# ARE LOCALLY FINITE POLYNOMIAL AUTOMORPHISMS LINKED TO LOCALLY FINITE DERIVATIONS?

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**Definition:** We define a polynomial map $F : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ to be locally finite if $deg(F^n)$ is bounded, i.e. $max(deg(F^n))$ is finite.

This definition is equivalent to "for each $g \in \mathbb{C}^{[n]}$ the vector space generated by $g, F(g), F^2(g), \ldots$ is finite dimensional", and also to "there exist $n \in \mathbb{N}, a_i \in \mathbb{C}$ such that $\sum_{i=0}^{n} a_i F^i = 0$".

Now, as we know, if $D$ is a locally finite derivation, then $F_D := exp(D)$ exists and is an automorphism of $\mathbb{C}^{[n]}$. It is also a locally finite polynomial automorphism: given $g \in \mathbb{C}^{[n]}$, we know that $g, D(g), D^2(g), \ldots$ is finite dimensional, which implies that $g, F_D(g), F_D^2(g), \ldots$ is finite dimensional. Now the obvious conjecture is: does the converse hold?

**Conjecture:** Is a locally finite polynomial automorphism an exponent of a locally finite derivation?

Note that it is conjectured that the exponents of locally finite derivations generate the automorphism group (this is equivalent to stating that the exponents of locally nilpotent derivations, plus the affine maps, generate the automorphism group).

## References

[FM] J-Ph. Furter, S. Maubach, *Locally finite polynomial endomorphisms* , to appear in J. of Pure and Appl. Algebra.