The Fellowship of Geometry and Quantum Theory

Genootschap voor Meetkunde en Kwantumtheorie

Summary

As a striking example of the cross-fertilization between mathematics and other fields, quantum theory has recently emerged as a unifying theme in the modern development of geometry. Ideas from quantum mechanics, quantum field theory, and string theory have transformed algebraic and symplectic geometry, and even inspired the creation of a new branch of mathematics (viz. noncommutative geometry). At the same time, progress in fundamental physics increasingly hinges on deep mathematical ideas. *The primary aim of this proposal is to make The Netherlands a major player in this development.*

Our group, based at the Universities of **Amsterdam**, **Nijmegen**, and **Utrecht** (the latter acting as the cluster center or 'hub'), plans to achieve this through the creation of a long-term educational infrastructure, combined with a new large-scale research program in mathematics. The former will nurture a generation of students who are, so to speak, bilingual in relevant areas of both pure mathematics and fundamental physics. This involves

1. Dual Bachelor degree programs in mathematics and physics (3 years);
2. Master programs in mathematical physics (2 years);
3. Specialized Master Classes on key cluster themes (1 year);
4. A PhD program in geometry and quantum theory (4 years).

Our research plans revolve around areas such as:

- Poisson geometry, quantization, and noncommutative geometry;
- Integrable systems, Frobenius manifolds, and the geometric Langlands program;
- Moduli spaces, mirror symmetry, and topological strings.

The scale and intensity of this proposal seem unprecedented in Dutch mathematics, yet a number of local circumstances appear to make our initiative timely and feasible. Indeed, part of the envisaged infrastructure already exists at the participating universities, much of the required scientific expertise is already scattered over the proposed cluster members, and most of us have a track record of management and collaboration.

The tenured faculty participating in the cluster includes ten full professors and thirteen other researchers. In addition, an active group of younger postdocs and PhD students will be involved in the cluster activities. Some of the most talented Dutch geometers working abroad will be appointed as Fellows of the cluster. The cluster will work in close contact with a Board of Advisors, which includes one Nobel Laureate and two Fields Medalists. Of the ten professors, three are members of the Royal Academy of Sciences (KNAW), and one has recently been appointed as the first Royal Academy Professor in mathematics. Of the younger professors, three have received prestigious PIONIER grants from NWO, and one is a recipient of the Spinoza Prize (the highest scientific distinction in The Netherlands). In addition, the work of some of the still younger tenured participants has already been recognized by the KNAW and NWO through fellowships and awards. Thus the continuity of our program seems guaranteed.

1 Research Plan

1.1 Introduction

> The more I have learned about physics, the more convinced I am that physics provides, in a sense, the deepest applications of mathematics. The mathematical problems that have been solved, or techniques that have arisen out of physics in the past, have been the lifeblood of mathematics…The really deep questions are still in the physical sciences. For the health of mathematics at its research level, I think it is very important to maintain that link as much as possible. (Michael Atiyah)

This year, the second Abel Prize has been awarded jointly to Atiyah and Singer "for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics." This citation, concerning arguably the most prestigious prize in mathematics, as well as the award of Fields Medals to e.g. Connes, Witten, and Kontsevich, confirms the remarkable fact that it is the frontiers of *pure* mathematics and *fundamental* physics that happen to be in close contact at the moment. This is, of course, not a new phenomenon, though it is significant that the three previous episodes where this happened marked some of the most significant revolutions in the history of science. Indeed, modern mathematics and physics were born together in the 17th century through the work of Newton, who created both the calculus and classical mechanics in intimate relationship to each other. Subsequently, Einstein's general theory of relativity (which replaced Newton's concepts of space, time, and gravity) was formulated on the mathematical basis of Riemannian geometry, in turn inspiring Weyl and Cartan to reshape the latter from a local to a global theory; cf. [23]. Third, through the work of Hilbert and von Neumann, quantum mechanics was an important source of the transition from classical analysis to its modern (abstract) form (see, e.g., [39]).

It appears, then, that we are currently witnessing another such episode, in which the autonomous development of geometry as a branch of pure mathematics (dating back at least to Euclid) is enriched by a remarkable flow of ideas from fundamental physics, notably quantum theory. It is primarily in this sense that we intend to realize the stated aim of NWO (the Dutch Research Council) and OOW (i.e., the combined Dutch inter-academic research schools in mathematics) to support research in the interface between mathematics and theoretical physics in The Netherlands. Furthermore, an important secondary effect of the cluster will undoubtedly be the enhancement of the opposite flow as well. The present proposal focuses on the interplay between *geometry*, including algebraic, symplectic, and noncommutative geometry, and *quantum theory*, incorporating quantum field theory, string theory, and quantization. While at first sight geometry seems a vast and diverse field, even when restricted to the three areas mentioned, our cluster achieves its coherence largely from the recent insight that these areas are related in remarkable new ways, often initially suggested by quantum physics.

As will become clear throughout this proposal, at present the national situation seems quite favourable to participate in this development. We possess considerable experience on all aspects of the present initiative, but - and this is the *raison d'etre* for the present cluster - our joint expertise has on the whole been kept separate so far. Thus our basic goal is to join forces in order to create an infrastructure in which the interplay between geometry and quantum theory can be exploited to maximal effect, primarily from a mathematical perspective.

1.2 Research area

For the benefit of the reader, we first sketch the historical background to our research area, restricting ourselves to those aspects that are immediately relevant to our own plans described in Section 1.4.

1.2.1 From quantum mechanics to noncommutative geometry

Quantum mechanics was born in 1925 with the work of Heisenberg, who discovered the noncommutative structure of its algebra of observables. The complementary work of Schrödinger from 1926, on the other hand, rather started from the classical geometric structure of configuration space. Within a year, their work was unified by Von Neumann, who introduced the abstract concept of a Hilbert space, in which Schrödinger's wave functions are vectors, and Heisenberg's observables are linear operators. The somewhat primitive notion of quantization that had been used by Heisenberg was rapidly put on a more mathematical footing by Dirac.

Weyl immediately recognized the notion of a Hilbert space as the appropriate setting for a theory of infinite-dimensional group representations, an area that has continued to interact with quantum theory ever since. For example, in the 1960s Kostant and Souriau related symplectic geometry to quantum mechanics and representation theory in a theory called *geometric quantization*, in which Kirillov's 'orbit method' is combined with the insights of Dirac and Weyl. In 1964 Dirac once again provided important ideas for this field through his work on constrained quantization, which inspired mathematical tools such as momentum maps and Marsden–Weinstein reduction in symplectic geometry. Dirac's combined influence culminated in the Guillemin–Sternberg conjecture, which states that geometric quantization commutes with symplectic reduction (cf. [19] for a recent overview).

A generalization of symplectic geometry, Poisson geometry, was defined in 1976 by Kirillov and Lichnerowicz. Almost by definition, Poisson geometry provides the mathematical setting for classical mechanics in Hamiltonian form, and accordingly for the classical theory of integrable systems. Weinstein introduced many fundamental ideas to this subject, most notably the relationship between Poisson manifolds and Lie groupoids (that is, objects that encode not only global symmetries, as Lie groups do, but also local ones, and have associated 'infinitesimal objects' known as Lie algebroids; cf. [37]). In addition, Poisson manifolds form the starting point of the notion of deformation quantization.

In the mid-thirties, von Neumann created the theory of operator algebras on Hilbert spaces, which extends the scope of his earlier mathematical formulation of quantum mechanics. A decisive technical contribution to operator algebras was also made from a purely mathematical perspective by Gelfand and Naimark in 1943 with their introduction of C^*-algebras. Almost simultaneously, Hodge considerably advanced the field of geometric analysis (initiated earlier by Weyl) with his introduction of topological and analytic methods in algebraic geometry. This field culminated in the index theorems of Atiyah and Singer in the 1960s, in which topological K-theory (a construction in algebraic topology due to Atiyah and Hirzebruch, who in turn were inspired by ideas of Grothendieck in algebraic geometry), also played an important role.

Around 1980, Connes incorporated operator algebras, geometric analysis, topological K-theory, Riemannian geometry, as well as a new construction in homological algebra called cyclic cohomology, in his formidable edifice of noncommutative geometry [6]. For one thing, this has led to vast generalizations of index theory, for example to noncompact and especially singular spaces. Connes not only explicitly acknowledged the role of quantum mechanics in the conceptual motivation for his theory; a decade after its incarnation this origin also resurfaced at a technical level, with Rieffel's recognition that an analytic version of deformation quantization could be defined within the technical framework of noncommutative geometry [41]. This move, then, at last also brought Poisson geometry into this framework. In recent years, noncommutative geometry has made connections with such diverse areas as quantum groups, modular forms (in algebraic geometry), and even number theory.

1.2.2 The impact of quantum field theory

In the preceding, the quantum theory involved was nonrelativistic. The conceptual revolution in algebraic geometry that emerged from 1990 onwards had its roots in two relativistic versions of quantum theory, viz. *quantum field theory* and *string theory*. Quantum field theory, combining quantum theory with *special* relativity, was first constructed in 1927, but was only completed as a physical theory in 1948 with the successful incorporation of renormalization and Feynman diagrams. Mathematically, quantum field theory largely remains mysterious, despite attempts to found the theory on the basis of Schwartz's theory of distributions (Wightman), von Neumann's operator algebras (Haag [21]), or on rigorous versions of Feynman's path integrals (Glimm & Jaffe). Indeed, this mystery is part of its current fascination among mathematicians.

At the classical level, it was noted in the 1970s by Yang and others that Yang–Mills theory, the field theory underlying the Standard Model of elementary particle physics (for which 't Hooft and Veltman got the Nobel prize in 1999), has a striking geometric nature. Moreover, around 1975 't Hooft (and independently Polyakov) discovered fascinating solutions to its equations, known as magnetic monopoles and instantons. These themes were picked up in 1977 by Atiyah and others at Oxford, who were thereby led to important new techniques for dealing with integrable systems [25]. The so-called Hitchin systems (i.e., integrable systems constructed from moduli spaces of principal bundles over Riemann surfaces) are a case in point. In 1982, Atiyah and Bott initiated a new approach to closely related moduli spaces, linking these to both Yang–Mills theory and symplectic geometry in a beautiful way. One of the highlight of this development was undoubtedly the new classification of four-manifolds by Atiyah's student Donaldson, which was directly based on Yang–Mills theory and instantons (Fields Medal 1986).

