Let $k[X]$ be the polynomial ring in n variables over a field k for some $n \in \mathbf{N}$, and $k(X)$ the field of fractions of $k[X]$. Then, *Hilbert's Fourteenth Problem* asks whether the k-algebra $L \cap k[X]$ is finitely generated whenever L is a subfield of $k(X)$ containing k. In 1958, Nagata solved this problem by giving a counterexample, where $k(X)$ is transcendental over L. In the case where $k(X)$ is algebraic over L, we gave a counterexample of extension degree $[k(X) : L] = d$ for each $d \geq 3$ when $n \geq 3$, and for each $d \geq 2$ when $n \geq 4$. On the other hand, $L \cap k[X]$ is always finitely generated if $n \leq 2$ due to Zariski. Clearly, $L \cap k[X] = k[X]$ if $[k(X) : L] = 1$, i.e., $L = k(X)$. However, the following problem remains open.

Problem Assume that $n = 3$, and L is a subfield of $k(X)$ containing k. Is the k-algebra $L \cap k[X]$ finitely generated if $[k(X) : L] = 2$?