

# Utrecht Graphs Workshop 2013

De Uithof, Utrecht University

31 October & 1 November

<http://www.staff.science.uu.nl/~Kang0002/UGW2013.html>

(Last updated 30 October 2013.)



We are pleased to welcome you to Utrecht for a workshop in graph theory.

This contains important practical and scientific information about the meeting. To conserve paper, we provide at the meeting printed copies of only the programme.

Note this document has links, via hyperref, to other parts of the document and to the web.

§ Practical matters § Programme § Invited lectures § Contributed talks § Participants §

## Practical matters

There are three locations for the talks. On Thursday, it is at Androclus Building (close to Heidelberglaan stop), room 101, and Buys Ballot Building (close to Botanische Tuinen and Padualaan stops), room 001. We have scheduled extra time during the lunch break to walk between the buildings. On Friday, we are only at Minnaert Building (close to Botanische Tuinen and Padualaan stops), rooms 211 and 208.

The campus has eduroam internet access. If you forget to configure it beforehand, there is also open access to the network UU-visitor, provided you accept the terms and conditions when you open your browser.

During the meeting, we provide caffeine, lunch and other refreshments at the right moments. There is a reception including food on Thursday, and drinks on Friday.

For non-local participants, Utrecht is easy to reach by train from anywhere in the Netherlands and is only 30 minutes from Schiphol airport. From the centre of Utrecht, the campus is about a 25-minute bus ride. (Take bus 11 or 12 from Utrecht Centraal train station to Botanische Tuinen, Padualaan or Heidelberglaan stops.) As this is in the middle of the academic term, be mindful of the student rush on buses arriving close to 09:00. We recommend <http://www.9292.nl/en> to check the overall route, by entering “De Uithof, Utrecht” as the destination. Note that limited free internet access is usually provided at Schiphol airport, major train stations, and on certain trains. To plan meals for Wednesday or Friday evening (if needed), participants are recommended to search on the website <http://www.iens.nl>.

## Programme

This is subject to change. Embedded into the schedule below are links to Google maps, as well as to abstracts. There is also the official campus map.

Thursday, 31 October

Morning session (Androclus Building, room 101)

08:30 – 09:00	Coffee
09:00 – 09:10	Opening remarks by Roberto Fernández
09:10 – 10:00	<b>Alex Scott</b> — Hypergraphs of bounded disjointness
10:00 – 10:30	Coffee
10:30 – 11:45	Short talks (Huynh, Harutyunyan, and Dutta)
11:50 – 12:40	<b>Louis Esperet</b> — Covering cubic bridgeless graphs ...

Lunch (Minnaert Building, cafeteria)

Afternoon session (Buys Ballot Building, room 001)

14:30 – 15:20	<b>Aart Blokhuis</b> — On the $q$ -analogs of the Kneser graph ...
15:20 – 15:50	Coffee
15:50 – 16:40	Short talks (Labbate and Nederlof)
16:40 – 17:30	<b>Maria Axenovich</b> — Twins in graphs and sequences

Reception (Hans Freudenthal Building, mathematics library, 7th floor)

Friday, 1 November

Morning session (Minnaert Building, room 211, with mid-session change)

08:40 – 09:10	Coffee
09:10 – 10:00	<b>Carsten Thomassen</b> — The weak 3-flow conjecture ...
10:00 – 10:25	Short talk (Patel)
10:25 – 10:50	Workshop photo and then coffee

Room change!  $\implies$  room 208

11:00 – 11:50	<b>Tom Kaiser</b> — Colouring quadrangulations of projective spaces
11:50 – 12:40	<b>Dan Král'</b> — FO limits of trees

Lunch (Minnaert Building, cafeteria)

Afternoon session (Minnaert Building, room 208)

14:00 – 14:50	<b>Hajo Broersma</b> — On the toughness of graphs
14:50 – 15:20	Coffee
15:20 – 16:10	Short talks (Regts and Aerts)
16:20 – 17:10	<b>Benny Sudakov</b> — Counting and packing Hamilton cycles

Closing drinks (The Basket)

## Invited lectures

Maria Axenovich (Karlsruhe Institute of Technology)

“Twins in graphs and sequences”.

Two identical disjoint subsequences of a given sequence  $S$  are called twins of  $S$ . Let  $t(n)$  be the largest integer  $k$  such that each binary sequence of length  $n$  contains twins of length  $k$  each. In a work with Y. Person and S. Puzynina we show that  $t(n) = \frac{n}{2}(1 - o(1))$ . I will discuss this result, its generalizations for sequences, and twin problem in graphs.

Aart Blokhuis (TU Eindhoven)

“On the  $q$ -analogs of the Kneser graph, their large cocliques and their chromatic number”.