So far, the only case where an appropriate mathematical theory is available in relativistic quantum field theory is in one space and one time dimension. The most prominent special case of this, *conformal field theory*, was effectively launched in 1984 by Belavin et al; cf. [9]. Conformal field theory gave a boost to a number of remarkably diverse areas of mathematics, which it to some extent integrated. These include infinite-dimensional Lie groups and Lie algebras (notably loop groups and the diffeomorphism group of the circle, with the affine Kac–Moody algebras and the Virasoro algebra as their Lie algebras), integrable systems and integrable hierarchies, algebraic topology, and even the theory of sporadic finite simple groups; Borcherds was awarded a Fields Medal in 1998 for establishing this link and the associated creation of the associated mathematical theory of vertex algebras.

1.2.3 String theory and mirror symmetry

For most of the subsequent developments, it is practically impossible to separate conformal field theory from topological quantum field theory and string theory, especially in the hands of Witten and Kontsevich, the theoretical physicist and the mathematician who (with Connes) dominated the geometry & quantum theory interface in the 1990s.

String theory started as a niche in high-energy physics in the early 1970s, but began to attract worldwide attention with the work of Green and Schwarz in 1984 on anomaly cancellation (cf. [9]), and its subsequent endorsement by Witten. It is an ambitious program that postulates that the smallest degrees of freedom in nature are not particles or fields but strings, and which has already succeeded in combining quantum theory with *general* relativity (this had turned out to be an impossible task in quantum field theory). Though it is not clear whether its central physical prediction, supersymmetry, will be experimentally verified, the relevance of string theory to pure mathematics is beyond any doubt. Since in string theory manifolds are studied through loops instead of points, it immediately suggests a natural generalization of ideas from classical geometry. More subtly, as a quantum theory it not only has Planck's constant $\hbar$ as a deformation parameter, but in addition contains the string parameter α'; cf. [9]. It is basically this two-parameter structure that lies behind the ability of string theory to relate seemingly diverse phenomena and theories in mathematics.

Perhaps the most unexpected such relation is what is known as *mirror symmetry*, originally discovered in 1991 in the setting of conformal field theory. Namely, the alleged equivalence of two seemingly different physical models turned out to imply a technique for solving problems in enumerative geometry in which mathematicians had made no progress for over a century. Specifically, a difficult enumerative problem on one manifold could be transformed to an easy problem about period integrals and hypergeometric functions on another, the so-called mirror partner. This is a special case of the phenomenon of *duality*, which plays a fundamental role in string theory. In the setting of so-called *topological strings*, mirror symmetry exchanges the A-model (whose degrees of freedom are Gromov–Witten invariants) and the B-model (which can be viewed as a quantization of the moduli space of complex structures).

Mirror symmetry poses a new conceptual framework for certain classical parts of algebraic geometry that consequently have returned to the forefront of research, such as toric varieties, Calabi–Yau varieties (especially of complex dimension 3, as first indicated by the needs of string theory compactification), K3-surfaces, and abelian varieties; see [26] for a recent survey. Mirror symmetry implies new and deep relations between algebraic and symplectic geometry, especially at the level of moduli spaces. In its most abstract and powerful formulation due to Kontsevich [30], mirror symmetry is an equivalence of certain categories defined by the two partners X and Y of the mirror pair. One, the so-called Fukaya category, is constructed in terms of the Lagrangian submanifolds of X (defined through a choice of a Kähler form), and the other is defined by the coherent sheaves on Y (associated to its complex structure) [30]. At the other side of the interface, further impetus from physics has come from the role of so-called D-branes in string theory, which are distinguished submanifolds defining appropriate moduli spaces that lie at the basis of higher forms of mirror symmetry.

Topological quantum field theory was invented by Witten in 1988 in an attempt to find a conceptual explanation from physics for various phenomena in geometry [46]. In dimension four, it contained Donaldson's theory as a special case, planting the seeds for the remarkable reformulation of the latter by Seiberg and Witten in 1994. In dimension three, it not only explained Jones's new knot invariant (which Jones, incidentally, discovered from the perspective of operator algebras, earning him a Fields Medal in 1990) in the said way, but in addition led to a whole new series of so-called 'quantum invariants' or 'Witten invariants' of three-manifolds. Efforts to make Witten's invariants mathematically rigorous branched into two directions. One defined topological quantum field theories through the geometric quantization of suitable moduli spaces (such as the moduli space of flat connections on a Riemann surface introduced by Atiyah and Bott [1]). The other used so-called braided tensor categories as the appropriate mathematical framework [2, 21].

Topological quantum field theory has had a comparable impact on mathematics in its two-dimensional version, in the context of quantum gravity and σ-models (whose basic variables are maps from a complex curve into a given manifold). These methods caused a watershed in Poisson geometry, algebraic geometry, and integrable systems, both independently, and in the recognition of how these areas are interrelated. As to the first (anti-chronologically), Kontsevich used ideas from topological σ-models in his remarkable proof in 1997 that any Poisson manifold admits a formal deformation quantization [27]. In its current formulation [29], this fundamental result (for which an analytic analogue remains to be found) is closely tied to operads and other aspects of modern homotopy theory.

1.2.4 Towards Frobenius manifolds and the geometric Langlands program

In the context of algebraic geometry, a paper by Witten in 1990 on the coupling of two-dimensional topological gravity to 'matter,' in the form of σ-models (and its subsequent mathematical reformulation and extension by Witten himself [47], Kontsevich and Manin [28], and others, along with independent work of Gromov in the setting of symplectic geometry and topology) led to the twin notions of Gromov–Witten invariants and quantum cohomology, introducing an entirely new tool in the study of moduli spaces. The essential structure defining the quantum cohomology of a given variety is intersection theory in a moduli space of suitable (i.e., stable) maps from curves into the variety. One important application is that the answers to certain questions in the enumerative geometry of a variety can be read off from the multiplication table of its quantum cohomology ring, so that the subject describes one half of a mirror symmetry pair.

In the same physical setting, Witten was led to an influential conjecture (later proved by Kontsevich) on the structure of the cohomology ring (more precisely, of the so-called tautological ring) of the moduli space of curves of given genus and number of marked points. Witten's conjecture states that the generating function of all top intersections of certain fundamental geometrically defined classes in this ring (the ψ-classes) satisfies a particular differential equation, which allows a recursive computation of the intersection numbers. Subsequently, still motivated by two-dimensional gravity and string theory, Dijkgraaf, E. Verlinde and H. Verlinde showed that Witten's conjecture has the form of an integrable hierarchy of partial differential equations, now known as the WDVV-equations. Witten's original generating function is then what in integrable theories jargon is called the τ-function of this hierarchy, which in the simplest case is the one associated to the KdV-equation. More recently, generalizations of the WDVV-equations have emerged in the context of Seiberg–Witten theory.

Thus the WDVV-equations establish a deep link between moduli spaces of algebraic geometry and integrable systems of the kind that originally were meant to describe phenomena in fluid mechanics such as shallow water waves! The original physical description of this link through quantum field theory in dimension two, while still providing essential intuition, has now been superseded by the mathematical concept of a *Frobenius manifold*, due to Dubrovin [12] (with important later contributions by Manin, Kontsevich, Givental, and others, see [36]). Such manifolds are locally described by the WDVV equations. In the original physics setting, a particular class of Frobenius manifolds came equipped with additional structure ultimately defined by a quantum cohomology ring. In this context, Manin and others have proposed that Frobenius manifolds provide a natural framework for the concept of mirror symmetry. In addition, there turns out to be a second class of Frobenius manifolds, having its origins in Saito's theory of unfolding isolated singularities of hypersurfaces [42]. The main examples of this theory come from constructions involving Lie-theoretic data, cf. [34]. This relates Frobenius manifolds and integrable systems to singularity theory, which generally studies the dependence of certain objects on parameters; see [43] for a recent survey. A third class of Frobenius manifolds, of equal interest to our cluster, comes from deformation theory, specifically from so-called differential Gerstenhaber–Batalin–Vilkovisky algebras. Note that the latter two authors discovered this structure in their work on the quantization of constrained systems.

More recently, a different perspective has emerged, which promises to link many of the ideas described above, notably those on integrable systems, conformal field theory, mirror symmetry, and geometric quantization. This is the *geometric Langlands program*, which adds representation theory to these areas as a basic tool [16]. The original Langlands program in number theory [4], dating from the 1960s, revolves around a profound correspondence between automorphic representations and Galois representations. Although a precise formulation of the general correspondence is not known, in a variety of nontrivial examples and special cases its predictions have been established in great detail. Some of the most exciting breakthroughs in modern mathematics fall under this umbrella, including Wiles's key result that implied Fermat's Last Theorem. Also, the Fields Medals awarded in 1990 to Drinfeld and in 2002 to L. Lafforgue recognized work in this area.

In the original setting, one starts with either a number field (i.e., a finite extension of $\mathbb{Q}$), or a function field (that is, the field of rational functions on a smooth projective curve C defined over a finite field $\mathbb{F}_q$). The geometric Langlands program provides a reformulation of the Langlands correspondence in the second case, where now $\mathbb{F}_q$ is replaced by the complex numbers $\mathbb{C}$. Thus the program is placed in the context of classical algebraic geometry, where it constructively interferes with the ideas from physics discussed so far in bringing both new tools and a unified perspective on the disciplines just mentioned. It is fair to say that on the one hand the expected interrelations are based on convincing and nontrivial examples, while on the other a general and satisfactory explanation or understanding is still missing.

1.3 Expertise in The Netherlands

Dutch mathematicians and mathematical physicists have already played a significant role in the developments described above, and the expertise of the cluster members in this area makes them well prepared to join forces in order to assume a leading role in the future.

As pointed out above, the discovery of magnetic monopole and instanton solutions by cluster advisor 't Hooft was instrumental in the establishment of the modern link between geometry and field theory (in the physics sense!) by Atiyah, Witten, and others from 1977 onwards. In 1988, 't Hooft's student and cluster advisor E. Verlinde wrote the paper on conformal field theory that probably had the single largest impact on pure mathematics, proposing both the algebra and the famous formula named after him. More generally, the theoretical physicists Dijkgraaf, H. Verlinde, and E. Verlinde emerged as central players in low-dimensional quantum field theory and string theory in the 1990s. Beyond this role, there is no question that Dijkgraaf in particular has been a pivotal figure in the communication between physicists and mathematicians working in this area.

The work of Duistermaat has played an important role in establishing the current link between symplectic geometry, quantum theory, analysis, and representation theory, firstly in the seventies with Hörmander and with Guillemin, secondly in the eighties with his student Heckman, and thirdly with his work on index theory and the Dirac operator in the nineties {13}. (Citations {. . . } refer to the list of Key publications in 2.6 below.)

