

Into to TQFT, Exercise sheet No 1

EXERCISE 1 *Let (V, a, μ) be a good triple and $Z(M)$ the (would-be) invariant of 2-manifolds constructed using it. Let $\gamma : V \rightarrow V^*$ be the isomorphism defined by $\gamma(x) = \mu(x, \cdot)$ and define $m = (\gamma \otimes \gamma \otimes \text{id})(a) \in V^* \otimes V^* \otimes V$ (i.e., m is a bilinear map $V \otimes V \rightarrow V$). Show that the number $Z(M)$ associated to a triangulated 2-manifold M is invariant under the 2-2 Pachner move iff m is associative (in which case (V, m) is an associative, not necessarily unital, algebra).*

EXERCISE 2 (a) *Let (V, a, μ) be a good triple and $Z(M)$ the (would-be) invariant of 2-manifolds constructed using it. Give a necessary and sufficient condition on (V, a, μ) for invariance of $Z(M)$ under the 1-3 Pachner move.*

(b) *(Bonus) Try to find an algebraic interpretation for the condition you have found (analogously to the preceding exercise).*

EXERCISE 3 *Give a triangulation of S^2 with minimal number of triangles (2-simplices), both in terms of a Δ -complex (geometric realization of a s.s.s.) and in terms of a geometric simplicial complex (i.e. a Δ -complex where no two triangles have the same set of corners).*