One of the famous results in graph coloring is Lovász’s proof of the Kneser conjecture concerning the chromatic number of the Kneser graph  $K(n, k)$  whose vertices are the  $k$ -subsets of an  $n$ -set being adjacent if they are disjoint, in other words ‘far away’ in the Johnson Scheme. Other famous results are the Erdős–Ko–Rado Theorem, and the Hilton–Milner Theorem that characterize the largest and second largest (maximal) cocliques in this graph.

In this talk I will give a number of recent extensions of these results to  $q$ -analogs of the Kneser graph — the most studied of these is the graph on the  $k$ -subspaces of an  $n$ -dimensional vector space (over a finite field), being adjacent if their intersection is  $\{0\}$ .

Hajo Broersma (Universeit Twente)

“On the toughness of graphs”.

The toughness of a graph measures the way a graph can fall apart into a number of components by deleting a set of vertices together with the incident edges. We will give the formal definition in the talk. Since its introduction in a paper due to Chvátal in 1973 many results have been obtained that relate the toughness of a graph to its cycle structure, but many open problems remain. In the talk we will discuss old and new results involving the toughness of graphs and we will discuss the progress on the main open conjectures.

Louis Esperet (CNRS, G-SCOP)

“Covering cubic bridgeless graphs with perfect matchings”.

A conjecture of Berge and Fulkerson (1971) states that every cubic bridgeless graph contains 6 perfect matchings covering each edge precisely twice, which implies that every cubic bridgeless graph has 3 perfect matchings with empty intersection (this weaker statement was conjectured by Fan and Raspaud in 1994). I will show that if  $4/5$  of the edges of every cubic bridgeless graph can be covered by 3 perfect matchings, then Fan-Raspaud conjecture holds, confirming a recent conjecture of W. Tang, C. Q. Zhang, and Q. Zhu. I will also prove that for any  $2 \leq t \leq 4$  and for any real  $\tau$  lying in some appropriate interval, deciding whether a fraction of more than (resp. at least)  $\tau$  of the edges of a given cubic bridgeless graph can be covered by  $t$  perfect matching is an NP-complete problem. This proves another conjecture of W. Tang, C. Q. Zhang, and Q. Zhu.

I will also mention a construction of an infinite family of snarks whose edge-set cannot be covered by 4 perfect matchings (only two such graphs were known beside the Petersen graph). This construction is also interesting in the study of the shortest cycle cover of snarks.

Joint work with G. Mazzuocolo.

Tomáš Kaiser (University of West Bohemia)

“Colouring quadrangulations of projective spaces”.

Youngs proved in 1996 that any quadrangulation of the projective plane is either bipartite, or 4-chromatic. We extend the definition of a quadrangulation to higher dimensions, and prove that any graph  $G$  which embeds as a quadrangulation in the real projective space  $P^n$  has chromatic number  $n + 2$  or higher, unless  $G$  is bipartite. The family of quadrangulations of projective spaces includes all complete graphs and all Mycielski graphs. We obtain a new proof of the Lovász-Kneser theorem by showing that certain projective quadrangulations are homomorphic to the Schrijver graphs. Finally, we show that (in contrast to the two-dimensional case) the chromatic number of quadrangulations of  $P^n$  is not bounded by any function of  $n$ . The talk is based on joint work with Matěj Stehlík.

Daniel Král' (University of Warwick)

“FO limits of trees”.

Nešetřil and Ossona de Mendez introduced a new notion of convergence of graphs called FO convergence. This notion can be viewed as a unification of notions of convergence for dense and sparse graphs. In particular, every FO convergent sequence of graphs is convergent in the sense of left convergence for dense graphs, and every FO convergent sequence of graphs with bounded maximum degree is convergent in the Benjamini-Schramm sense. FO convergent sequences of graphs can be associated with a limit object called modeling. Nešetřil and Ossona de Mendez showed that every FO convergent sequence of trees with bounded depth has a modeling. We extend this result to all FO convergent sequences of trees and discuss possibilities for further extensions.

The talk is based on a joint work with Martin Kupec and Vojtech Tuma.

Alexander Scott (Oxford University)

“Hypergraphs of bounded disjointness”.

A  $k$ -uniform hypergraph is said to be intersecting if no pair of edges is disjoint. The maximal size of an intersecting  $k$ -uniform hypergraph with a given groundset is given by the beautiful and well-known theorem of Erdős, Ko and Rado.

A  $k$ -uniform hypergraph is  $s$ -almost intersecting if every edge is disjoint from exactly  $s$  other edges. Gerbner, Lemons, Palmer, Patkós and Szécsi made a conjecture on the maximal number of edges in such a hypergraph. We prove a strengthened version of this conjecture and determine the extremal graphs. We also give some related results and conjectures.