Dutch algebraic geometers have made important contributions to the study of moduli spaces; cf. [11, 14]. Looijenga developed an invariant theory for generalized root systems with applications to moduli spaces [34], proved the surjectivity of the period map for Kähler K3 surfaces, proved the Zucker conjecture, and contributed to the compactification theory of moduli spaces, as well as to motivic integration. Current research on the tautological ring on the moduli spaces $\mathcal{M}_g$ of curves is largely driven by Faber's conjectures (see [13, 33, 45]), whereas van der Geer determined the tautological ring of the moduli space $\mathcal{A}_g$ of abelian varieties and found the formulas for the cycle classes of the Ekedahl–Oort stratification [17]. Further contributions to this analysis were made by Moonen. Other themes where progress was made by Dutch geometers include the Schottky problem (van Geemen, van der Geer), Torelli theorems (Oort, Steenbrink, Peters), and the study of Shimura varieties (van der Geer, Oort, Moonen). Cornelissen contributed to the study of Mumford curves in positive characteristic, in particular to equivariant deformation theory. Finally, cluster Fellow De Jong is widely regarded as one of the world's leading algebraic geometers.

The so-called 'Dutch school of singularity theory,' led by Looijenga, Siersma and Steenbrink, emerged in the 1980s as a potent force in this field, contributing to the deformation theory of weakly normal (non-isolated) singularities), to discriminant spaces, and to the study of sheaves of vanishing cycles. For example, Steenbrink's results on Calabi–Yau threefolds with isolated hypersurface singularities [38] are well known.

Crainic recently solved some of the most important open problems in Lie groupoids and Poisson geometry (cf. {12} and [7]). Landsman, originally a theoretical physicist, proved in 1998 that noncommutative spaces defined by Lie groupoids arise from the quantization of the underlying Lie algebroids. His work on axiomatic quantum theory has been used by researchers in areas ranging from quantum gravity to the philosophy of physics [31]. Moerdijk is best known for his work on topos theory and on groupoids. In 1988, he solved a conjecture of Haefliger on the cohomology of the classifying spaces of foliation groupoids. Recently, his work with Berger on the existence of Quillen homotopy model structures on categories of operads {11} attracted considerable attention. Jointly, Crainic, Landsman, and Moerdijk have made the intersection between Poisson geometry, noncommutative geometry, Lie groupoids, quantization, and deformation theory a Dutch specialty.

Apart from his renowned work with Duistermaat on localization in symplectic geometry, Heckman proved quantum integrability of the Calogero-Moser system in the context of general root systems, found the eigenfunctions of these systems (with Looijenga), and studied the root system generalizations of the quantum integrable system describing the boson gas on the real line with delta function interaction (with Opdam) {7}. Last year, Opdam found an explicit Plancherel formula for general Iwahori-Hecke algebras, and also constructed a highest weight category for rational Cherednik algebras {3}. Stokman made well-known contributions to quantum integrable systems and (with Koelink) to noncompact quantum groups.

1.4 Proposed research

We now describe our concrete research plans. It should be taken into account that these plans incorporate ideas by 23 people, to be carried out by a sizeable additional group of PhD students and postdocs as well. Hence, rather than describing specific research problems in great detail, we have preferred to isolate a number of areas and emphasize their interrelations. For the sake of concreteness, however, certain topics have been marked in italics, for instance as the subject of PhD theses.

1.4.1 Poisson geometry, quantization, and noncommutative geometry

As we have seen in the general overview, noncommutative geometry is closely related to Poisson geometry through the notion of quantization. One of our ambitions is to relate noncommutative geometry to algebraic geometry as well, in the following fashion. One of the original goals of noncommutative geometry was to provide new tools for the study of singular spaces, such as the K-theory and cyclic cohomology of an appropriate noncommutative algebra associated to a quotient space. Connes himself successfully applied his toolkit to foliated spaces, Penrose tilings, and certain other examples [6]. However, the application of noncommutative geometry to some other important classes of singular spaces, namely those that have traditionally been studied using algebraic geometry, is still in its infancy. Here we are thinking, for example, of orbifolds, certain types of moduli spaces, and symplectic quotients (cf. [32]). Remarkably, it therefore seems that such spaces may alternatively be studied using either the tools of commutative algebra (in the setting of Grothendieck-style algebraic geometry), or of noncommutative algebra (in the context of noncommutative geometry). *The comparison of these methods (in the context of suitable examples like the ones listed) is bound to lead to new insights;* cf. [5]. This would combine the joint expertise of at least half of the cluster members.

Another pertinent interdisciplinary topic is the *functoriality of quantization*, in the sense recently proposed in $\{9\}$. The most immediate concrete consequence of this functoriality principle is an extension of the Guillemin–Sternberg conjecture in geometric quantization (which is a theorem now for compact Lie groups acting on compact symplectic manifolds, cf. [19]) to the noncompact case. Proving this, or else limiting the scope of the conjecture through the discovery of counterexamples, would combine the expertise of Duistermaat, Heckman, Landsman, Van den Ban, and others, as it links symplectic and noncommutative geometry with index theory and representation theory. In the singular case, also stratification techniques from algebraic geometry will enter. Furthermore, functoriality of quantization needs to be concretely developed through examples involving Lie groupoids and algebroids. This includes the establishment of a general index theorem for Lie groupoids, generalizing the ordinary and family index theorems of Atiyah and Singer, the index theorem for noncompact groups of Connes and Moscovici, as well as the index theorem for foliated spaces of Connes and Skandalis. Another necessary ingredient would be *the K-theory and representation theory of Lie groupoids*, which will be taken up from the perspective of a generalized orbit correspondence (identifying the coadjoint orbits in the dual of a Lie algebra with its symplectic leaves, which notion immediately generalizes to Lie algebroids).

Parallel to this, we intend to study *deformations of Lie groupoids that are Hopf algebroids*, relating the subject to dynamical quantum groups and Yang-Baxter equations. In fact, the precise relationship between the quantum analogues of semisimple noncompact Lie groups and the concepts of noncommutative geometry remains to be clarified; here one might think of relating the Haar weight to the Dixmier trace, and the Duflo–Moore operators to the corresponding modular operators. In this effort, the combined expertise of Van den Ban, Crainic, Koelink, Opdam, Stokman, and Moerdijk will be relied upon.

Modern deformation theory heavily relies on the concept of an operad (originally invented in topology in the 1970s by Boardman–Vogt, May, Stasheff, and others), cf. [29]. In addition, operads relate to various other research topics in this cluster, notably to moduli spaces (cf. [18, 35]) and configuration spaces (in the sense of algebraic geometry). In the context of mirror symmetry for Calabi–Yau manifolds (cf. Section 1.2.3), the Fukaya category is a so-called A_∞-category, which means that its composition structure is modeled on some operad and needs 'higher compositions of morphisms' to compensate for a lack of straightforward associativity. Operad structures also occur in various (topological or conformal) quantum field theories. Apart from developing their unifying role, we aim to address several important open problems, e.g., *the question to what extent the topological Boardman-Vogt resolution can be applied to non-topological operads*. The leading figures in this research will be Moerdijk.

Another notion that is central to the research topics mentioned so far is that of a gerbe. Gerbes were originally introduced in the 1960s by the Grothendieck school in algebraic geometry (in particular, by Giraud) in the context of non-abelian cohomology. In the 1990s, gerbes resurfaced in geometric quantization as well as in mirror symmetry, where they enter in the description of the mirror partner of a Calabi–Yau 3-fold in terms of the moduli space of Lagrangian submanifolds equipped with gerbes (cf. [24]). In addition, a gerbe over a manifold enables one to 'twist' the K-theory of this manifold. First introduced in algebraic topology by Donovan and Karoubi in 1970 (and subsequently shown by Rosenberg to be a special case of C^*-algebraic K-theory), twisted K-theory made a striking reappearance in 1998 in string theory [48]. Subsequently, Freed, Hopkins and Teleman observed that the Verlinde algebra of the Wess–Zumino–Witten model of conformal field theory (or, mathematically, the appropriate representation category of the underlying loop group LG) coincides with the suitably twisted K-theory of G.

Our plan to understand the representation theory and the K-theory of Lie groupoids by combining techniques from equivariant algebraic topology and from C^*-algebra theory (cf. [44]), is partly motivated by examples coming from this development. Indeed, the gerbes occurring in this context can be described as extensions of Lie groupoids, and the K-theory of such a central extension is closely related to the twisted (by the gerbe) K-theory.

In a more categorical direction, we will attempt to relate the new approach to quantum probability and second quantization recently initiated by Guta and Maassen [20] to the general setting described in this section. Since their work is based on Joyal's combinatorial theory of so-called species of structure, which touches on a number of the themes discussed so far, this seems a realistic goal.

Interesting problems remain, of course, even at the purely classical level. We will focus on a generalization of the notion of a Poisson structure, called a Dirac structure (originating in Dirac's work on constrained systems). It was recently shown by Bursztyn and Crainic that Dirac structures are closely related to the group valued momentum maps of Alekseev et al, but in this relationship much remains to be understood (such as the precise relationship to Manin pairs and quasi-Poisson Lie groups). In addition, an exciting *link between this generalized Poisson geometry and mirror symmetry* was recently uncovered by Hitchin, who showed that complex versions of Dirac structures naturally appear in the theory of mirror symmetry and Calabi–Yau manifolds. This poses, of course, an attractive area of research in our cluster (Crainic, Dijkgraaf, Van der Geer, Looijenga, Stienstra).

1.4.2 Integrable systems, Frobenius manifolds, and the geometric Langlands program

Integrable systems and representation theory (or Lie theory) are closely related to each other, as well as to algebraic geometry and quantization. Thus the area is ideally suited for the proposed cluster. The main researchers will be Cushman, Duistermaat, Heckman, Helminck, Koelink, Van de Leur, Opdam, and Stokman, relying on the knowledge of mirror symmetry, moduli spaces, conformal field theory, and quantization of practically all other cluster members.

As we have seen, some of the pertinent relationships are codified by the notion of a Frobenius manifold, others by the geometric Langlands program. The starting point of the geometric Langlands correspondence is the moduli space Bun_G of principal G-bundles over a smooth projective curve C. In the context of the Langlands program, one associates a group ${}^LG$ (the Langlands dual group) to a given complex semisimple algebraic group G. We intend to study the recent conjecture of Hausel and Thaddeus [22] that the moduli spaces Bun_G and $\mathrm{Bun}_{^LG}$ (with certain additional data) are in an appropriate sense relative mirror partners (in the sense of Strominger–Yau–Zaslow, cf. [26]). This is related to the conjectured existence of a general Fourier–Mukai transform underlying the geometric Langlands duality (see [16]). We plan to *investigate whether there is a geometric Langlands correspondence for the moduli space $\mathrm{Bun}_{G,S}$ of G-bundles with parabolic structure at a finite list S of marked points of C, and local systems with ramifications at the elements of S* (this has been done in positive characteristic by Drinfeld for $GL(2)$, and by Heinloth for $GL(3)$). This raises further questions about the "categorification" of the full Iwahori Hecke algebra, and is also related to the work of Varchenko and coworkers on the Bethe Ansatz, a subject well familiar to the researchers listed above.

