_{r+s-1}))$
- page 142 exercises 5 and 6: Add: Put $P = \max(\mathcal{O}_K) \setminus \{\mathfrak{p}\}$. Replace $K_\mathfrak{p}$ and $K_\mathfrak{p}^*$ by K_P and K_P^* respectively.
- page 142 exercise 8: Dedekind domain~~s~~
- page 152 line 1: $3\gamma + \operatorname{Tr}_{\mathbb{Q}}^{L}(\gamma) \in \mathbb{Z}[\zeta_3] + \mathbb{Z}[\alpha] + \mathbb{Z}[\zeta_3\alpha] + \mathbb{Z}[\zeta_3^2\alpha]$
- page 209 line -7: is the next lemma.
- page 320 line -1: Delete the $\{$: $\ldots$ for all $\mathfrak{p} \mid \mathfrak{m}_0$, (is followed by an unfortunate page break.)
- page 479 line 4: if $\sigma \in U$
- page 479 line -10: $\displaystyle\sum_{\substack{H^* \in \Upsilon(G) \\ H^* \supseteq H}}$
- page 485: a more detailed proof of Lemma 18.37:

  Proof. By Proposition 18.15

$$\sum_{U \in \Sigma(G)} n_U(Z \cap U) \in \mathrm{NR}(Z).$$

For $V \in \Sigma(Z)$ put $m_V = \sum_{\substack{U\in\Sigma(G)\\ Z\cap U=V}} n_U$. Then by Lemma 18.33 for each $d \mid (Z : T)$:

$$0 = \sum_{\substack{V\in\Sigma(Z)\\ (VT:T)=d}} m_V \#(V) = \sum_{\substack{V\in\Sigma(Z)\\ (V:(V:T))=d}} \sum_{\substack{U\in\Sigma(G)\\ Z\cap U=V}} n_U \#(V)$$
$$= \sum_{\substack{U\in\Sigma(G)\\ (Z\cap U):(T\cap U)=d}} n_U \#(Z \cap U). \qquad \square$$

- page 485 line -9: of right cosets of U in G.
- page 485 line -8: a partition of $U \setminus G$ into orbits of cosets.
- page 488: The proof of Proposition 18.43 refers to Theorem 18.34. However, this theorem applies only to abelian groups. For the following proof this condition is not needed.

Proof. For $U \in \Sigma(G)$ we have $r_U + 2s_U = [L^U : \mathbb{Q}]$. So

$$\sum_{U\in\Sigma(G)} n_U(r_U + 2s_U)\#(U) = \sum_{U\in\Sigma(G)} n_U[L^U : \mathbb{Q}][L : L^U] = [L : \mathbb{Q}] \sum_{U\in\Sigma(G)} n_U = 0$$

In the proof of Theorem 7.53 the splitting of a prime ideal in an intermediate field of a Galois extension is used. This applies equally well to the splitting of infinite primes. Let $\mathfrak{q}$ be an infinite prime of L. Note that for infinite primes the inertia groups coincide with the decomposition groups. In Theorem 18.38 it is shown that $\sum_U n_U\#(U)t_{\mathfrak{p},U} = 0$, where t_U is the number of prime ideals of $\mathcal{O}_{L^U}$ above a given prime ideal $\mathfrak{p}$ of K with a given residue class degree. For infinite primes the same holds, all residue class degrees being 1. Summation over all infinite primes of K yields

$$\sum_{U\in\Sigma(G)} n_U\#(U)(r_U + s_U) = \sum_{U\in\Sigma(G)} n_U\#(U) \sum_{\mathfrak{p}\in\mathcal{P}_\infty(K)} t_{\mathfrak{p},U}$$
$$= \sum_{\mathfrak{p}\in\mathcal{P}_\infty(K)} \sum_{U\in\Sigma(G)} n_U\#(U)t_{\mathfrak{p},U} = 0. \qquad \square$$