Joint work with Elizabeth Wilmer.

Benny Sudakov (UCLA and ETH Zürich)

“Counting and packing Hamilton cycles”.

We present a general method, based on estimates for permanents of matrices, for counting and packing Hamilton cycles in dense graphs and oriented graphs. We utilize this approach to prove several new extremal results, and also to derive new and conceptually simple(r) proofs of some known results in this area.

Joint with A. Ferber and M. Krivelevich

Carsten Thomassen (Technical University of Denmark)

“The weak 3-flow conjecture and some applications”.

Tutte’s 3-flow conjecture says that every 4-edge-connected graph has an orientation such that, for each vertex  $x$ , the indegree of  $x$  equals the outdegree of  $x$  modulo 3. In 1988 Jaeger suggested a weaker conjecture obtained by replacing 4 by a larger (universal) number and called that the weak 3-flow conjecture. He also suggested a stronger conjecture, called the circular flow conjecture.

In this talk we indicate a proof of the weak circular flow conjecture (and hence also the weak 3-flow conjecture) and discuss its applications to graph decomposition, group flow, and factors modulo  $k$ .

## Contributed talks

Nieke Aerts (Technische Universität Berlin),  
“Straight line triangle representations”.

A *straight line triangle representation* is a plane drawing of a graph such that all edges are represented by straight line segments and all faces are triangles. We want to characterize graphs that admit such a representation. I will present necessary and sufficient conditions based on flat angle assignments, i.e. assignments of angles that will have size  $\pi$  in the representation. The drawback of this characterization is that we are not aware of an efficient way to check whether a graph has a flat angle assignment that satisfies the conditions.

Kunal Dutta (Max–Planck–Institut für Informatik, Saarbrücken),  
“On lower bounds on independence numbers of hypergraphs”.

Yair Caro and Zsolt Tuza (1991) gave lower bounds on the independence numbers of  $k$ -uniform hypergraphs in terms of their degree sequence, using extremal techniques. D., Mubayi, and Subramanian (2012) answered some their open questions — obtaining degree sequence based lower bounds on the independence numbers of  $K_r$ -free graphs and linear hypergraphs, using probabilistic ideas. A curious coincidence seems to be the fact that a step in the D.–Mubayi–Subramanian lower bound for *linear* hypergraphs gives exactly the same expression as the Caro–Tuza lower bound for *general* hypergraphs, though they come from completely different approaches. In this talk, we shall briefly mention some of these results and related questions, and explain how to extend the D.–Mubayi–Subramanian partial lower bound from the linear to the general case, thus providing a ‘combinatorial explanation’ for the seemingly magical lower bound of Caro and Tuza.

Ararat Harutyunyan (Oxford University),  
“Some problems on the dichromatic number of digraphs”.

Given a digraph  $D$ , the *dichromatic number* of  $D$  is the smallest number of colors needed to color the vertices of  $D$  such that each color class induces an acyclic subdigraph. One long-standing open conjecture in the area is whether every oriented planar digraph  $D$  has dichromatic number at most 2. I will discuss some other open problems and present some results that show the relationship this invariant has with the graph chromatic number.

Tony Huynh (Sapienza – Università di Roma),  
“A directed version of a conjecture of Gyárfás”.

Gyárfás conjectured that if the edges of  $K_n$  are coloured with  $t$  colours, then there exists a partition of  $V(K_n)$  into  $t$  monochromatic paths. This conjecture has received quite a bit of attention, which we will review. We will then consider a directed analogue of Gyárfás’s Conjecture, and prove a weakened form of our conjecture.

This is joint unfinished work with Matt Devos, Frédéric Havet, Jessica McDonald, Bojan Mohar, and Robert Šámal.

Domenico Labbate (Università degli Studi della Basilicata – Potenza),  
“Odd 2-factored snarks”.

A *snark* is a cubic cyclically 4-edge connected graph with edge chromatic number four and girth at least five. We say that a graph  $G$  is *odd 2-factored* if for each 2-factor  $F$  of  $G$  each cycle of  $F$  is odd. Some of the authors conjectured that *a snark  $G$  is odd 2-factored if and only if  $G$  is the Petersen graph, Blanuša 2, or a Flower snark  $J(t)$ , with  $t \geq 5$  and odd.* Brinkmann et al. have obtained two counterexamples that disprove this conjecture by performing an exhaustive computer search of all snarks of order  $n \leq 36$ .

In this talk, we present a method for constructing odd 2-factored snarks. In particular, we independently construct the two odd 2-factored snarks that yield counterexamples to the above conjecture. Moreover, we approach the problem of characterizing odd 2-factored snarks furnishing a partial characterization of cyclically 4-edge connected odd 2-factored snarks. Finally, we pose a new conjecture regarding odd 2-factored snarks.