The link between the geometric Langlands program and Hitchin's integrable systems (cf. [16, 25]) beautifully fits in the cluster theme, and will be examined in detail. The point of departure is a remarkable result of Hitchin, which says that the symplectic space $T^*\mathrm{Bun}_G$ is a completely integrable system (assuming the curve C has genus $g \geq 2$; for $g = 2$ and $G = SL(2)$ the Hitchin system is related to the classical Neumann system, a relationship we plan to investigate for other low genus and small rank cases). In a monumental unpublished paper, Beilinson and Drinfeld [3] have recently proved a special case of the geometric Langlands correspondence through the quantization of the Hitchin system, involving infinite-dimensional Kac–Moody algebras, as well as the $\mathcal{W}$-algebras first encountered in conformal field theory.

This breakthrough poses all sorts of questions, and suggests various generalizations. For example, *the relationship between Beilinson and Drinfeld's notion of quantization and deformation or geometric quantization ought to be established*. As another example, deformation quantization suggests that one should be able to *diagonalize the pertinent $*$-algebra by means of a suitable spectral decomposition*. In an analytic setting, the semiclassical (WKB) approximation could be applied (here as well as in other integrable models). The geometric quantization of the Hitchin system, on the other hand, has at least two interesting aspects. Firstly, the appropriate Guillemin–Sternberg conjecture should be proved; the classical reduction procedure leads to the well-known integrable systems named after Schlesinger. Secondly, the situation is analogous to the quantization of the Atiyah–Bott moduli space of flat connections over a curve [1], and leads to similar links with conformal field theory (as established in detail by Laszlo). A number of important open problems remain here, most notably *the unitarity of the representation of* *the mapping class group of C*, defined by either the geometric quantization procedure or the underlying conformal field theory. This is closely related to the construction of an appropriate braided tensor category describing, from the conformal field theory perspective, the charge sector structure of the model [21], or, from the loop group point of view, the pertinent representation category.

Quantization also provides a link with quantum (elliptic) Calogero–Moser–Sutherland integrable models and the special functions related to these; in particular, the hypergeometric function for root systems belongs to this family. There are many research issues related to these integrable systems and their root system generalizations, see for instance [40]. Similarly, cyclotomic Hecke algebras arise, with many open questions remaining; cf. $\{3\}$. The role of $\mathcal{W}$-algebras in the construction of Beilinson and Drinfeld leads, via the classical Drinfeld-Sokolov Hamiltonian reduction procedure, to a direct link between Hitchin systems and integrable hierarchies of partial differential equations. For example, the cases ${}^{L}G = SL_n$ give rise to the so-called generalized KdV hierarchy. *The quantization procedure of Beilinson and Drinfeld then suggests a quantization of this hierarchy* (and its generalizations), which we plan to study in detail, again also in connection with the issue whether quantization commutes with reduction.

Integrable hierarchies will also be studied in connection with Frobenius manifolds, where we wish to relate four existing developments [36]: firstly, Barannikov's construction of Frobenius manifolds inspired by mirror symmetry, secondly, their construction from the KP hierarchy, thirdly, their origin in Saito's theory of isomonodromic transformations, and finally, the construction of 'almost Frobenius manifolds' from generalized WDVV equations. The first three of these topics involve a geometric construction using admissible planes within an infinite-dimensional Grassmannian, and we propose to view these constructions on an equal footing. The third approach turns out to be closely related to the geometric Langlands program.

In this context, our main research questions are as follows. Which of the various Frobenius manifolds constructed from integrable hierarchies have a similar description like the ones of Barannikov, i.e., as a family of planes in the Grassmannian satisfying some additional restraints? To what extent can geometrical Darboux transformations be found that relate Frobenius manifolds to each other? Another research issue involves *the solutions of the generalized WDVV equations in the coordinate free setting of the perturbative Seiberg-Witten prepotentials*. Finally, we intend to use deformations of connections in the construction of Frobenius manifolds.

To close this section, we announce a quite novel plan to relate the geometric Langlands program to noncommutative geometry. This will be done through the so-called the Baum–Connes conjecture (1982) in the latter field (cf. [6], Ch. II). This conjecture describes the K-theory of a (reduced) group C^*-algebra $K_0(C_r^*(G))$ in terms of a 'topological' K-theory group $K^0_{\rm top}(G)$. (The underlying toolkit is heavily used in the study of the functoriality principle for quantization described in the preceding section, and, indeed, the conjecture itself may be formulated in terms of deformation quantization [6], $\{8\}$.) The Baum–Connes conjecture was proved for a large class of groups in 1999 by V. Lafforgue. This class includes all reductive groups over a p-adic field, which implies that all discrete series representations of such groups can be realized as the index of an equivariant Fredholm operator defined on the Bruhat-Tits building of G.

Building on the expertise of Van den Ban, Heckman, Landsman, Opdam, and Stokman, our plan is *to examine the relation of $K^0_{\rm top}(G)$ to the structure of the Langlands dual group for G reductive and p-adic*. A related problem is the study of so-called index functions, partly in connection with important open questions about the structure of the category of tempered representations. For example, is it true that discrete series representations of G are projective (and thus injective by duality) in the category of tempered representations? Transposing these matters to the representation theory of the affine Hecke algebra [10] leads to interesting formulas for index functions and to the following conjecture: the K-theory of the reduced C^*-algebra completion of an affine Hecke algebra H is independent of the deformable parameters defining H.

1.4.3 Moduli, mirrors, and topological strings

As mentioned in the general overview, algebraic geometry has greatly benefited from the input of physics, and our research themes reflect this. Our guiding idea is that the connections revealed by this input are merely the tip of an iceberg. Cluster members involved in what follows would be Cornelissen, Dijkgraaf, Van der Geer, Van der Kallen, Looijenga, Moonen, Steenbrink, and Stienstra, drawing on the expertise of other cluster members in relevant areas.

As a case in point, mirror symmetry will be an important theme in our cluster. Although this subject initially dealt with complex manifolds, a number of ingredients are well defined in a purely algebraic setting, like counting of curves and variation of filtrations on the Rham cohomology. *It is therefore tempting to ask to what extent the notion of mirror symmetry is meaningful in a purely algebraic setting.* A related question is, of course, what consequences mirror symmetry might have in positive characteristic. For example, is there such a notion for varieties defined over finite fields and if so, what does it imply? These questions are certainly difficult, but on the other hand, since explicit computations are possible here they leave ample room for experimentation. For example, moduli of Calabi-Yau varieties, both in characteristic zero and in positive characteristic, lend themselves for this purpose. In particular, we would like to explore newly observed phenomena in positive characteristic, like non-liftability. For elliptic curves and K3-surfaces there is a beautiful theory of moduli in positive characteristic due to Serre, Tate and Dwork, which can easily be extended to Calabi–Yau 3-folds in positive characteristic. It then shows remarkable analogies with what physicists have discovered about the space of complex moduli of Calabi-Yau 3-folds near the large complex structure limit, such as p-adic integrality properties of the mirror map. In any case, any insight into these matters might contribute also to a better understanding of mirror symmetry in characteristic zero, and might also have profound applications to arithmetic geometry, for example for questions on rational points on varieties defined over number fields.

As a second focus for study we propose the cohomology (and Chow rings) of moduli spaces of stable maps. The cohomology of moduli spaces of abelian varieties, and possibly also those of curves, can be described in terms of automorphic forms. For example, in recent work of Faber and Van der Geer moduli of curves over finite fields were used to obtain information on vector valued Siegel modular forms of genus 2. A geometric study of the moduli spaces both in characteristic zero as well as in positive characteristic could give concrete information on automorphic forms in higher genus. *An interesting question is whether the cohomology of M_g for $g \geq 4$ can be described in terms of Siegel modular forms, or whether other automorphic forms are needed.* Concretely, we propose to work on the tautological rings of moduli of stable maps; on stratifications on moduli spaces of stable maps, both in characteristic zero and positive characteristic and their implications for the cohomology (in positive characteristic these stratifications are connected with subtle phenomena in the de Rham cohomology, a largely unexplored territory). Furthermore, we want to study the cohomology of local systems on these moduli spaces and their relations with Siegel modular forms. For example, *an enticing question is what the zeta function of M_g over a finite field should be.*

Thirdly, a most interesting recent development has been the increased interest in non-archimedean aspects of algebraic/arithmetical geometry in connection with non-commutative geometry, involving Connes, Manin, Marcolli, and others. This includes, for example, a reinterpretation of the correspondence between Mumford curves and the graph of their uniformizing group in the Bruhat-Tits tree as a holography correspondence in the sense of 't Hooft and Süsskind, the association of spectral triples as defined in noncommutative geometry to such Mumford curves, and the treatment of (enlarged) boundaries of classical modular curves as non-commutative space in the sense of Connes. Our cluster members seem well prepared to enter this game, as all expertise is at hand. Concretely, we would like to *introduce and understand better orbifold versions of the holography correspondence for Mumford 'orbifold curves,' and explore their physical meaning*. We wish to generalize holography and spectral aspects of the theory to rigid analytic uniformization to higher dimensions, where the theory of buildings will start to play an increasingly important role, and to the case of positive characteristic. Building on work of Faber, Van der Geer, and Zagier, we are interested in *studying zeta functions of curves over finite fields using modular forms*.

Finally, led by Dijkgraaf, we will study topological strings, of which a comprehensive theory now seems close. For a large class of Calabi–Yau manifolds (basically including all toric cases), exact solutions of the B-model have been found in the form of matrix models. This gives a direct relation with integrable hierarchies such as the KP and Toda hierarchy. The corresponding A-models can be physically interpreted as quantum crystals, and mathematically there are promising relations with seven- and eight-dimensional manifolds with exceptional holonomy groups G_2 and Spin(7). Particularly interesting is the *study of D-branes in the A-model and B-model, leading to special Lagrangian and holomorphic calibrations respectively*. Moreover, Kontsevich' derived category interpretation of mirror symmetry yields a powerful reformulation of certain aspects of the geometric Langlands program in terms of quantum field theory, relating G-bundles and $\mathcal{D}$-modules for the Langlands dual moduli space. This link is, of course, an ideal cluster theme.

