IS $\mathrm{SAut}\,R[t][x_1,\cdots,x_n] \to \mathrm{SAut}\,R[t]/(t^m)[x_1,\cdots,x_n]$ SURJECTIVE?

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The ring R is commutative with unity and t is an indeterminate. The notation SAut stands for the Special Automorphism Group, i.e. the group of automorphisms with Jacobian determinant equal to one. The application in question in the title is the morphism of groups induced by the canonical epimorphism $R[t] \to R[t]/(t^m)$. Note that for $m = 1$ the answer is trivially yes.

This question has positive answer for any ring containing $\mathbb{Q}$ and any $m, n \geq 1$ (see [vdEMV]). When R has positive characteristic p, or more generally when $pr = 0$ for some $r \in R \setminus \sqrt{(0)}$ and for $n = 1$ then the automorphism $\alpha : x_1 \mapsto x_1 + rtx_1^p \in \mathrm{SAut}\,R[t]/(t^m)[x_1]$ furnishes a negative answer $\forall m \geq 2$.

So the remaining cases are: $n \geq 2$ and $\mathbb{Q} \nsubseteq R$.

The motivation for the question comes from [Ve04] where the non-surjectivity case given above is at the origin of the construction of some "bad" objects (see the ref.).

References

[vdEMV] A. van den Essen, S. Maubach, S. Vénéreau, *The special automorphism group of $R[t]/(t^m)[x_1,\cdots,x_n]$ and coordinates of a subring of $R[t][x_1,\cdots,x_n]$*, Journal of Pure and Applied Algebra (2006), doi:10.1016/j.jpaa.2006.09.013

[Ve04] S. Vénéreau, *New bad lines in $R[x,y]$ and optimization of the Epimorphism Theorem*, Journal of Algebra 302 (2006) 729-749.