Joint work with M. Abreu, R. Rizzi, J. Sheehan.

Jesper Nederlof (Utrecht University),  
“Fast Hamiltonicity checking via bases of perfect matchings”.

We study the rank of the Matchings Connectivity matrix  $H_t$  defined as follows. For an even integer  $t$  at least 2,  $H_t$  is a matrix that has rows and columns both labeled by all perfect matchings of the complete graph  $K_t$  on  $t$  vertices; an entry  $H_t[M1, M2]$  is 1 if  $M1 \cup M2$  is a Hamiltonian cycle and 0 otherwise. We will show that  $H_t$  has rank  $2^{t/2-1}$  over  $GF(2)$ . We will briefly discuss algorithmic consequences of this result for the Hamiltonicity and Traveling Salesman problems.

The talk is based on joint work with Marek Cygan and Stefan Kratsch.



Viresh Patel (University of Birmingham),

“A conjecture of Thomassen on Hamilton cycles in highly connected tournaments”.

A conjecture of Thomassen from 1982 states that for every  $k$  there is an  $f(k)$  such that every strongly  $f(k)$ -connected tournament contains  $k$  edge-disjoint Hamilton cycles. A classical theorem of Camion, that every strongly connected tournament contains a Hamilton cycle, implies that  $f(1) = 1$ . Until now, even the existence of  $f(2)$  was open. In this talk, I will discuss a proof of Thomassen’s conjecture. Our methods in fact allow us to show that  $f(k) = O(k^2 \log^2 k)$ , which is best possible up to a logarithmic factor.

This is joint work with Daniela Kühn, John Lapinskas, and Deryk Osthus.

Guus Regts (Centrum Wiskunde & Informatica),

“Compact orbit spaces, graph limits and limits of edge-coloring models”.

Let  $G$  be a group of orthogonal transformations of a Hilbert space  $H$ . Let  $R$  and  $W$  be bounded  $G$ -stable subsets of  $H$ . Let  $\|\cdot\|_R$  be the seminorm on  $H$  defined by  $\|x\|_R := \sup_{r \in R} |\langle x, r \rangle|$  for  $x \in H$ . If  $W$  is weakly compact and the orbit space  $R^k/G$  is compact for each  $k \in \mathbb{N}$ , then the orbit space  $W/G$  is compact when  $W$  is equipped with the norm topology induced by  $\|\cdot\|_R$ . In this talk I will discuss a few consequences of this result: the compactness of the graphon space and the existence of limits of edge-coloring models.

This is based on joint work with Lex Schrijver.

## Participants

Invited speakers:

Maria Axenovich (Karlsruhe), Aart Blokhuis (TU Eindhoven), Hajo Broersma (Twente), Louis Esperet (CNRS), Tom Kaiser (West Bohemia), Dan Král’ (Warwick), Alex Scott (Oxford), Benny Sudakov (UCLA and ETH), and Carsten Thomassen (DTU).

Registered participants:

Aida Abiad (Tilburg), Marien Abreu (Potenza), Nieke Aerts (TU Berlin), Nikhil Bansal (TU Eindhoven), Evan DeCorte (TU Delft), Josse van Dobben de Bruyn (Leiden), Kunal Dutta (MPI Saarbrücken), Roberto Fernández (Utrecht), Murat Firat (VU Amsterdam), Dion Gijswijt (TU Delft), Marinus Gottschau (TU München), Grzegorz Gutowski (Jagiellonian), Willem Haemers (Tilburg), Ararat Harutyunyan (Oxford), Tony Huynh (Sapienza), Leo van Iersel (CWI), Lotte de Jonker (Amsterdam), Judith Keijsper (TU Eindhoven), Steven Kelk (Maastricht), Domenico Labbate (Potenza), Jan van Leeuwen (Utrecht), Nela Lekic (Maastricht), Anshui Li (Utrecht), Killian Matzke (TU München), Jesper Nederlof (Utrecht), Neil Olver (VU Amsterdam), Viresh Patel (Birmingham), Murray Patterson (CWI), Christos Pelekis (TU Delft), Helena Peña (Greifswald), Rudi Pendavingh (TU Eindhoven), Teresa Pivovsan (CWI), Jorn van der Pol (TU Eindhoven), Guus Regts (CWI), Mayank Singhal (CAU Kiel), Cristian Spitoni (Utrecht), Matěj Stehlík (Grenoble), Leen Stougie (VU Amsterdam), Siamak Taati (Utrecht), Christoph Temmel (VU Amsterdam), Antonios Varvitsiotis (CWI), Kees de Vreugd.

Organisers: Ross Kang (Utrecht) and Tobias Müller (Utrecht).