The study of the mathematics of topological strings also has important implications for physics. It has been shown that these models compute the vacuum structure of various four-dimensional supersymmetric gauge theories. This gives a promising framework to settle longstanding open problems in the dynamics of gauge theories, perhaps even quark confinement. Recently, topological strings have been used to calculate the entropy of black holes in supergravity and string theory. This has profound implications for our understanding of quantum gravity.

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2 Quality of the research team

2.1 General

The team consists of ten full professors and eleven other researchers based at one of the three cluster locations, plus two associated researchers from other institutions. They are supplemented by a number of PhD students and Postdocs. This group of researchers has been formed with utmost care: we wanted our senior participants not only to be prominent researchers and scientific leaders, but we also weighed their ability and inclination to interact and collaborate. All of the 23 researchers involved have collaborated with others in the group in the past, and their present research falls naturally within the scope of the cluster. They are all strongly committed to this enterprise, and keen to develop new lines of interaction.

As indications of excellence and viablity, we mention that of the ten professors, three are members of the Royal Academy (KNAW), one has recently been appointed the first Royal Academy Professor in mathematics, three have received prestigeous PIONIER grants from NWO, and one is a recipient of the Spinoza Prize (the highest scientific distinction in this country). The last four are all relatively young, in their fourties, so continuity is guaranteed. In addition, there is a wealth of talent among the younger (in their thirties) tenured Faculty. In particular, Cornelissen, Crainic, Moonen and Stokman already have an excellent international reputation, as confirmed by their KNAW-Fellowships and VIDI-grants.

The team will collaborate actively with a group of Fellows of the cluster, and will work in close contact with its Board of Advisors (see also Section 3 about the cluster structure).

2.2 Composition of the research team

Senior Researchers:

Utrecht:

Prof Dr J.J. Duistermaat (geometric analysis)

Prof Dr E.J.N. Looijenga (geometry)

Prof Dr I. Moerdijk (topology)

Prof Dr D. Siersma (singularity theory)

Amsterdam:

Prof Dr R.H. Dijkgraaf (mathematical physics)

Prof Dr G. van der Geer (algebraic geometry)

Prof Dr E.M. Opdam (representation theory)

Nijmegen:

Prof Dr G.J Heckman (Lie Theory)

Prof Dr N.P. Landsman (mathematical physics)

Prof Dr J.H.M. Steenbrink (algebraic geometry)

Other Tenured Faculty:

Utrecht:

Dr E. van den Ban (UHD, Lie groups)

Dr G. Cornelissen (UD, algebraic geometry)

Dr M. Crainic (UD, KNAW-Fellow, Poisson and noncommutative geometry)

Dr R. Cushman (UHD, symplectic geometry, integrable systems)

Dr J. van de Leur (UD, Lie groups)

Dr W. van der Kallen (UHD, algebraic groups)

Dr J. Stienstra (UD, algebraic geometry)

Amsterdam:

Dr B. Moonen (UD, algebraic geometry)

Dr J. Stokman (UD, KNAW-Fellow, quantum groups)

Nijmegen:

Dr F. Clauwens (UHD, algebraic topology)

Dr J. Maassen (UHD, mathematical physics)

Associated Researchers:

Dr G.F. Helminck (UD, Lie groups)

Dr H.T. Koelink (UD, Quantum Groups)

These are two additional members of the research team who hold tenured positions at the Technical Universities of Twente and Delft, respectively.

Nontenured members of the research team:

There are at present 5 postdocs and 10 PhD students at Amsterdam, 2 postdocs and 5 PhD students at Nijmegen, and 6 postdocs and 6 PhD students at Utrecht whose research falls within the themes of the cluster.

2.3 Advisors and Fellows

Board of Advisors:

Prof Dr G ’t Hooft (Theoretical physics, Utrecht)

Prof Dr V. Kac (MIT, USA)

Prof Dr M. Kontsevich (IHES, France)

Prof Dr A.N. Schellekens (Theoretical physics, Nijmegen, and NIKHEF)

Prof Dr E. Verlinde (Theoretical physics, Amsterdam)

Prof Dr A. Weinstein (UC Berkeley, USA)

Prof Dr E. Witten (Princeton, USA)

Fellows:

Prof Dr C.F. Faber (KTH Stockholm, Sweden, algebraic geometry)

Prof Dr A.J. de Jong (MIT, USA, algebraic geometry)

Prof Dr L.N.M. van Geemen (Milan, Italy, algebraic geometry)

Prof Dr R. Sjamaar (Cornell University, USA, symplectic geometry)

Prof Dr D. van Straten (University of Mainz, Germany, singularity theory)

2.4 Curricula Vitae of Senior Researchers

2.4.1 Robbert Dijkgraaf

Robbert Dijkgraaf (1960) holds the chair of Mathematical Physics at the University of Amsterdam since 1992 (and is since 1998 Faculty Professor in the Faculty of Science). He studied theoretical physics and mathematics in Utrecht, where he obtained his PhD cum laude under supervision of Gerard ’t Hooft in 1989. Subsequently he held a postdoctoral position at Princeton University and was a long-term member at the Institute for Advanced Study. He has been a visiting professor in Berkeley, MIT, IAS, among others. Dijkgraaf research group works in string theory, quantum gravity, and the interface of mathematics and particle physics. He manages the FOM programs “Mathematical Physics” and “String Theory and Quantum Gravity.”

Dijkgraaf gave an invited lecture at the ICM in Berlin (1998) and was a plenary lecturer at the International Congress of Mathematical Physics (London, 2000) and the European Congress of Mathematics (Barcelona, 2000).

Dijkgraaf is a member of the Royal Netherlands Academy of Arts and Sciences (KNAW) and the Koninklijke Hollandse Maatschappij van Wetenschappen. He was the recipient of the 2001 Physica Prize of the Dutch Physical Society. In 2003 he was awarded the Spinoza Prize, the highest scientific award in the Netherlands.

Dijkgraaf is editor of Nuclear Physics B, Journal of Differential Geometry, Journal of Geometry and Physics, Advances in Theoretical and Mathematical Physics, International Mathematical Research Notices, Journal of Mathematical Physics, Reviews of Mathematical Physics, Elsevier Mathematical Library, Academische Boekengids, and was an editor of Communications in Mathematical Physics from 1992 to 2002. Dijkgraaf was a director of the spring school at the ICTP Trieste (1992-1996) and has served on various international scientific committees among other for the Isaac Newton Institute for Mathematical Sciences in Cambridge, Max-Planck-Institut für Mathematik in Bonn, Erwin Schroedinger Institut für Mathematische Physik in Vienna, and the International Review of UK Mathematics.

2.4.2 Hans Duistermaat

J.J. (Hans) Duistermaat (1942) studied mathematics at Utrecht University from 1959-65 and obtained his PhD degree there in 1968. After a postdoctoral year 1969-70 in Lund (Sweden), where he learned Fourier integral operators from Hörmander, he went in 1971-74 to Nijmegen, where he became full professor in 1972. In 1974 he returned to Utrecht on the chair of professor Freudenthal, where he has stayed until now.

He became member of the KNAW (Royal Dutch Academy of Arts and Sciences) in 1982, and Academy Professor in 2004, which means that he is supposed to do research without being distracted by administrative duties until his retirement.

He has been 'promotor' of 17 PhD students, of which 10 as the main thesis advior. Several of these were NWO projects, and one was research paid by Shell.

Duistermaat's current interests include classical mechanics, symplectic differential geometry, high-frequency asymptotics of solutions of linear partial differential equations, the differential geometric theory of arbitrarily nonlinear partial differential equations, and stochastically perturbed dynamical systems. Apart from 43 articles in refereed international journals, he has written 7 books, of which probably the introduction to Fourier integral operators is the most well known. His best known research is probably his article with Guillemin on spectra of elliptic operators and periodic bicharacteristics, his article with Heckman on the Duistermaat-Heckman formula, and his article with Grünbaum on the bispectral problem.

At the moment his main editing task is being co-ordinating editor of Indagationes Mathematicae, the mathematics journal of the KNAW.

2.4.3 Gerard van der Geer

Gerard van der Geer (1950) studied mathematics at the University of Leiden. He received his PhD from that university in 1977. Subsequently he worked at the Sonderforschungsbereich at Bonn University and then got a position at the University of Amsterdam, where he has been full professor in Algebra since 1987. He spent long visits at research institutes like MSRI at Berkeley and the Max-Planck-Institut at Bonn, and foreign universities like Harvard, the University of Tokyo and Kyoto University.

Van der Geer has been managing editor of Compositio Mathematica for more than ten years and is editor of Geometriae Dedicata and of the EMS Monograph series. He is member of the scientific committees of the Max-Planck-Institut fuer Mathematik in Bonn and the Research Institute in Oberwolfach. He has successfully supervised seven PhD theses (including those of C. Faber and G. Farkas) and is currently supervising another three. He was one of the initiators of the big NWO projects "Moduli" and "Algebraic curves and Riemann surfaces". He started the well-known series of Texel conferences.

Van der Geer has worked on Hilbert modular surfaces, on which he wrote the well-known volume "Hilbert Modular Surfaces" in the Ergebnisse series of Springer, on the Schottky problem, where he contributed with van Geemen a conjectural solution, on moduli of curves and abelian varieties, and on curves over finite fields. His current research deals with cohomology of local systems on moduli spaces and with moduli of Calabi-Yau varieties. He has published over 50 research papers in refereed journals.

2.4.4 Gert Heckman

Gert Heckman (1953) studied mathematics at the University of Leiden, where he obtained his PhD in 1980. After a period of 2 years as postdoc at MIT, he returned to Leiden as assistant professor until 1988, with a half year interruption as visiting associate professor at Universite Paris 7. From 1989 until now he has been at the University of Nijmegen, from 1999 on as professor of pure mathematics. He has trained 3 PhD students.

Heckman' s research interests include symplectic geometry and geometric quantization, algebraic geometric analysis (hypergeometric functions, differential Galois theory), and representation theory of reductive groups. About his joint work with Eric Opdam he was invited to give lectures at Seminair Bourbaki (1997) in Paris, and Current Developments in Mathematics (1996) at Harvard.

2.4.5 Klaas Landsman

N.P. (Klaas) Landsman (1963) studied theoretical physics and mathematics at the University of Amsterdam, and got his PhD degree cum laude from the same institution in 1989. He worked at the University of Cambridge from 1989-1997, initially as a Research Assistant in theoretical physics and subsequently as a 5-year Advanced Research Fellow in mathematics. He interrupted his stay at Cambridge for a year in 1993-94 to work in Hamburg. He returned to Amsterdam in 1997 as a KNAW Fellow, and was appointed full professor of mathematical physics in 2002. From September 2004 he will be a professor of analysis at the University of Nijmegen.

