Platonism or Psychologism in Mathematics: A False Dilemma?

(Frege, Husserl, Russell, and Wittgenstein on the Philosophy of Mathematics)

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1. Introduction

a. Technical and Philosophical Philosophy of Mathematics (TPM and PPM). Core questions of TPM: e.g. completeness, intuitionistic logic, &c. Core questions of PPM: what is mathematics about? What is the source of its necessary truths? How do we get acquainted with its objects? The Psychologism/Platonism Dilemma (PPD) in PPM. But: why should we stroke Plato's beard again? Can't we eliminate PPM by doing TPM?
b. Intertwinings of TPM and PPM in Frege: the Kantian background of Frege's logicism.
c. Did TPM eclipse PPM in Frege (as the anti-philosophical but philosophically minded mathematician might hope)? Or in Russell? It did not, for acc. to Frege, even logic is <u>about</u> "*ein Gebiet des Objectiven, Nichtwirklichen*" (*Vorwort* GGA: xviii). Nelson Goodman or William V.O. Quine on nominalism vs. Platonism. Quine's criterion for 'aboutness' or ontological commitment: "To be is to be the value of a bound variable". The Platonic beard of PPM cannot be shaved off easily by TPM. Cf. Quine, "Logic and the Reification of Universals" in *From a Logical Point of View* (1953).
d. An illustration of **(c)** by Frege's *Vorwort* to *Grundgesetze der Arithmetik* (1893).

2. From Psychologism to Platonism: Frege and Husserl

a. What is wrong with the psychologism of John Stuart Mill et al.? Frege (*Vorwort*) and Husserl, *Prolegomena zur reinen Logik* (*Logische Untersuchungen* I, 1900).
* Mathematical propositions are necessarily true, not contingently.
* Mathematics is *a priori*, not empirical. Mathematicians don't do psych. research.
* Props. of math and logic not vague (like psych. laws) but exact.
* Psychologism leads to radical skepticism (Husserl) or idealism (Frege).
* Laws of logic are *Denkgesetze* only in the normative sense, not in the descriptive sense.
* Ambiguity of *Vorstellung*. *Es ist unmöglich, jedem Menschen seine eigne Eins zuzuweisen* (*Vorwort*: xviii).

b. Should we then be Platonists? Frege, Husserl, Russell, Quine, Gödel etc. all say: YES! But, if so, what is the ontological status of Platonic objects? How can we know them? These are the eternal & unanswerable questions of PPM! So: TPM did not succeed in eliminating PPM/Plato's beard, whereas the philosopher cannot solve PPM's typical problems either! So: it was time for a radically new start, which has been pioneered by Wittgenstein.

3. Wittgenstein's elimination of the PPD

a. *Tractatus Logico-Philosophicus* and later works (*Philosophische Untersuchungen* (PU) and *Bemerkungen über die Grundlagen der Mathematik* (BGM). I focus on the later writings.

b. The typical problems of PPM arise because we misunderstand the forms/uses of our language. Cf. PU I §111: "Die Probleme, die durch ein Mißdeuten unserer Sprachformen entstehen, haben den Charakter der *Tiefe*". Empirical propositions describe (possible) states of affairs. We think that, analogously, propositions of logic and math. must *describe* something. But what? How can descriptions be necessarily true? Are the objects of math. and logic perhaps eternal, "Grenzsteine in einem ewigen Grunde befestigt" (Frege)? Now we get entangled in the irresolvable difficulties of PPM. So: the very questions PPM asked are mistaken. The main elements of W's view are the following:

(1) There are <u>many different types</u> of necessary propositions, e.g. logical, arithmetical, metaphysical, explications of meaning. We should study their *use* in order to understand their "necessity", instead of asking what they are *about*. Their use is *not* to describe something (in that case, their negation should also describe something!), but to give rules of substitution, addition, construing valid arguments, etc. In one sense, 2+2=4 is "about" numbers, but in another sense, saying this is deeply misleading. And it is equally misleading to say that necessary propositions are propositions "true in all possible worlds" (as if they describe necessary features of each possible world).

(2) Use is often deceptively masked by grammatical form. Tokens of the same sentence type (e.g. 'this is one meter long', 'this is black') may be used as descriptions but *also* as stipulative definitions or explanations of meaning. So we should not be deceived by sentential form (like Quine and many other physicalistic students of logic) whenever we want to study the roles or uses of sentences. Technique: to get an overview! In other words: we should not analyse the difference between necessary and contingent propositions by attempting to make distinctions between sentence-types, such as the distinction between analytic and synthetic sentences (cf. Logical positivists, Quine).

(3) Necessary truths typically are expressions of rules or conceptual connections. Examples: 'water is H_2O' (an empirical discovery may become hardened into a rule, but Kripke's idea of *de re* necessity is confused); 'if p and if p implies q, then q'; '2+3=5'; 'red is darker than orange'.

(4) We call necessary propositions 'true' and, surely, the predicate 'true' is not ambiguous (to say that *p* is true is to assert that *p*). But what is it for a necessary proposition to be true? Surely not: correspondence to the facts! Sometimes, the denial of a necessary proposition is just nonsense ('red is not a colour'). Sometimes, it has a specific use, such as in a *reductio ad absurdum* proof ('there is a greatest prime number').

(5) Learning necessary truths is learning a *skill* (to use a specific conceptual apparatus). The epistemological question ('how do we know that *p* is necessarily true?') is misconceived. The ontological question ('what kind of objects is mathematics about?') is equally misconceived.

4. Conclusion. The 'eternal problems' of PPM should not be eliminated by doing TPM but by doing philosophy better.