GALOIS THEORY 2015/2016 EXERCISE SHEET 8

(1) Give an example of a field extension $L|\mathbb{Q}$, and subextension F, such that $F|\mathbb{Q}$ and $L|F$ are both Galois, but $L|\mathbb{Q}$ is not Galois (hint: you can take $[F:\mathbb{Q}] = [L:F] = 2$.

(2) Let $L|K$ be a Galois extension with group G, and let $h(X)$ be a separable polynomial in $L[X]$. Let F be the splitting field of h. Show that $F|K$ is Galois if and only if for all σ in G, F contains the roots of $h^\sigma(X)$.

(3) Let L be the field $\mathbb{C}(T_1, T_2)$. Define automorphisms σ and τ in $\mathrm{Aut}_{\mathbb{C}}(L)$ by

$$\sigma(T_1) = T_2$$
$$\sigma(T_2) = T_1$$
$$\tau(T_1) = -T_1$$
$$\tau(T_2) = T_2.$$

Let G be the group of automorphisms of L generated by σ and τ. Show that $L^G = \mathbb{C}(T_1^2 + T_2^2, T_1^2 T_2^2)$.

Comments, corrections, questions etc to netandogra@gmail.com.