His research Awards include an SERC Advanced Fellowship, an Alexander von Humboldt Fellowship a KNAW Fellowship, and an NWO Pioneer Grant of 1 ME. Over the last five years he held four additional project grants from NWO and/or FOM. He has been a Board Member of the Dutch Association for Mathematical Physics since 2000, and has been running a Master's Degree Program in Mathematical Physics at Amsterdam since 2001. He supervised four PhD students at Cambridge and Amsterdam, and is currently training three more.

Landsman's active research interests include noncommutative geometry, geometric and deformation quantization, index theory, Lie groupoids and algebroids, particularly in connection with each other. He is the author of the acclaimed monograph Mathematical Topics Between Classical and Quantum Mechanics (Springer, New York, 1998), and is the author of more than 50 refereed papers. He founded a series of conferences on the quantization of singular Poisson spaces at Oberwolfach and elsewhere. He is an editor of the International Journal of Geometric Methods in Physics, and an Honorary Member of the British Society for the Philosophy of Science.

2.4.6 Eduard Looijenga

Eduard Looijenga (1948) obtained his Masters's degree in mathematics at the University of Amsterdam in 1971. From 1971 till 1973 he stayed as a junior fellow at the Institut des Hautes Études Scientifiques and in 1974 he took his doctoral degree at the University of Amsterdam. After holding a postdoc position at the University of Liverpool (1974-75), he was appointed Professor at the University of Nijmegen (1975). From 1987 till 1990 he was at the University of Amsterdam and in 1991 he took his current position at the University of Utrecht. He held visiting positions at Yale (1980), U. of North Carolina at Chapel Hill (1985), Columbia U. (1987), U. of Michigan at Ann Arbor (1990), U. of Utah (1991).

His research started in singularity theory, but migrated via Torelli problems (often related to rational surfaces and K3 surfaces) to locally symmetric varieties, then to mapping class groups and moduli spaces of curves, while his recent work is concerned with automorphic forms with poles along Heegner divisors and (jointly with Heckman and Couwenberg) generalizations of Lauricella functions.

Looijenga was an invited speaker at the ICM in 1978 and at the ECM in 1992. He was on the selection panel for Algebraic Geometry of the ICM in 1994, the Prize Committee of the ECM in 2000 and the Scientific Committee of the ECM in 2004. Since 1995 he is an ordinary member of the Royal Netherlands Academy of Arts and Sciences (KNAW). He is currently editor of Comp. Math., Michigan Math. J. and the J. of the Eur. Math. Soc..

2.4.7 Ieke Moerdijk

Izak (Ieke) Moerdijk (1958) studied mathematics, philosophy and general linguistics at the University of Amsterdam. He received his PhD in Mathematics from the same institution in 1985, with the distinction Cum Laude. Subsequently he worked at the University of Chicago and at the University of Cambridge, before joining the Mathematics Department of the University of Utrecht in 1988, where he has been a Professor of Topology since 1996. Moerdijk was awarded a Huygens Fellowship from NWO in 1986 and a PIONIER grant, again from NWO, in 1995. Moerdijk held visiting positions in Cambridge (St John's College), Montreal (McGill University), Sydney (University) and Aarhus, among others. He was an invited speaker at the ECM 2000.

At Utrecht, Moerdijk has successfully supervised nine PhD theses, and is supervising another three at present.

Moerdijk's current research interests include algebraic and differential topology (operads, Lie groupoids, ...), and applications of topological structures in mathematical logic. He is the coauthor of several well-known books, including "Sheaves in Logic and Geometry" with S. Mac Lane (Springer-Verlag, 1992, 1994), and "Introduction to Foliations and Lie Groupoids" with J. Mrcun (Cambridge UP, 2003). He has published over 60 research papers in refereed journals. Together with C. Berger, he recently provided a solution to the problem of the existence of homotopy model structures for operads and their algebras.

Moerdijk is editor of The Annals of Pure and Applied Logic, of The Journal of Pure and Applied Algebra, and of Theory and Applications of Categories, and is a member of the Advisory Board of North-Holland Mathematical Library.

2.4.8 Eric Opdam

Eric M. Opdam (1960) studied mathematics at the University of Leiden. He received his PhD in Mathematics in 1988, also at the University of Leiden. He worked at the University of Utrecht and at the Massachusetts Institute of Technology before accepting a permanent position at the University of Leiden in 1989. He stayed in Leiden until 1999 when he was appointed as professor in Mathematics at the University of Amsterdam.

Opdam has held positions as a visiting professor at several occasions in Ann Arbor (MI, USA), Paris, Marseille and Kyoto. He was invited speaker at the European mathematical congress in 2000. In 2000 he was awarded a prestigious Pionier grant from NWO. He has successfully supervised 2 PhD students, and he is currently training three more. In 2001 he was honorary promotor when Ian Macdonald was granted an honorary doctorate degree at the University of Amsterdam.

Opdam's research interests include representation theory, Lie groups and algebraic groups, Hecke algebras, integrable systems, special functions, and operator algebras. In his work he has paid special attention to applications of techniques across traditional borders. This has led to active contacts with researchers in various disciplines, ranging from algebraic combinatorics to Langlands philosophy.

2.4.9 Dirk Siersma

Dirk Siersma (1943) studied mathematics and meteorology at the University of Amsterdam. After a teaching position at a secondary school he returned to this university , where he received a PhD in 1974. His supervisor was Nicolaas H. Kuiper. He became associate professor in Utrecht in 1976 and full professor in 1980.

Siersma's active research interest is singularity theory and applications. His principal work includes classification of singularities, geometry and topology of non-isolated singularities, behaviour of singularities at infinity and more recently the study of the conflict set of the distance function. He was one of the founding members of the Dutch Singularity School. He has approximately 30 refereed research papers and supervised 11 PhD students.

Siersma has many East-European contacts: he has been coordinator of three consecutive INTAS programs with the former Soviet union and two NWO-programs with Russia. Moreover he has been main organizor of the Singularity Semester at the Newton Institute in Cambridge (Fall 2000) and (co)organizor of many international scientific meetings in his field, e.g. in the framework of the European Singularity Network. Recently he was invited guest at IHES (2 months), Banach Center (1 month) and the University of Lille (1 month).

Siersma was the first scientific director of the Mathematical Research Institute (MRI) in The Netherlands and the initiator of its scheme of international Master Classes.

2.4.10 Joseph Steenbrink

Joseph Steenbrink (1947) studied mathematics at the University of Nijmegen, where he got his degree in 1969. He received his PhD at the University of Amsterdam in 1974, where Frans Oort was his supervisor. Subsequently he spent a year at the IHES at Bures sur Yvette, invited by Pierre Deligne. He was supported by an NWO stipend. He became assistant professor at the University of Amsterdam and full professor at Leiden University in 1978. Since 1988 he has the chair in geometry at the University of Nijmegen. He supervised nine PhD students, several of whom (Van Straten, Stevens, de Jong) now are full professor. His main research interest is algebraic geometry, where he has developed tools in mixed Hodge theory and applied these to singularity theory. He was one of the leaders of the successful NWO-projects in Singularity Theory and Arithmetic Algebraic Geometry. He was invited speaker at many international events, notably at the ICM 1990 in Kyoto. He has been Managing Editor of Compositio Mathematica from 1982 till 1993, and is a member of the Advisory Boards of North-Holland Mathematical Library and Epsilon Uitgaven. He was dean of the Faculty of Mathematics and Informatics during six years, and scientific director of the Mathematical Research Institute. His current research interests are: geometry of moduli spaces and of certain special threefolds. He published 50 research papers in refereed journals.

2.5 Expertise in project management

Our ability to manage a collaboration of the type foreseen here, and, indeed, to make it an overall success, may be illustrated by previous large projects the applicants have led in the research area in question. These include:

- *Singularities* (Looijenga, Siersma and Steenbrink; 1981–1985)
- *Riemann surfaces and algebraic curves* (Dijkgraaf, Faber, Van der Geer, Looijenga and Oort; 1993–1998)
- *Moduli* (Van der Geer, Oort, Peters; 1994–1999)
- *Lie theory and special functions* (Heckman, Helminck, Koornwinder, Opdam; 1994–1999)
- *The geometry of logic* (Moerdijk; 1995–2001)
- *Operads in geometry and physics* (Looijenga and Moerdijk; 1998–1999)
- *Mathematical physics* (Broer, Dijkgraaf, Landsman and Van Enter; 1999–2006)
- *Symmetry and symmetry breaking in mathematics and mathematical physics* (Opdam; 2000–2005)
- *Quantization, noncommutative geometry and symmetry* (Landsman; 2002–2007)
- *String theory and quantum gravity* (Dijkgraaf; 2002–2009)

as well as dozens of smaller ones. In addition, over the last decade one-year national seminars on topics such as automorphic forms, geometry and quantization, Hodge theory, Lie groupoids, moduli [15], modular curves, motivic integration, mathematical structures in field theory, noncommutative geometry, and tensor categories, as well as the yearly Lie group conference at Enschede (Helminck) have paved the way for the proposed cluster.

2.6 Key Publications

Amsterdam:

{1} R. Dijkgraaf and C. Vafa, Matrix Models, Topological Strings, and Supersymmetric Gauge Theories, Nucl. Phys. B644, 3-20 (2002).

{2} G. van der Geer and T. Katsura, On a stratification of the moduli of K3 surfaces, J. Eur. Math. Soc. 2, 259–290 (2000).

{3} N. Guay, V. Ginzburg, E.M. Opdam, R. Rouquier, On the category $\mathcal{O}$ for rational Cherednik algebras, Invent. Math. 154, 617–651 (2003).

{4} B. Moonen, Serre-Tate theory for moduli spaces of PEL type, Ann. Sci. Ec. Norm. Sup. 37, 223-269 (2004).

{5} J.V. Stokman, Difference Fourier transforms for nonreduced root systems, Sel. Math., New. ser. 9, 409-494 (2003).

Nijmegen:

{6} G. Heckman and E. Looijenga, The Moduli Space of Rational Elliptic Surfaces, Adv. Studies in Pure Math. 36, 185-248 (2002).

{7} G.J. Heckman and E.M. Opdam, Yang's system of particles and Hecke algebras, Ann. Math. 145, 139-173 (1997).

{8} N.P. Landsman, Deformation quantization and the Baum-Connes conjecture, Commun. Math. Phys. 237, 87-103 (2003).

{9} N.P. Landsman, Functorial quantization and the Guillemin–Sternberg conjecture, arXiv:math-ph/0307059, 15 p (2003).

{10} C.A.M. Peters and J.H.M. Steenbrink, Degeneration of the Leray spectral sequence for certain geometric quotients, Moscow Math. J. 3 (2003).

Utrecht:

{11} C. Berger and I. Moerdijk, Axiomatic homotopy theory of operads, Comm. Math. Helv. 78, 805-831 (2003).

{12} M. Crainic and R. Fernandez, Integrability of Lie brackets, Ann. of Math. (2) 157, 575–620 (2003).

{13} J. J. Duistermaat, The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator (Birkhäuser, Boston, 1996).

{14} E. Looijenga, Compactifications defined by arrangements I, II, Duke Math. J. 118, 151–187 (2003) and Duke Math. J. 119, 527–588 (2003).

{15} D. Siersma and M. Tibar, Deformations of polynomials, boundary singularities and monodromy, Moscow Math. J. 3, 661-679 (2003).

3 Cluster structure

3.1 Location

The cluster will have one main location (the 'hub') at the University of Utrecht, and two other nodes, at the Universities of Amsterdam and Nijmegen.

3.2 Management and organisation

3.2.1 Leadership

The cluster will be directed by an Executive Committee (EC) consisting of three members (one from each location), together with a Managing Director (MD). The MD is not formally a member of the EC. The Managing Director:

1. plans and calls the meetings of the EC and chairs these meetings. The EC takes decisions about issues brought up by one of its members or by the MD. The MD does not have voting right in this context.
2. maintains the contacts with NWO, and is the first representative of the cluster where it concerns external contacts. He may replace himself on any particular occasion by a member of the EC.
3. writes a yearly report (assisted by the EC), which includes a financial report as well as a budget plan for the coming year. The report will be made available to NWO and to the Board of Advisors.
4. controls the budget on a daily basis, and is responsible for its being spent according to the plan.
5. calls a yearly meeting of the EC with all 23 cluster participants, at which this report and plans for the coming year are presented. At this meeting, the MD asks for agreement with possible deviations from earlier plans proposed by the EC.

3.2.2 Office

The Cluster will have an administrative office, staffed by an administrative assistant at Utrecht.

3.2.3 Financial organisation

The funds will be spent along two distinct lines: the salaries for long term appointments (postdocs, PhD students, etc.) will be payed by NWO directly to one of the three cluster institutions. The budget for central activities will be managed at the admistrative office, under the responsibility of the MD. (For more details, see Section 4 below.)

3.3 Scientific Activities of the Cluster

3.3.1 Teaching

The cluster will organize a one-year Master Class on a yearly basis. Moreover, its members will participate in joint mathematics-physics programs, such as the Master programs in Mathematical Physics at Amsterdam and Nijmegen, as well as the Bachelor TWIN program at Utrecht. For more details, see Section 9 below.

3.3.2 Weekly Research Seminars

Within the cluster several weekly or biweekly seminars will be organized, where advanced literature on and recent developments in one of the subjects within the scope of the cluster will be presented. The themes of these Research Seminars will to a large extent agree with those of the Master Class (see Section 9) of the preceding year, so as to make these seminars accessible to young researchers. Although these seminars are expected to attract a significant number of participants to the cluster, they are not necessarily attended by all, and several subgroups may meet simultaneously in different seminars. Distinct weekly seminars may also be organized in alternating weeks.

3.3.3 Monthly Cluster Colloquia (MCC)

Once a month the cluster will organize a day on which several survey lectures of a more general kind than the research seminars will be presented. The aim of these lectures is to bring all cluster members together, and present some of the new developments in the areas in which the various subgroups are working. Many of the lectures by visitors from abroad will take place in the context of the MCC. The lectures at the MCC are more independent from each other than those at the research seminars. The cluster colloquium will contribute to the coherence of the cluster as a whole and to the interaction between the various subdisciplines. (Meetings discussing organisational aspects of the cluster will take place on the same day as the MCC.) The MCC is also a natural occasion to invite one of our Advisors to deliver a lecture. The MCC will be followed by a reception, as well as, in case of a guest speaker, a dinner.

3.3.4 International Workshops and Conferences

See Section 9.

With this set-up, smaller research groups within the cluster will meet on a weekly basis, while the cluster as a whole will meet at least once a month. In this way, the cluster members will not only become familiar with the latest results in their field of research, but the seminars will also facilitate and stimulate existing and new joint research between participants of the cluster. Many of the cluster members have proved in the past that this type of collaboration can be productive and successful; see 1.3 above.

4 Viability

At the “hub-location” Utrecht, 4 chairs and 7 other permanent members of the department are involved in the present cluster. It is a general long term policy of the department to maintain a group of researchers of roughly this size working in the various fields involved in the cluster. This policy is supported by earlier commitments already made, to NWO in relation to Moerdijk’s PIONIER grant, and to the KNAW in relation to the appointment of Duistermaat to a Royal Academy Chair. More specifically, in relation to the latter appointment the department has committed itself to the investment in younger personnel while Duistermaat is holding the chair, while succession of Duistermaat is also guaranteed. This succession will be within the research area of the cluster. At the more junior level, the department has made long term commitments with respect to the VIDI grant of Cornelissen and the KNAW Fellowship of Crainic.

The Faculty of Mathematics and Computer Science at Utrecht attaches great importance to research and teaching in the fields of the cluster. The TWIN program (dual maths-physics Bachelor’s degree) is an important part of the curriculum at Utrecht, and will be even more so in the new context of the “Federation of Science Faculties” which stimulates programs on the border lines of two or more disciplines. The Master’s Degree in Mathematics has recently been named one of Utrecht’s “Prestige Master” programs by the Board of the University, and as a consequence the Board has provided extra research resources for the department to complement the high level teaching.

If the cluster Geometry and Quantum Theory is granted, it is to be expected that the Faculty will make even more investments in the direction of the cluster topics; in fact, this would be a natural continuation of existing policies. For example, the Faculty intends to install one or two Personal Chairs (“bijzondere leerstoelen”) in fields falling within the cluster, while one or two new appointments to be made in the near future will also be allocated to the area of the cluster. In particular, the Faculty plans to create a tenure position for a young mathematician in an area related to mathematical physics.

At the University of Amsterdam, the relationship between mathematics and theoretical physics plays a dominant role in the Science Faculty. Dijkgraaf holds a Faculty Chair, and is the personification of the intimate relations between the departments of mathematics and of theoretical physics. Apart from the other cluster members Opdam and Van der Geer and cluster advisor E. Verlinde, professors of theoretical physics such as Bais and De Boer, and mathematics professor Koornwinder have always had a strong interest in the interaction between geometry and quantum theory. The University has started a Master’s program in Mathematical Physics in 2001, which is currently under redevelopment in order to secure an optimal connection to the cluster themes.

The Mathematics Department has made long-term commitments in the cluster area of research, related to the PIONIER grant of Opdam and the KNAW Fellowships of Moonen and Stokman. In relation to the cluster, the Faculty will also create a new Chair in Geometry and Quantum Theory, initially partly financed through the cluster. Both the research and the teaching activities of the appointee will entirely take place in the area of the cluster. Furthermore, an UHD due to retire in 2007 will already be succeeded in 2004 or 2005 by a mathematician or mathematical physicist working in the area of the cluster.

At Nijmegen, the vacant Chair in Analysis (previously held by A. van Rooij) will be occupied from 1 September 2004 by cluster member N.P. Landsman. In this context, the remaining three years of the PIONIER grant of Landsman will be transfered from the UvA to the KUN, including the substantial matching obligation to the host university. Landsman will retire in 25 years. Cluster members Heckman and Steenbrink will retire in 15 and 8 years, respectively. The chairs of Heckman and Landsman are structural. The succession of Steenbrink is not excluded, depending in part on the success of the present cluster and the influx of students.

The KUN is currently developing a Master Program in mathematical physics. All this guarantees both the commitment of the KUN to the research area of the cluster, as well as its continuity. Moreover, the vacant UD position in analysis is expected to be filled by a researcher in geometry and quantum theory as well, whereas the vacant UD position in geometry and algebra might be occupied likewise. Finally, the Dean of the Science Faculty has expressed his willingness to continue the 0.2 fte chair in Geometry and Quantum Theory mentioned above after the cluster has ended (subject to performance and availability of funds).

For formal statements on these plans, we refer to the letters of the Deans and/or Rectors of the three universities involved, to be sent under separate cover.

5 Added value for Dutch mathematics

The proposed cluster will incorporate direct investments in people (like PhD students, postdocs, and visitors), as well as effects on the long-term policies of the three Universities involved. Taken together, these will primarily help to

- Nurture a new generation of researchers who are fully equipped to participate in the spectacular developments in the interface of mathematics and physics described in Section 1.2 above;
- Maintain the high level of certain areas of mathematics that traditionally have been strong in The Netherlands, like algebraic geometry;
- Protect and expand areas that currently lack critical mass in The Netherlands, notably algebraic topology, differential geometry, and noncommutative geometry.

These goals are to some extent inseparable, but our main ambition is the first: to breed a generation of young researchers not hampered by the differences between the languages spoken by physicists and mathematicians. We wish to educate students who are truly 'bilingual' in geometry and quantum theory, and hence capable of crossing the bridges between these two disciplines with little effort. If successful, the long-term rewards for Dutch mathematics will be great.

On the research side, in large parts of mathematics history shows that it is precisely through exchanges of this type that significant progress is to be expected (cf. Section 2). Conversely, without the proposed investments the Dutch mathematical community would be in serious danger of being left out of some of the most beautiful and important developments currently taking place at the frontier of mathematical science.

From an educational perspective, the cluster presents exceptional opportunities, both for PhD students and postdocs, and for our current permanent staff. It will enable them to acquire or strengthen a broad and flexible view of mathematics, and to gain a deep understanding of the interrelations between the various subdisciplines involved in the cluster. In our experience, such a broad vision makes for the best lecturers and researchers in mathematics.

Provided our enterprise is amply supported, we are quite confident that we will accomplish this goal through the following long-term educational infrastructure:

1. Dual Bachelor degree programs in mathematics and physics (3 years);
2. Master programs in mathematical physics (2 years);
3. Specialized Master Classes on key cluster themes (1 year);
4. A PhD program in geometry and quantum theory (4 years).

Hence one of the principal benefits of our cluster to Dutch mathematics would be the firm establishment and maintainance of this system. We refer to Section 9 for further particulars.

Algebraic topology and differential geometry are two of the cornerstones of modern mathematics. Thus they are not only of central importance to our cluster themes, but also to many areas of mathematics and adjacent areas (like physics and theoretical computer science) practiced by scientists outside our cluster. Yet, at the moment differential geometry is not even represented by a chair in our country, whereas also algebraic topology is currently underrepresented in The Netherlands. Thus the support to these fields through the proposed cluster will also have important secondary benefits to Dutch mathematics as a whole.

Noncommutative geometry (also cf. Section 1.4.1), on the other hand, is a much younger field, originally starting as a small niche. However, its depth and relevance to practically all areas of mathematics (and beyond) is now beginning to be appreciated, especially by students: of all advanced mathematics courses offered in Holland, those in noncommutative geometry are among the best attended (typical classes sporting an audience of about thirty). Also researchwise, The Netherlands ought to step up its activities in this area.

Some other areas relevant to Dutch mathematics that would almost certainly benefit from a national stronghold in geometry and quantum theory include noncommutative (or 'quantum') probability theory and arithmetic algebraic geometry. The former is a new field of research, in which cluster member Maassen is a renowned expert. The strengthening of his links with geometric aspects of quantum theory will also help other Dutch mathematicians and mathematical physicists working in this new area. The latter is an area with a strong Dutch presence, which would be strengthened and enhanced by research in this cluster (cf. our research plans on zeta-functions, etc.).

6 Added value for other scientific disciplines

Our theme of Geometry and Quantum theory is obviously an interdisciplinary one, connecting mathematics and theoretical physics. Although our research proposal is largely focused on the benefits of this connection to mathematics, there is no question that physics (and hence young physicists in particular) will profit from the teaching and research activities of the cluster as well. Indeed, in the past a number of our specific cluster themes have already led to remarkable advances in the fundamental understanding of Nature.

For example, through its application to so-called anomalies (that is, the possibility that conservation laws in classical physics may no longer hold in quantum theory), the Atiyah–Singer index theorem has decisively clarified such phenomena as baryogenesis in the early Universe and the selection of viable string theories through anomaly cancellation. As another case in point, the application of noncommutative geometry to the Standard Model of elementary particle physics has brought a new perspective to the specific choice of the fundamental symmetry group of Nature. At the moment, in The Netherlands professional mathematical expertise in the pertinent areas seems strictly limited to cluster members. We intend to make this expertise available to as wide an audience as possible, both at a technical and at a popular level (also cf. Section 9 below).

Looking ahead, one may realistically expect the mathematical results produced by this cluster to apply to string theory and quantum cosmology. The former will presumably benefit from our progress in algebraic geometry (see Section 1.4.3), whereas the latter (an area in which despite its obvious importance a certain stagnation may currently be observed) will probably receive a boost from new techniques in the quantization of singular systems we intend to develop in the cluster (cf. Section 1.4.1). Thirdly, it goes without saying that the physics side of integrable systems, with their numerous applications from hydrodynamics to space travel, will benefit; see Section 1.4.2. In fact, some areas of physics appear to be literally *waiting* for input from mathematics. But on the whole, it should be clear that the finest future applications of geometry to physics will be completely unexpected.

On the technological (or R& D) side of physics, two emerging areas of considerable future importance to Society immediately come to mind as suitable research areas for students trained in the cluster. Although nanotechnology (one of NWO's current central themes) is at present largely an experimental science, its theoretical foundations are built on quantum theory and its interface with classical physics. For example, expertise in quantization theory as developed in our cluster is clearly relevant to transport phenomena at the nanoscale. But also, those educated in our cluster will be well prepared to do theoretical work in this area. The second is quantum computation and quantum information (where cluster member Maassen is an expert), which by definition is a merger of quantum theory and the science and technology of information. Compared to the first, this field has so far been of a much more theoretical nature, rendering the relevance of our cluster area self-evident.

While the potential spin-off of our cluster research to physics (a geometrical science ever since Descartes) is hardly surprising, we also expect other fields of science that use some kind of geometry to benefit, such as ('classical') computer science (especially geometric models for programming, and visualisation or imaging) and medicine. The point here is that research in this cluster encompasses many seemingly different aspects of geometry, one of its central aims being to transfer established knowledge from one kind of geometry to another, less well understood one. Thus any science that uses some kind of geometry might profit from a better understanding of geometry as achieved in this cluster.

Yet there are even more indirect and unexpected applications of geometry to science and technology. A remarkable example is coding theory, an area that at present thrives on results in abstract algebraic geometry of the kind studied in our cluster, and which therefore may be expected to benefit from it (e.g., through cluster member Van der Geer, a well-known expert and textbook author in this area). Perhaps an even more surprising example comes from the oil industry, where, in a collaboration with scientists from Shell, cluster member Duistermaat has recently developed new seismographic techniques on the basis of geometric insights, which turned out to be of immediate practical relevance.

Finally, even outside the context of academic or industrial research, those trained in the cluster will be broad-minded thinkers prepared to work in any area of Government, Consultancy, Finance, or Industry where flexible and cross-disciplinary thought is required. Thus we are confident that our graduates and PhD students will be sought after in those areas.

7 Knowledge transfer

7.1 Extra-academic

As far as the transfer of knowledge is concerned, the most general expected outcome of the project will be greater accessibility and popularity of geometry and quantum theory in The Netherlands. The achievement of this aim will be much facilitated by the fact that both subjects have immediate appeal and fascination to lay people and experts alike. Thus we feel a special responsibility towards the problem of increasing the dramatically declined number of university students in mathematics (and to a lesser extent also in physics) in The Netherlands. We intend to step up already existing activities by some cluster members contributing to this goal, such as masterclasses and other activities for teachers (including performances at the 'National Mathematics Days'), popular talks (even to children of primary school age, or at unexpected venues like museums), interviews, columns, book reviews and letters to the editor in the Press, a (forthcoming) popular science book on quantum theory, etc.

7.2 Academic

We have already mentioned our basic educational infrastructure in Section 7, on which we now expand. It goes without saying that most of these efforts hinge on the funding of the proposed cluster, for at present means do not nearly suffice to maintain schemes of this kind.

7.2.1 Dual Bachelor degree programs in mathematics and physics

At the moment, Utrecht offers the so-called TWIN program in this respect, which is quite popular among students, and has produced some of the best and most enthusiastic PhD students in The Netherlands. In a more informal manner, the University of Amsterdam features a similar program, which tends to attract the best students in their year (though fewer in number than at Utrecht). Recent appointments at Nijmegen make it realistic to start a similar scheme there as well. See also Section 6.

7.2.2 Master programs in mathematical physics

As already mentioned (cf. Section 6), the University of Amsterdam has started a Master program in Mathematical Physics in 2001. The University of Nijmegen will do so in the academic year 2004–5. (At the moment, Utrecht is considering its options in this respect.) Despite the inevitable element of competition for students between the three cluster locations, these Master programs are intended to complement each other. Since each of them is entirely controlled by members of the proposed cluster, we will be able to fine-tune the programs, guarantee credits for courses taken at other universities than the home one, and more generally stimulate exchanges so as to achieve the goals spelled out in Section 7. For example, a joint student seminar of pertinent master's students of all cluster locations would contribute towards this aim.

7.2.3 Specialized Master Classes on key cluster themes

The cluster plans to organize a yearly 'Master Class' (MC), with the same format as the Master Classes that have been organized for over a decade now by the Mathematical Research Institute MRI, often in collaboration with the Stieltjes Institute. (These are so-called Research Schools in Mathematics, through which the various Dutch universities collaborate, mainly in the education of PhD students.) In particular, all three cluster locations have extensive experience with this format.

An MC of the type in question is a one-year program for students who are at the end of their Master's Degree and have not yet started on a PhD project. The aim of the MC is to train a small group of students in a specific subdiscipline, and prepare them for PhD research in this area. Through the cluster funds we will make a number of student grants available, to attract the most excellent students from abroad. The MC lectures will also be open to students in mathematics and theoretical physics who are completing their Master's Degree at one of the cluster locations. The most talented students from the MC will be recruited as PhD students in the cluster. The theme of the MC will differ from year to year.

The following one-year programs for the MC have been planned (this preliminary list is subject to possible change):

MC1: Quantum Groups and Conformal Field Theory

MC2: Calabi–Yau Geometry and String Theory

MC3: Poisson geometry, Lie groupoids, and Quantization

MC4: To be determined

7.2.4 PhD program in geometry and quantum theory

We intend to create 6 PhD positions during the cluster period (cf. Section 4.2), to which others will probably be added from other sources. Although international recruitment will always be an option, our experience shows that an infrastructure of the kind described so far will suffice to attract excellent students to this PhD program. Our PhD students will benefit from the coherence of the cluster in general, as well as from all of the specific activities to be listed now.

7.2.5 Spring Schools

This year, 6 of the 23 cluster participants were involved in the MRI Spring School and Workshop on *Lie Groups in Analysis, Geometry and Physics*, which drew more than 50 applications from abroad (and was thereby sizeably overbooked). The cluster intends to participate with similar intensity in future MRI Spring Schools. Furthermore, next year Van der Geer and Moonen will organize a Spring School on Abelian Varieties.

7.2.6 Research Seminars

The Master Class will be followed by a research seminar around the same theme in the following year, see 3.3.2.

7.2.7 Workshops

The cluster plans to organize a yearly International Workshop (of 3–5 days), preferably in one of the standard conference centers in The Netherlands. At such a workshop, international experts will be invited to deliver lectures, and it will be a natural occasion to meet the Advisors and Fellows of the cluster. The (main) theme of such a workshop will be the subject of the MC of the preceding year. In this way, there will be a two-year track that will bring young researchers to the front of research in a particular domain: a Master Class in one year, followed by Research Seminars in the next, culminating in an International Workshop.

The long-standing yearly Lie group conference at Enschede will be increasingly devoted to cluster themes; already this year, integrable hierarchies and the geometric Langlands program will play a prominent role.

7.2.8 International Conference

Towards the end of the four year cluster period, the cluster plans to organize a major international conference, having more participants and a wider scientific scope than the yearly workshops. At this conference, we expect to show that through the cluster activities, The Netherlands has reached the forefront of international research in geometry and quantum theory.

7.2.9 Student Prizes

The cluster will install a prize of 1000 euro for the best Master's thesis in a field related to Geometry and Quantum Theory. We aim at the master's level and not at the PhD level here, because this will attract young students to the field, stimulating them to do high-level work and preparing them for PhD work in geometry and quantum theory (within the cluster or elsewhere).

7.3 Website

The cluster will maintain a professional and attractive website at which all conceivable information on the cluster and its activities will be posted. In addition, the website will contain further information on and links to various individual cluster themes.